# L1, L2, Kalman Filter and Time Series Analysis in Deformation Analysis

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**Key words**: L1, L2, Kalman Filter, Time Series, Deformation Analysis.

## ABSTRACT

This paper discusses the use of minimum norm (L1), least squares (L2), Kalman filtering, and time-series analysis (Box-Jenkins models) in extracting the true signals in deformation measurement and analysis, especially in the presence of outlying observations and environmental disturbances due to wind, rain and temperature changes.

Minimum norm, or the L1 method, and the least squares, or L2 method, with robust estimation techniques are the traditional methods used in pre-deformation analysis. The inherent weakness in these two traditional methods in dealing with continuous deformation measurement using motorized total station (also known as surveying robot) is the non-consideration of the correlation in the data, which is basically a time series.

The autoregressive and autocorrelation nature of the continuous monitoring measurements could be explored using the Box-Jenkins time series models. Filtering and prediction done using the sophisticated Autoregressive Integrated Moving Average (ARIMA) techniques or its subset is however not suited for automated real-time deformation analysis.

This study focused on the use of Kalman filter in filtering out outlying data and noisy data. The known systematic effect of pillar rotation, scale change and change in refraction as well as the horizontal directions, slope distances and zenith angles were modeled as the state vector in a forward Kalman filtering and backward smoothing. It is proven in this study that it is feasible to implement an automated real-time Kalman filter in deformation analysis. Test carried out on a 'stable' building at Nanyang Technological University, Singapore, and processed using Kalman filter confirm the 'movement' of the building of less than 1 mm.

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# L1, L2, Kalman Filter and Time Series Analysis in Deformation Analysis

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## 1. INTRODUCTION

This paper discusses the use of the minumum norm (L1), least squares (L2), Kalman filtering and time series analysis in extracting the true signals in deformation measurement and analysis.

The L1, L2 and time series analysis will be briefly covered before the deliberation on Kalman filtering which was found to be superior in eliminating outliers whether they are due to erroneous observations or disturbances to the instrument arising from adverse environmental conditions - the wind and the rain and the heat.

Data were gathered from a test site setup in Nanyang Technological University, Singapore, using both Motorised Electronic Total Station, also known as Surveying Robot, and Global Positioning System (GPS). Data from GPS were processed using a commercial software and served as an independent results for comparison purpose.

## 2. THE L1 METHOD

Caspary and Borutta (1987) adopted Iterative Reweighted Least Squares (IRLS) in implementing the robust method that minimises the sum of the absolute values of the residuals. The $L_1$ method that implements the robust method is not the same as IRLS (Harvey, 1993). The procedure of the $L_1$ method is to set up one equation per observation as in parametric least squares. If there are u unknown parameters, a simple approach is to try every possible combination of u equations and select the one with the smallest sum of the absolute residuals. This approach is very inefficient. A better method is to use Dantzig's Simplex algorithm, the standard linear programming algorithm (Branham, 1990). There are other developments based on the simplex algorithm, such as those of Barrodale and Roberts (1974) and Branham (1990).

Gass (1985) states the general linear programming problem which is to find a vector **x** ($x_1$, $x_2$,…, $x_n$) which minimizes the linear form **Cx** (the objective function) subject to the linear constraints **Ax**=**b** and $x_j \geq 0$ (j=1,2,…, n). **A** is a m x n constant matrix, **b** and **x** are a m x 1 constant vectors with m < n.

One feature of the $L_1$ method is that u of the n equations will be satisfied exactly giving zero residuals for the corresponding observations. Many of the (n-u) observation equations are virtually not used in the solution of the u parameters. This is one of the reasons Gauss preferred least squares to $L_1$ method (Brahman, 1990).

Harvey (1993) suggests a simple modification of the $L_1$ method to account of the different units and varying precision in survey observations. The normal $L_1$ method solves

$$\mathbf{Ax} = \mathbf{b} \quad (1)$$

so that the sum of the absolute values of the residuals (v) is a minimum. **A** is the coefficient matrix, **x** is a vector of parameters to be solved and **b** is a vector representing the difference between observations and the values calculated from estimates of the parameters.

The modified methods solves

$$\mathbf{P}^{1/2}\mathbf{Ax} = \mathbf{P}^{1/2}\mathbf{b} \quad (2)$$

so that $\sum\left|\frac{v}{s}\right|$ is a minimum, where $\mathbf{P}^{1/2}$ is the weight matrix with inverse of the standard deviation as its diagonal elements.

Marshall and Bethel (1996) illustrated the application of the $L_1$ method to adjustment of horizontal networks in the form of $\mathbf{v} + \mathbf{Ax} = \mathbf{b}$. Weight matrix $\mathbf{P}^{1/2}$ is applied to each to each equation.

Further generalisation was made in this study to solve for,

$$\mathbf{P}^{1/2}\mathbf{Ax} + \mathbf{P}^{1/2}\mathbf{Cv} = \mathbf{P}^{1/2}\mathbf{b} \quad (3)$$

where **C** is design matrix with respective to the observations and $\mathbf{P}^{1/2}\mathbf{Cv}$ is minimized in the linear programming. In this study, **C** is a identity matrix.

