# On the Data Processing Methods of Surface Antenna's Inspection

**LI Zong-chun, Prof. LI Guang-yun and JIN Chao, China P. R.**

**Key words:** Inspection; Coordinate Transformation; Surface Free Fitting; Common Point; CAD Surface Model.

## ABSTRACT

The data processing methods of surface antenna's inspection are deeply discussed in this paper. There are two steps from 3D measuring coordinates to useful information (for example, surface standard deviation): coordinate transformation and surface standard deviation calculation. The first step is more important. Three methods (surface free fitting, common point coordinate transformation and CAD surface fitting) are discussed in this paper. The mathematic models and formulas of these methods are deduced and some results are given finally. According to the theoretical analysis and actual calculation we can see that the CAD surface model fitting is the best method because it needs neither the surface equation nor common point but the CAD surface and its result is reliable. If there is no CAD surface model but a number of common points, then the common point coordinate transformation is a better method, but its result is influenced by the accuracy of common points. If there exists neither CAD surface model nor common point, the surface free fitting is also a good method, which has a high surface precision but the calculated surface may be shifted.

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## 1 INTRODUCTION

The electronic theodolites measuring system and laser tracker measuring system are widely used in modern surface antenna's manufacture, which brings a revolution for it. In this revolution there are two very important problems, the one is how to obtain the antenna's 3D coordinates, the other is how to process the measuring data in scientific way in order to get useful information. The latter is mainly discussed in this paper with the example of a rotating parabolic surface antenna's inspection.

## 2 THE CONCEPTS OF TWO COORDINATE SYSTEMS

The measuring tasks are put up on definite space-time, so the coordinate system is a very important concept. There are two kinds of coordinate systems in surface antenna's inspection: the measuring coordinate system and the design coordinate system.

The measuring coordinate system is defined by the hardware of industrial measuring system. For example, in theodolites measuring system, we usually choose the center point of the first theodolite as the origin, the projected line between the two theodolites center points in the horizontal plane as the X-axis, the vertical line through the first theodolite as Z-axis, and the Y-axis is determined by right hand rectangular coordinate system. It is shown in Fig.1.

The design coordinate system of the rotating paraboloid is shown in Fig.2. The rotating axis is Z-axis, the vertex of paraboloid is the origin, the X-axis is vertical to the Z-axis and its direction is free, the Y-axis is determined by right hand rectangular coordinate system. In this coordinate system, Z-axis is the rotating axis that has physical meanings and the formula of paraboloid is very simple. Given the formula of a parabola is $z = f(x)$, and then the formula of rotating paraboliod is $z = f(\sqrt{x^2 + y^2})$.

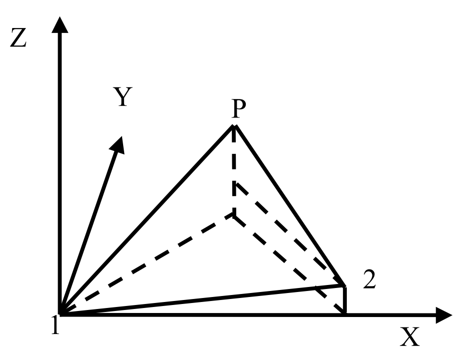


Fig.1 Measuring Coordinate System

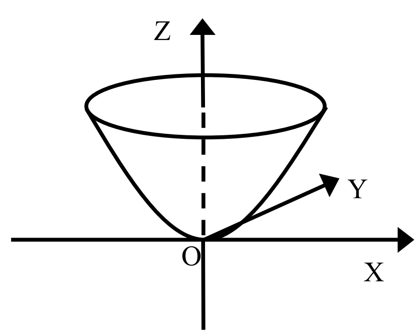


Fig.2 Design Coordinate System

## 3 DATA PROCESSING METHODS OF SURFACE ANTENNA

From the above discussion we can see that the antenna's formula is very simple in design coordinate system, whose parameters have significant meanings. But it is more complex and less meanings in measuring coordinate system. So the data processing must be divided into two steps: the first is to transform the measuring coordinate system to design coordinate system, the second is to calculate the surface standard deviation and/or adjustment values in its design coordinate system.

There are three kinds of transformation methods: the first is surface free fitting, the second is common point coordinate transformation and the third is CAD surface model fitting.