As the Simplex algorithm deals only with positive numbers, each residual v and parameter x are respresented by two positive numbers (Marshall and Bethel, 1996), i.e.

$$v_i = v_{i1} - v_{i2} \quad \text{and } x_i = x_{i1} - x_{i2} \quad (4)$$

The gross errors present in the observations will have attracted larger residuals in the $L_1$ method. The standardised residual will thus have a magnitude greater than $3\sigma$. The arbitrary magnitude of $5\sigma$ as proposed by Sprent (1998) was used in this study.

LINDO, a callable library of the Lindo System Inc. for optimisation, was used in a Visual Basic program to calculate the parameters with the minimum norm of the residuals.

## 3. THE L2 METHOD

The same coefficient matrices **A, C** and **b** used in the $L_1$ method could be used in the $L_2$ (least squares) method. In constrast to $L_1$ method, the gross errors present in one observation will smear other good observations. Robust estimation in deformation analysis is thus essential to provide accurate displacement vectors since least squares adjustments are easily influenced by a single gross error. Conventional robust estimations, for instance using the M-estimator, the Least Absolute Sum method or the Danish method, attempt to detect the gross errors one at a time and reassign weights to the suspected outliers in an iterative least squares adjustment (Caspary and Borutta, 1987) .

Caspary and Robutta (1987) illustrated that the Danish Method is most robust with respect to outlying observations, followed by the LAS estimation. Huber's M-estimation is a compromise between the extremely robust Danish Method and the non-robust LS method.

The following weight functions are used in the Danish Method:

$$w_{v+1} = w_v f(\delta_v), \qquad v = 1,2,\ldots \tag{5}$$

$$f(\delta) = \begin{cases} 1 & \text{for } \dfrac{|\delta|\sqrt{w_1}}{\sigma} < c \\ \exp-\left(\dfrac{|\delta|\sqrt{w_1}}{c\sigma}\right) & \text{else} \end{cases} \tag{6}$$

With these weights the LS computation is repeated, leading to new residuals and by virtue of Eq. (6) to new weights. The process of re-weighting and adjustment is repeated until convergence is achieved. The weights of outlying observation become small, thus being of minor influence on the parameter estimation. In this sense, the method can be classified as a robust one in respect of gross residuals. The value of c adopted in this study is based on the tau-statistics, which in this case has a value of close to 3.

Harvey (1993) as well as Marshall and Bethel (1996) suggested using $L_1$ method to detect outliers and then using least squares (the $L_2$ method) on the 'cleaned' data.

## 4. TIME SERIES ANALYSIS

A series of successive and regularly taken measurements is called a time series. This is applicable to measurements taken using survey robots in monitoring survey. Each epoch of measurement data, however, is often processed in commercial software as independent set of data. The special feature of time-series data is the fact that successive observations are usually not independent and that the analysis must take into account the time order of the observations. When successive observations are dependent, future values may be predicted from the past observation. If a time series can be predicted exactly, it is said to be deterministic. But most time series are stochastic in that the future values are only partly determined by past values, so that exact predictions are impossible and must be replaced by the idea that future values have a probability conditioned by knowledge of past values.

Program packages are now available that can handle time series using various models. AutoRegressive Integrated Moving Average (ARIMA) and the sub-models ARMA, AR and MA, due to the book by Box and Jenkins (1970), are the popular models to analyse the characteristics of a time series.

Detection of outlying observations in a time series is not easy (Alba and Zartman, 1980). Spuriosity may be due to innovation or additive effects. Innovation spuriosity is the error or random shock that affects the model at each time period; it stays in the system and affects subsequent observations. Additive effect spuriosity does not affect observations after the outlier. Fox (1972) first studied the detection of outliers in time series. He considered the problem of detecting the spurious observations in time series in the context of a p-th order autoregressive model. He derived a likelihood ratio test assuming that a single specific observation is spurious. Other references on this topic include Abraham and Box (1979), Martin (1980), Chang and Tiao (1983), Chang, Tiao and Chen (1988).

Various researchers studied the analysis of time series in great depth. For instasnce, instance time series regression analysis postulates a structural equation model adn tests it using time series data (Ostrom, 1990); application of bootstrapping to time series data (Efron and Tibshirani, 1986); Grey Model in time series (Chen and Tang, 1993). The analysis is quite complex even for an univariate ARIMA model. The application of the time series analysis in a multivariate ARIMA model would have been too tedious, even without additive or innovation outliers. Furthermore, the human intervention required for the identification of a suitable model makes the analysis unsuitable for real-time applications in deformation analysis. The use of Grey Model in time series analysis, though simple to apply, is not a robust method. The data series derived by summation will propagate the outlier that is present in the original time series data. The next section will look into the Kalman Filter, a different algorithm used in filtering, smoothing and predicting time series data.

## 5. KALMAN FILTER

"Filtering, smoothing and prediction techniques are used for problems in which the parameters being estimated by least squares process vary with time" (Cross, 1994). If $t_i$ is the present time and $t_j$ is the time at which we want to estimate the position of a dynamic or moving platform, we are filtering if $t_i = t_j$; smoothing if $t_i > t_j$ and predicting if $t_i < t_j$. The methods are also valid for situations in which the parameters have temporal variations even though there may be no movement. The Kalman Filter, first derived by Kalman (1960) for use in electrical control systems, is a widely used method for filtering, smoothing and prediction. In principle, it is feasible to apply the Kalman filtering technique to deformation monitoring data that are subject to temporal variation due to changes in temperature, noises due to wind and rain and other shocks.

Krakiswsky (1975), Cross (1994), Strang and Borre (1997) and Han (1998) had shown that the Kalman Filter can be derived from the standard least squares adjustment. In fact recursive least squares estimation could be re-formulated as a Kalman filtering technique for non-dynamic or static positioning.

Strang and Borre (1997) highlighted four key points of the Kalman filter, namely that the process is recursive, that there are many forms of the update formula, that the error covariance matrix $P = \sum_{\hat{x}}$ must also be updated and that the Kalman Filter suits dynamic problems.

### 5.1 Dynamic Updates

The Kalman Filter suits dynamic problems where, in the Kalman notation, a state vector $\mathbf{x}_k$ (e.g. the rotation of survey pillar) at epoch k is not the same as the state vector $\mathbf{x}_{k-1}$ at epoch k-1. The state is changing (the pillar is rotating). If we assume a linear equation for a dynamic change of state with its own error $\boldsymbol{\varepsilon}_k$ and with new measurements $\mathbf{b}_k$ and the errors $\mathbf{e}_k$ in these measurements, we get

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} + \boldsymbol{\varepsilon}_k \qquad (7)$$

and

$$\mathbf{b}_k = \mathbf{A}_k\mathbf{x}_k + \mathbf{e}_k \qquad (8)$$

The state of the system is modelled by the state vector $\mathbf{x}_k$ This vector changes with time. The known matrix $\mathbf{F}_{k-1}$ in Eq. (8) is called the state transition matrix. Equation (7) is the state equation for the dynamic process. The $\boldsymbol{\varepsilon}_k$ in the state equation is the so-called 'system noise' which represents the unknown error in the state model. This unknown error must be estimated to carry out the Kalman Filtering process.

The update could be expressed in two stages, namely by a prediction using Eq. (9) and by a filtering using Eq. (10):

*Prediction:* $$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1|k-1} \qquad (9)$$

*Filtering:* $$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{b}_k - \mathbf{A}_k\hat{\mathbf{x}}_{k|k-1}) \qquad (10)$$

The update of the error covariance matrix **P,** that is required to compute the gain matrix, also comes in two stages:

*Prediction:* $$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k-1}^{t} + \Sigma_{\varepsilon,k} \qquad (11)$$

*Filtering:* $$\mathbf{P}_{k|k} = (\mathbf{I}-\mathbf{K}_k\mathbf{A}_k)\mathbf{P}_{k|k-1} \qquad (12)$$

The gain matrix $\mathbf{K}_k$ required in Eqs. (10) and (12) is shown in Eq. (13) :

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{A}_k^t\left(\mathbf{A}_k\mathbf{P}_{k|k-1}\mathbf{A}_k^t + \Sigma_{e,k}\right)^{-1} \qquad (13)$$

### 5.2 Smoothing

The forward process of filtering produces $\hat{\mathbf{x}}_{k|k}$ is the best estimate of the state vector up to epoch k. Smoothing produces a better estimate $\hat{\mathbf{x}}_{k|N}$ for the state vector at epoch k by using the observations up to the latter time N. The smoothed estimate of $\mathbf{x}_k$ is always better than the filtered estimate. There are three types of smoothers – fixed interval, fixed-point and fixed lag. The last two types may be used in real time. The fixed-interval smoothing, only for postprocessing, seeks $\hat{\mathbf{x}}_{k|N}$ for k = 0 to N, while the fixed-point smoothing seeks $\hat{\mathbf{x}}_{k|j}$ for a fixed k and j = k+1, K+2, etc. The fixed-lag smoothing seeks $\hat{\mathbf{x}}_{k-n}$ for a fixed lag n. The fixed-lag smoother estimates the system state at epoch k-n, given the measurements up to epoch k (usually the current epoch). The memory requirements for fixed-lag smoothers increase with n, because the intermediate Kalman Filter values must be saved. The expression is also rather complex. In consequence, it is not an attractive option for smoothing the estimates.

For fixed-interval smoothing, after filtering forward from Epochs 0 to N, the results $\hat{\mathbf{x}}_{k|k-1}$, $\hat{\mathbf{x}}_{k|k}$, $\Sigma_{k|k-1}$ and $\Sigma_{k|k}$ are kept. Filtering backward from Epoch N to 0 is carried out with the recursive formulae (starting from k=N-1):

$$\hat{\mathbf{x}}_{k|N} = \hat{\mathbf{x}}_{k|k} + \mathbf{A}_k (\hat{\mathbf{x}}_{k+1|N} - \hat{\mathbf{x}}_{k+1|k}) \tag{14}$$

where $\mathbf{A}_k$ is the smoothing gain and

$$\mathbf{A}_k = \Sigma_{k|k} \mathbf{F}_k^t \Sigma_{k+1|k}^{-1} \tag{15}$$

for

k = N-1, N-2, … , 0

The algorithm presented is due to Rauch, Tung and Striebel and is referred to as the RTS algorithm by Brown and Hwang (1997).