### 3.1 Surface Free Fitting

There are three translation parameters and three rotation parameters between measuring coordinate system and design coordinate system, which is $(X_0, Y_0, Z_0, \varphi, \theta, \psi)$. Due to the special character of the rotating paraboloid (the X-axis of design coordinate system can be fixed at any direction), we can fix $\psi$ as constant ($\psi = 0$), so only three translation parameters and two rotation parameters exist, i.e. $(X_0, Y_0, Z_0, \varphi, \theta)$. Suppose the coordinate of antenna is $(x, y, z)$ in design coordinate system, and $(X, Y, Z)$ in measuring coordinate system, and then the formula of it in design coordinate system can be written as:

$$F = z - f(\sqrt{x^2 + y^2}) = 0 \tag{1}$$

Because $(x, y, z)$ is the function of $(X_0, Y_0, Z_0, \varphi, \theta)$, so $F$ is also the function of $(X_0, Y_0, Z_0, \varphi, \theta)$.

The transformation (Li , 1994) between design coordinate system and measuring coordinate system is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi\cos\theta & \cos\varphi\cos\theta & \sin\theta \\ \sin\varphi\sin\theta & -\cos\varphi\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} \tag{2}$$

Having n measuring points $(X_i, Y_i, Z_i)$ $i = 1, 2, \cdots n$, we can use iterative LS method (Li, 2000) to solve the coordinate parameters $(X_0, Y_0, Z_0, \varphi, \theta)$ and to calculate the coordinates of measuring points in design coordinate system.

This method can achieve the highest surface precision of the three. In fact, the actual antenna is only a small part of rotating paraboloid, as it's difficult to fix the boundary conditions, the result may be shifted. That is why we call it the surface free fitting method.

### 3.2 Common Point Coordinate Transformation

The precondition of using this method is that there exist common points. So there are three translation parameters and three rotation parameters, i.e. $(X_0, Y_0, Z_0, \varepsilon_x, \varepsilon_y, \varepsilon_z)$. The coordinate of a common point is $(X, Y, Z)$ in measuring coordinate system, and $(x, y, z)$ in design coordinate system, the relationship (Li, 1994) between two coordinate systems is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} \tag{3}$$

Where:

$$\left.\begin{aligned} a_1 &= \cos\varepsilon_y \cos\varepsilon_z \quad , \quad b_1 = \cos\varepsilon_y \sin\varepsilon_z \quad , \quad c_1 = -\sin\varepsilon_y \\ a_2 &= -\cos\varepsilon_x \sin\varepsilon_z + \sin\varepsilon_x \sin\varepsilon_y \cos\varepsilon_z \ , b_2 = \cos\varepsilon_x \cos\varepsilon_z + \sin\varepsilon_x \sin\varepsilon_y \sin\varepsilon_z \ , c_2 = \sin\varepsilon_x \cos\varepsilon_y \\ a_3 &= \sin\varepsilon_x \sin\varepsilon_z + \cos\varepsilon_x \sin\varepsilon_y \cos\varepsilon_z \ , b_3 = -\sin\varepsilon_x \cos\varepsilon_z + \cos\varepsilon_x \sin\varepsilon_y \sin\varepsilon_z \ , c_3 = \cos\varepsilon_x \cos\varepsilon_y \end{aligned}\right\}$$

Differentiating formula (3), it is easy to obtain:

$$\left.\begin{aligned} dx &= -a_1 dX_0 - b_1 dY_0 - c_1 dZ_0 + d_1 d\varepsilon_x + e_1 d\varepsilon_y + f_1 d\varepsilon_z \\ dy &= -a_2 dX_0 - b_2 dY_0 - c_2 dZ_0 + d_2 d\varepsilon_x + e_2 d\varepsilon_y + f_2 d\varepsilon_z \\ dz &= -a_3 dX_0 - b_3 dY_0 - c_3 dZ_0 + d_3 d\varepsilon_x + e_3 d\varepsilon_y + f_3 d\varepsilon_z \end{aligned}\right\} \tag{4}$$

Where:

$$\left.\begin{aligned} d_1 &= 0 \\ d_2 &= a_3(X - X_0) + b_3(Y - Y_0) + c_3(Z - Z_0) \\ d_3 &= -a_2(X - X_0) - b_2(Y - Y_0) - c_2(Z - Z_0) \\ e_1 &= -\sin\varepsilon_y \cos\varepsilon_z (X - X_0) - \sin\varepsilon_y \sin\varepsilon_z (Y - Y_0) - \cos\varepsilon_y (Z - Z_0) \\ e_2 &= a_1 \sin\varepsilon_x (X - X_0) + b_1 \sin\varepsilon_x (Y - Y_0) + c_1 \sin\varepsilon_x (Z - Z_0) \\ e_3 &= a_1 \cos\varepsilon_x (X - X_0) + b_1 \cos\varepsilon_x (Y - Y_0) + c_1 \cos\varepsilon_x (Z - Z_0) \\ f_1 &= -b_1(X - X_0) + a_1(Y - Y_0) \\ f_2 &= -b_2(X - X_0) + a_2(Y - Y_0) \\ f_3 &= -b_3(X - X_0) + a_3(Y - Y_0) \end{aligned}\right\} \tag{5}$$