Brown and Hwang (1997) also proposed and illustrated the use of the RTS algorithm for fixed-interval smoothing in fixed-lag smoothing by first filtering up to the current measurement and then sweeping back a fixed number of steps with the RTS algorithm. If the number of backward steps is small, it is a simple and effective way of doing fixed-lag smoothing. Thus, it is possible to use fixed-interval smoothing in real-time smoothing in deformation measurement and this approach is adopted in this study with N=3, i.e. filtered backwards by 3 epochs.

## 6. NTU TEST SITE

The layout of the Test Site is depicted in Figure 1. The Surveying Robot Station (Fig. 2) and the GPS Reference Station (Fig. 3) are situated at the rooftop of Block N1 (about 35 m above ground level) of NTU. G2 to G5 in Figure 1 denote the monitoring points comprising the GPS antenna and the prism target mounted on a customised bracket (Fig. 4). G2, G3, G4 and G5, all installed at the rooftop, are about 120 m, 145 m, 110 m and 95 m, respectively, from the Surveying Robot Station and the GPS Reference Station. G2 and G3 are about 5 m lower than the two reference stations whilst G4 and G5 are at the same height with the reference stations. Block N1 and N2 are supposedly stable buildings.

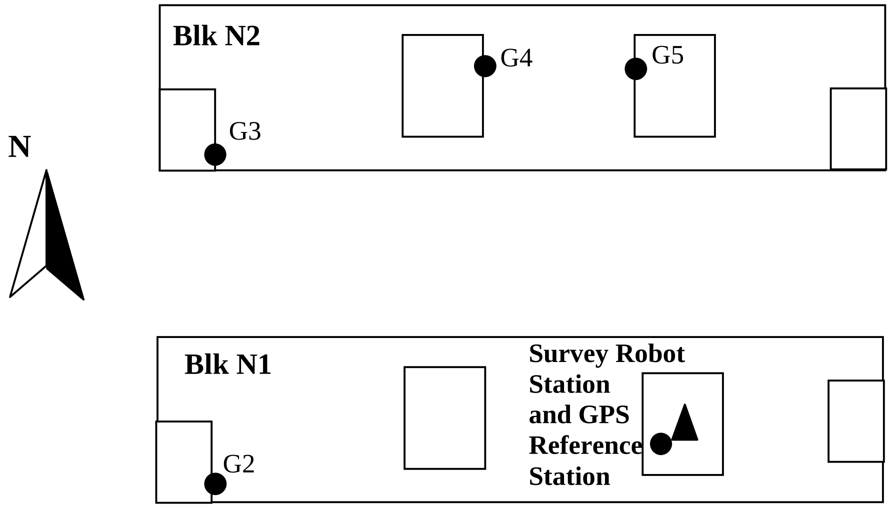


Figure 1. Layout of the Surveying Robot Station, GPS Reference Station and monitoring points G2 to G5

Figure 2. Surveying Robot Station situated at the rooftop of Block N1, NTU

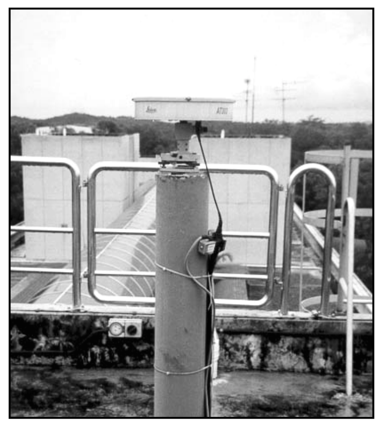

Figure 3. GPS Reference Station at the rooftop of Block N1, NTU


Figure 4. Wall-mounted bracket with GPS antenna and prism target

## 7. OBSERVATION AND RESULTS

### 7.1 Surveying Robot Observations

The surveying robot observations using a Zeiss S10 began on 03.00h (GMT) 28th July 2001 and ended on 22.00h (GMT) 14 August 2001. 428 sets of observations were acquired and taken at hourly interval to the prism targets at the monitoring points G2 to G4. Temperature and pressure readings were also taken by the Zeiss S10 besides the horizontal directions, slope angles and slope distances.

#### 7.1.1 Results of L1, L2 and Kalman Filtering

Changes with time in the orientation, due to pillar rotation, scale correction and refraction, due to changes in temperature, was found evident in an earlier study (Tor, 1999). The orientation (z-rotation or kappa), scale correction and refraction correction were solved as unknown parameters in the L1 and L2 solutions. For the Kalman filtering, the state vectors include the three parameters, their first and second order differentials, and the adjusted horizontal directions, zenith angles and slope distances. The system noises of the z-rotation, scale correction and refraction correction are function of their third differentials (i.e. the uncertainties in the respective accelerations). The adjusted observations are also subjected to unknown system noises.

Figure 5 shows the results of the z-rotation, scale correction and refraction correction, respectively. The red line shows the smoothed results of the Kalman filtering and the blue and green lines show the results of the L1 and L2 solutions, respectively.

The application of the corrections obtained in the L1 and L2 solutions to the observations gave the adjusted X-, Y- and Z-coordinates. For the Kalman filtering, the adjusted observations were obtained as the smoothed state vectors together with the corrections to the observations. Figure 6 shows the X, Y and Z-coordinates of Point G3 computed using the three methods. Again the red lines represent the Kalman results, blue for the L1 and green for the L2 results. The observed coordinates are shown in grey.

#### 7.1.2 Results of Time Series Analysis

Raw observations (shown as blue lines), time-series filtered and predicted observations (shown as red lines) of G3 are depicted in Figure 7. The observations in Figure 7 were found to be a time series of the sophisticated ARIMA (Autoregressive Integrated Moving Average) model (also known as "Box-Jenkins" model) with both 1st order autoregressive and 1st order moving average with periodicity of 24 hours. The filtered results fit well with the observations. The predicted results for 6 days also exhibit the cyclical (periodic) nature of the time series

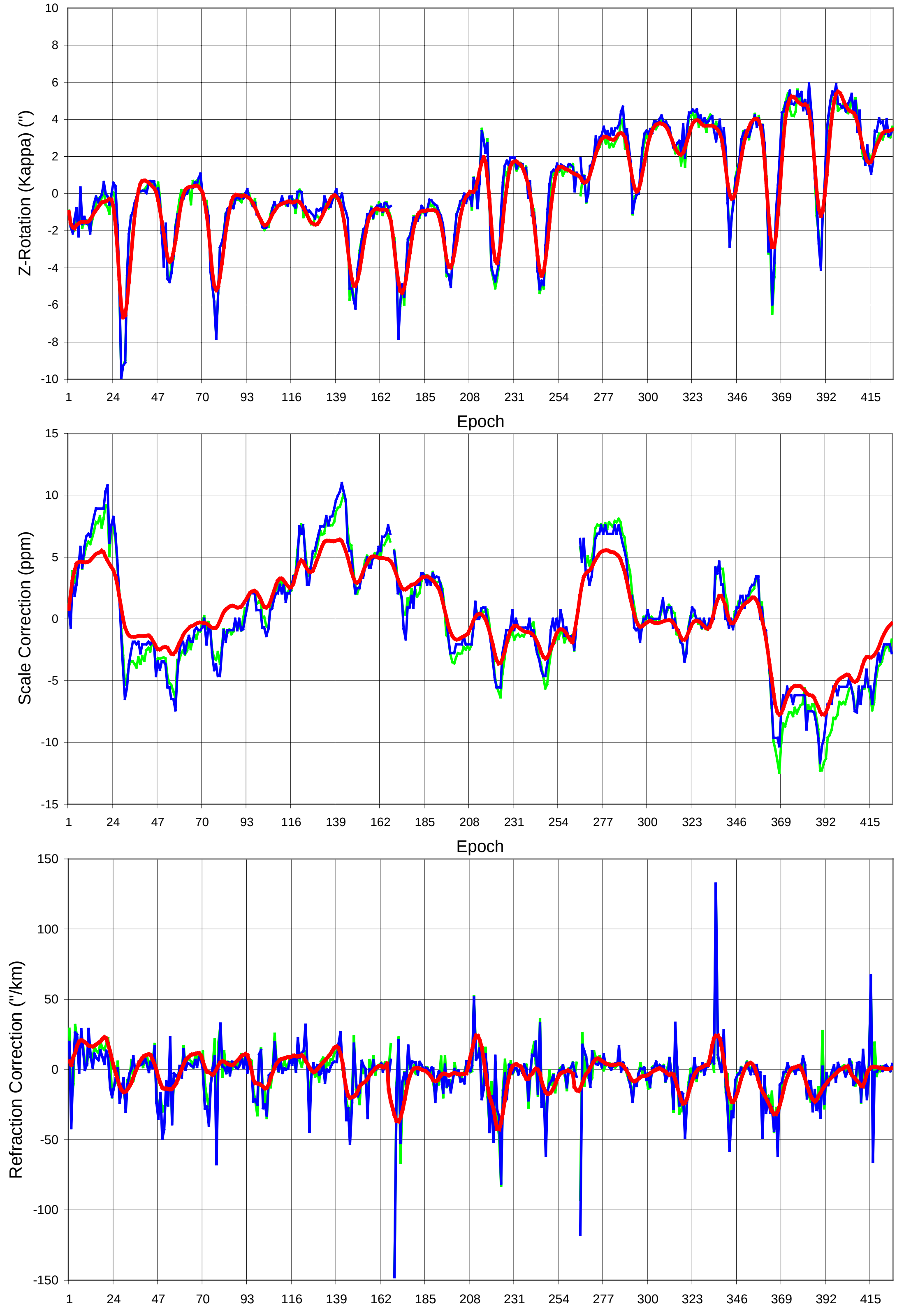


Figure 5. L1, L2 and Kalman filtered solutions of z-rotation, scale correction and refraction correction as shown in blue, green and red, respectively