From formula (3) we can get the fitting equation:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{pmatrix} \tag{6}$$

Let

$$t = (X_0, Y_0, Z_0, \varepsilon_x, \varepsilon_y, \varepsilon_z) \tag{7}$$

The fitting equation of the $i$ common point is:

$$v_i = A_i dt + F_i \tag{8}$$

Where:

$$A_i = \begin{pmatrix} -a_1 & -b_1 & -c_1 & d_1 & e_1 & f_1 \\ -a_2 & -b_2 & -c_2 & d_2 & e_2 & f_2 \\ -a_3 & -b_3 & -c_3 & d_3 & e_3 & f_3 \end{pmatrix} \tag{9}$$

$$F_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} - \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} X_i - X_0 \\ Y_i - Y_0 \\ Z_i - Z_0 \end{pmatrix} \tag{10}$$

Formula (8) can be written in matrix form:

$$V = Adt + F \tag{11}$$

Given the approximation $(X_0^0, Y_0^0, Z_0^0, \varepsilon_x^0, \varepsilon_y^0, \varepsilon_z^0)$ of the $t$, we can use iterative LS method to solve the parameter $t$.

Theoretically this method can be used if there exist more than 3 common points. The space distributing of the common points should be well concerned. If there exists scale error between the design value and measuring value (caused by tempreture), we can use this method to slove 7 parameters including scale factor. As the coordinate accuracy of common point can not be controlled very well in manufacturing process, so the final result may be influenced in this situation.

### 3.3 CAD Surface Model Fitting

CAD surface model fitting is to use measuring point and design CAD surface model to do transformtion and comparison. The disadvantage of surface free fitting is that it cannot control the boundary of surface. To the common point transformation, its result is influenced by the accuracy of common points. So these two methods are not very good. CAD surface model fitting can elimnate these two disadvantages. That is, on the one hand it can fix the boundary of the surface, on the other hand its common points have no accuracy lost. So it can

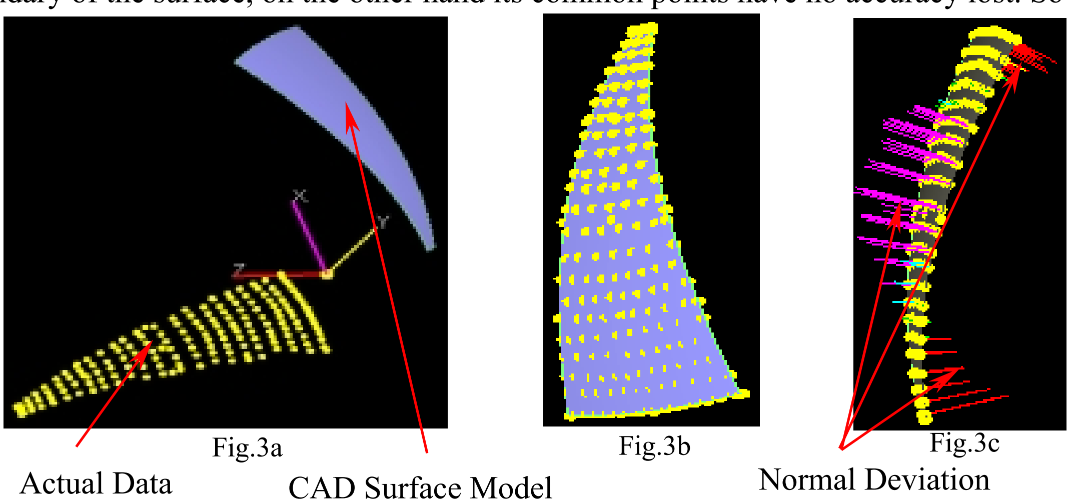


Fig.3 The Process of CAD Surface Fitting

be called surface-fitting-with-boundary-condition or no-accuracy-lost-common-point- -coordinate-transformation. If there is no boundary conditions in CAD surface model then it equals to surface free fitting. Its calculation process is shown in Fig.3. First, the CAD surface model and measuring data are loaded in one window (Fig.3a), then the two kinds of data are adjusted in order to find the approximate coordinate transformation parameters. Once the approximate coordinate transformation parameters are found, the software can solve automatically and stop when the square sum of the distance between measuring points and its projection is minimum (Fig.3b). Fig.3c is a needle error map which shows that the two ends of the actual surface are concave and the central part is convex according to design surface. Practically we should find "good" coordinate transformation parameters which can reduce the iterative times. The result of this method is reliable, the whole process is visualized, so this method is the best method of the three.