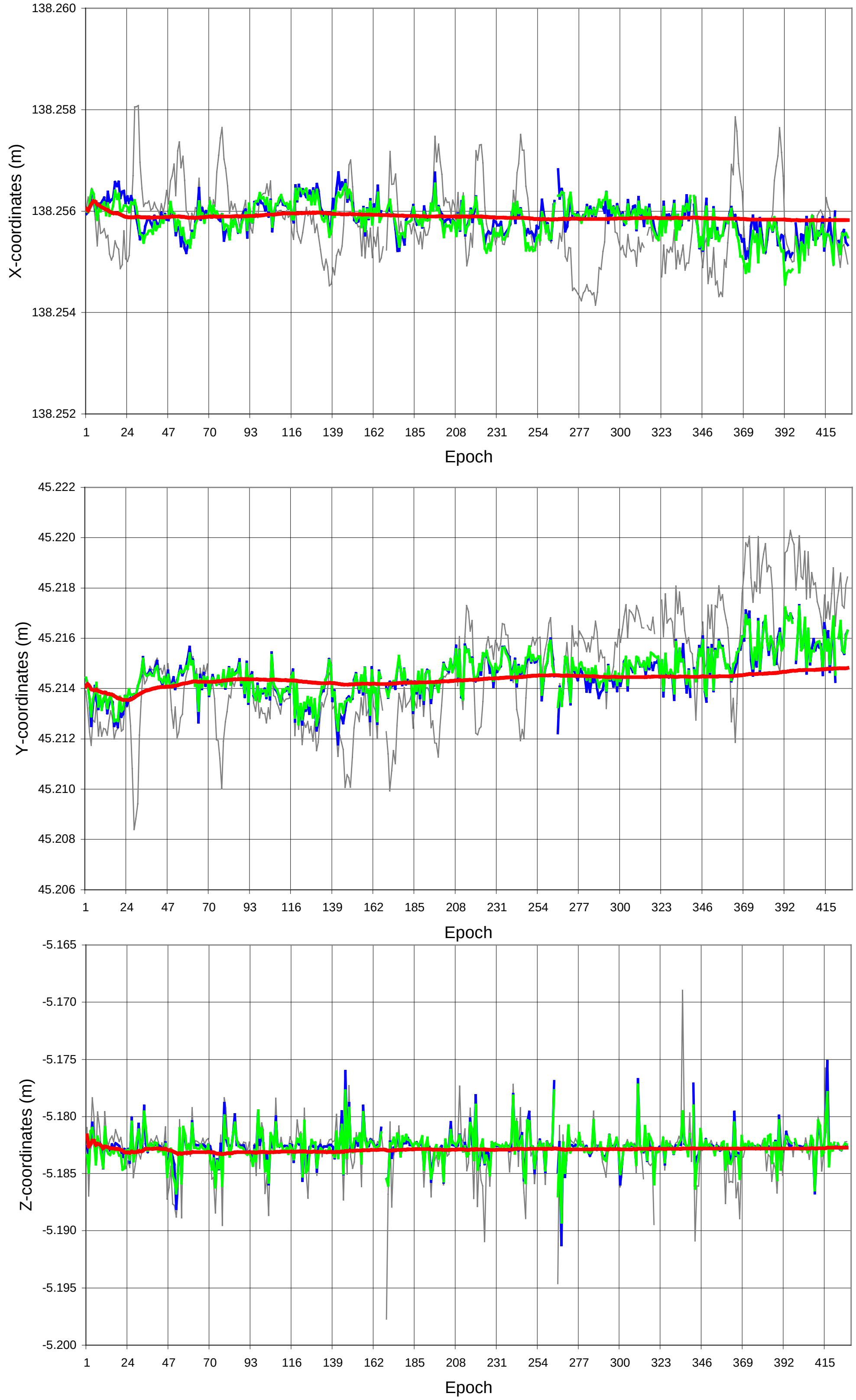


Figure 6. X, Y and Z-coordinates of G3 computed using L1(blue), L2(green), Kalman filtered(red) and the observed coordinates (grey)

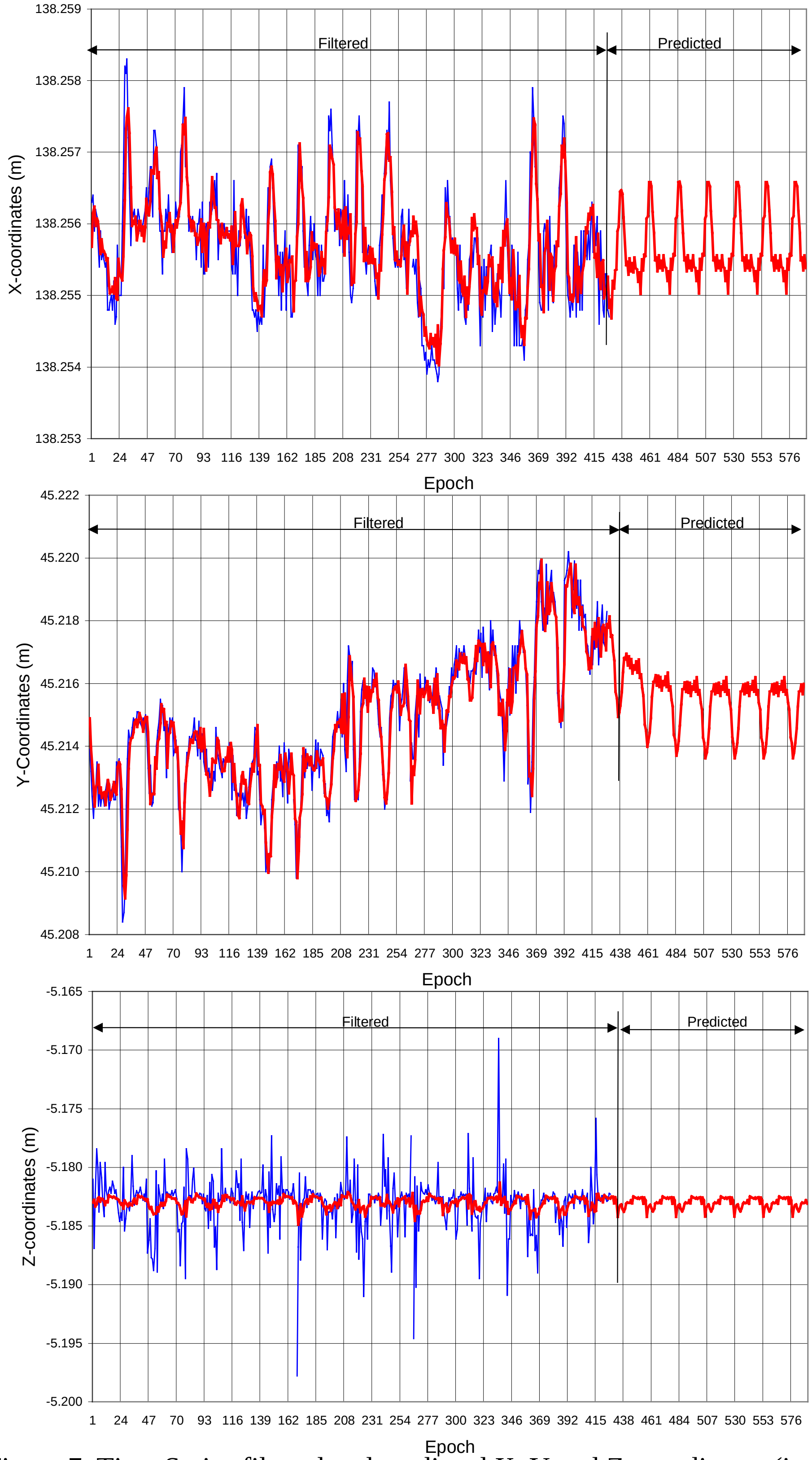


Figure 7. Time Series filtered and predicted X, Y and Z-coordinates (in red) and the raw X, Y and Z-coordinates (in blue) of Point G3

### 7.2 GPS Observations

Two sets of GPS observations were acquired, each with approximately 48 hours. The first began on 08.00h (GMT) 3rd August 2001 and ended on 07.15h (GMT) 5th August 2001. The second set began on 06.00h (GMT) 13 August 2001 and ended on 08.00h (GMT) 15 August 2001. The sampling rate of the GPS observations is fifteen seconds, i.e. one reading every 15 seconds

The changes in the easting(X), northing(Y) and height(Z) coordinates of the GPS antenna at G3 computed at hourly interval are shown in Figure 8. The height shown is the WGS84 ellipsoidal height. Due to the small extent of the test site, the changes in the ellipsoidal height, for practical purpose, is taken as the same as the changes in the orthometric height. The horizontal line in each graph indicates the mean of the changes in the respective coordinate. The numerical value of the mean and the root mean square (RMS) of the results are shown as well. The results were processed using the Motion Tracker module of the Trimble GeoGenious program.