### 3.4 Surface Standard Deviation Calculation

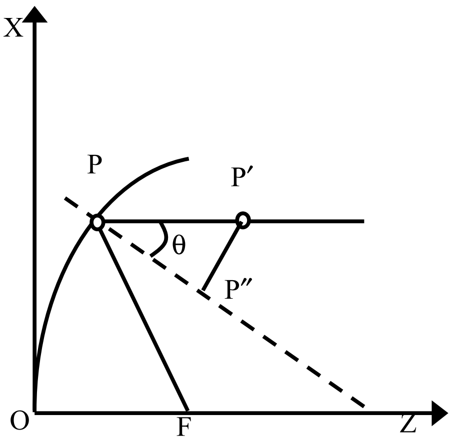


Fig.4 Surface Standard Deviation Calculation

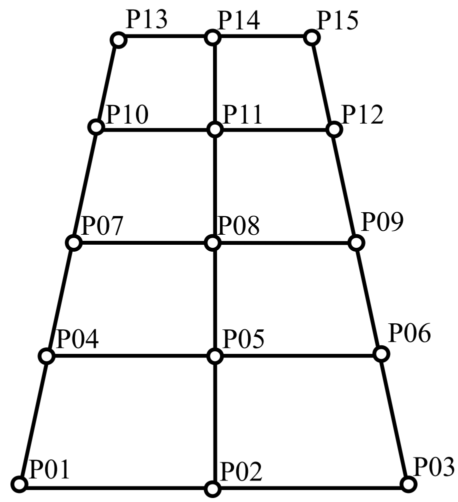


Fig.5 The Distribution of Common Points

As shown in Fig.4, for standard parabola $x^2 = 4fz$, OZ is the rotating axis, $F$ is focus point, $P'$ is actual measured point, P is the theoretically point of P′, PP″ is the normal of P. According to the character of the parabola (Li, 2000), the angle between normal PP″ and z-axis is θ, which can be derived from:

$$tg\theta = \frac{x}{2f} \tag{12}$$

Then the axial deviation of P′ can be solved by:

$$\Delta_z = z - \frac{x^2}{4f} \tag{13}$$

Finally the normal deviation PP″ can be gotten:

$$\Delta = \Delta_z \cos\theta \tag{14}$$

## 4 EXAMPLES

In an antenna model, 170 points are measured and 15 points (P01~P15, which is shown in Fig.5) are chosen as common points (which have design coordinates). Theoretically, if there exist no manufacture error, surface deformation, and etc., these methods should have the same results. In fact these errors cannot be eliminated, so the results are different. The results are shown in Tab.1, Tab.2 and Tab.3.

Tab.1 Result of Surface Free Fitting

| Common point ID | Coordinate value after transformation | | Design coordinate value | | Normal deviation (mm) |
|---|---|---|---|---|---|
| | x (mm) | z (mm) | x (mm) | z (mm) | |
| P01 | 3092.486 | -84.905 | 3100.000 | -78.7 | 0.072 |
| P02 | 3090.884 | -86.185 | 3100.000 | -78.7 | 0.117 |
| P03 | 3090.992 | -85.457 | 3100.000 | -78.7 | 0.606 |
| P04 | 2480.723 | -542.984 | 2489.200 | -537.076 | -0.090 |
| P05 | 2479.982 | -543.389 | 2489.200 | -537.076 | -0.020 |
| P06 | 2479.490 | -543.548 | 2489.200 | -537.076 | 0.118 |
| P07 | 1828.608 | -911.185 | 1839.200 | -906.100 | 0.155 |
| P08 | 1827.669 | -911.718 | 1839.200 | -906.100 | 0.070 |
| P09 | 1827.073 | -911.870 | 1839.200 | -906.100 | 0.185 |
| P10 | 1077.066 | -1177.164 | 1088.800 | -1174.700 | 0.344 |
| P11 | 1076.149 | -1177.693 | 1088.800 | -1174.700 | 0.042 |
| P12 | 1075.541 | -1178.101 | 1088.800 | -1174.700 | -0.213 |
| P13 | 297.902 | -1268.608 | 310.000 | -1268.750 | 0.119 |
| P14 | 296.238 | -1268.725 | 310.000 | -1268.750 | -0.005 |
| P15 | 295.190 | -1268.445 | 310.000 | -1268.750 | 0.270 |
| Surface standard deviation (average normal deviation of 170 points) □0.106 mm | | | | | |

Tab.2 Result of Common Point Transformation

| Common point ID | Coordinate value after transformation | | Design coordinate value | | Normal deviation (mm) |
|---|---|---|---|---|---|
| | x (mm) | z (mm) | x (mm) | z (mm) | |
| P01 | 3101.299 | -77.322 | 3100.000 | -78.7 | 0.228 |
| P02 | 3100.088 | -78.062 | 3100.000 | -78.7 | 0.438 |
| P03 | 3100.226 | -77.130 | 3100.000 | -78.7 | 1.060 |
| P04 | 2490.758 | -537.029 | 2489.200 | -537.076 | -0.664 |
| P05 | 2490.486 | -536.999 | 2489.200 | -537.076 | -0.489 |
| P06 | 2490.057 | -536.995 | 2489.200 | -537.076 | -0.243 |
| P07 | 1839.622 | -906.967 | 1839.200 | -906.100 | --0.713 |
| P08 | 1839.220 | -907.179 | 1839.200 | -906.100 | 0.733 |
| P09 | 1838.713 | -907.210 | 1839.200 | -906.100 | -0.545 |
| P10 | 1088.789 | -1174.947 | 1088.800 | -1174.700 | -0.240 |
| P11 | 1088.458 | -1175.286 | 1088.800 | -1174.700 | -0.492 |
| P12 | 1087.957 | -1175.623 | 1088.800 | -1174.700 | -0.702 |
| P13 | 309.846 | -1268.466 | 310.000 | -1268.750 | 0.284 |
| P14 | 308.809 | -1268.531 | 310.000 | -1268.750 | 0.218 |
| P15 | 307.869 | -1268.234 | 310.000 | -1268.750 | 0.514 |
| Transformation error (15 points included): 1.384 mm<br>Surface standard deviation (average normal deviation of 170 points)□0.512 mm | | | | | |

Tab.3 Result of CAD Surface Fitting

| Common point ID | Coordinate value after transformation | | Design coordinate value | | Normal deviation (mm) |
|---|---|---|---|---|---|
| | x (mm) | z (mm) | x (mm) | z (mm) | |
| P01 | 3101.145 | -77.090 | 3100.000 | -78.7 | 0.503 |
| P02 | 3099.979 | -77.985 | 3100.000 | -78.7 | 0.567 |
| P03 | 3100.166 | -77.189 | 3100.000 | -78.7 | 1.057 |
| P04 | 2490.552 | -536.728 | 2489.200 | -537.076 | -0.300 |
| P05 | 2490.303 | -536.822 | 2489.200 | -537.076 | -0.240 |
| P06 | 2489.903 | -536.928 | 2489.200 | -537.076 | -0.108 |
| P07 | 1839.375 | -906.593 | 1839.200 | -906.100 | -0.269 |
| P08 | 1838.976 | -906.896 | 1839.200 | -906.100 | -0.373 |
| P09 | 1838.483 | -907.008 | 1839.200 | -906.100 | -0.264 |
| P10 | 1088.511 | -1174.488 | 1088.800 | -1174.700 | 0.271 |
| P11 | 1088.171 | -1174.882 | 1088.800 | -1174.700 | -0.032 |
| P12 | 1087.670 | -1175.267 | 1088.800 | -1174.700 | -0.288 |
| P13 | 309.558 | -1267.920 | 310.000 | -1268.750 | 0.829 |
| P14 | 308.506 | -1268.000 | 310.000 | -1268.750 | 0.749 |
| P15 | 307.564 | -1267.717 | 310.000 | -1268.750 | 1.031 |
| Transformation error (170 points): 0.419 mm<br>Surface standard deviation (average normal deviation of 170 points)□0.366 mm | | | | | |

From the above results we can see that the standard deviation of surface free fitting is the highest, the result of common point coordinate transformation and CAD surface fitting is lower than that of surface free fitting. If we compare the difference between the measuring coordinate and design coordinate, the results of CAD surface fitting and common point coordinate transformation are much better than that of surface free fitting. In surface antenna's data processing, the surface position is more important, surface free fitting gets a mathematical surface which departure the design surface and is useless. Although both common point coordinate transformation and CAD surface fitting can control the boundary, the result of the common point coordinate transformation is influenced by the accuracy of common points, so the CAD surface fitting is the best method of the three.

## 5 CONCLUSIONS

1) CAD surface fitting needs neither surface equation nor known common points but the CAD surface model is a perfect method, which has a reliable result.
2) If there exists no CAD surface model but common points, common point coordinate transformation is a better method, but whose results is influenced by the accuracy of common points.
3) If there exists neither CAD surface model nor common point, there is no choice but surface free fitting, which has a high surface precision but the calculated surface may be shifted.

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