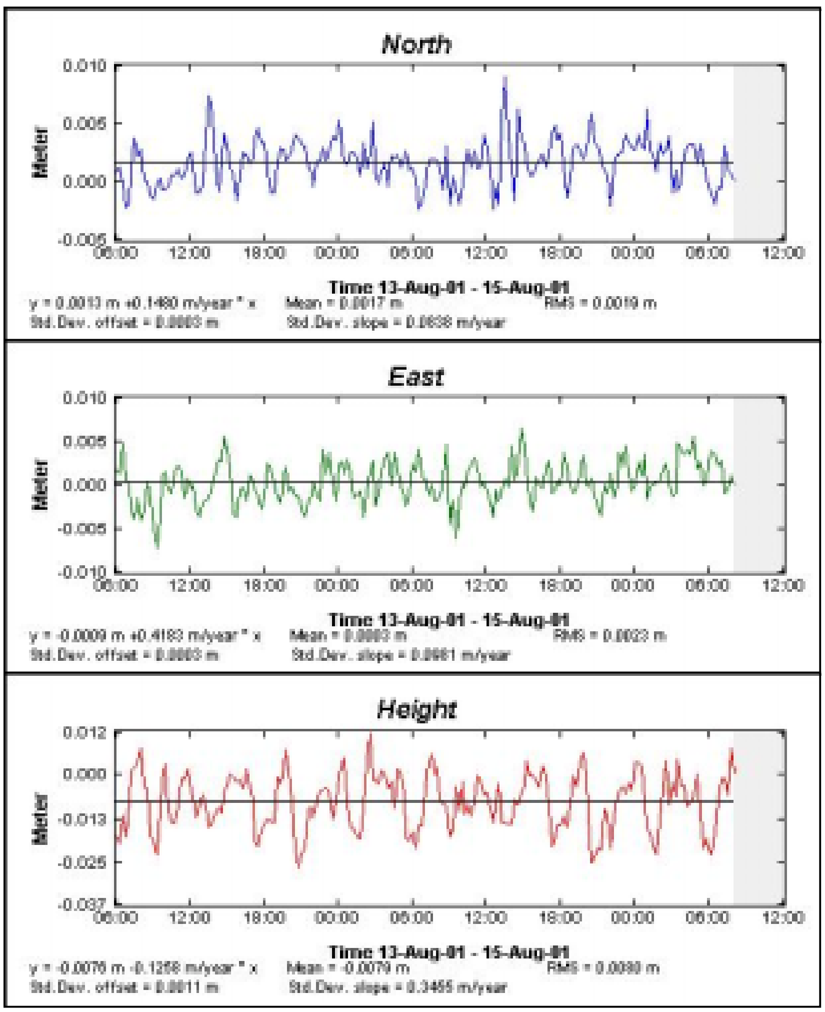


Figure 8. Changes in the northing, easting and ellipsoidal height of monitoring point G3 at Block N2 computed at hourly interval with reference to the GPS reference station at Block N1.

## 8. ANALYSIS OF RESULTS

As shown in Figure 5, the results of the L1 solution using the Simplex algorithm and L2 solution with Danish method of outlier detection are found to be similar in magnitude. The Kalman filtering and smoothing solution was found to be able to smooth the sharp peak, especially in the refraction corrections which are strongly dependent on the changes in the zenith angles. It is thus an effective low pass filter which removes the high frequency signals from the data. The effect of wind is the likely cause of the noisy zenith angles.

The adjusted X, Y and Z-coordinates as depicted in Figure 6, further validate the advantage of using Kalman filtering and smoothing to extract the true signal of the state of deformation, in this case of a supposedly stable building – Block N2 (Figure 1). The changes in the three-dimensional coordinates of G3, as well as G2 and G4, are 1 mm or less! The diurnal changes in the coordinates, due to the pillar rotations, changes in scale and changes in refraction, all related to the changes in the temperature (Figure 9), are successfully removed with a 3-step fixed-interval smoothing which is a good approximation for a fixed-lag smoothing. Figure 10 shows the Kalman smoothed (in red) and the observed (in blue) horizontal directions, zenith angles and slope distances. It is noted that the Kalman-smoothed values start to converge after 24 epochs (1 day) of observation.

The constant mean level of the GPS results as shown in Figure 8 is a good independent verification of the stability of the Block N1. The fluctuations in the planimetric position of about 7 mm and height from the mean levels of about 15 mm are substantially larger than that of the surveying robot. The two days of processed results exhibit diurnal repetitions in the changes in the coordinates and this phenomena could be attributed to the multipath effect suffered by the GPS signals.

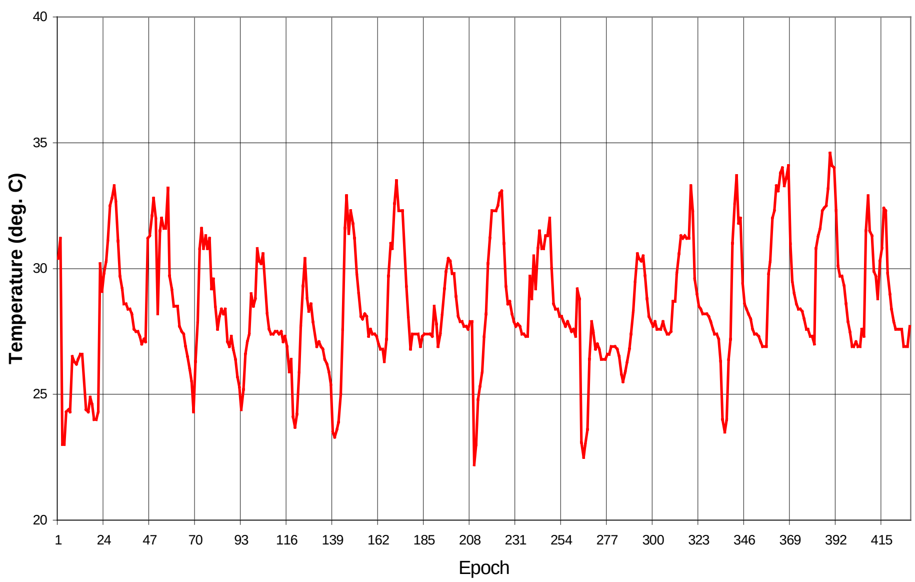


Figure 9. Changes in temperature (the sudden dips in the graph – e.g. epochs 51, 210, 265, 336 - indicate cooling of temperaute due to rainfall)

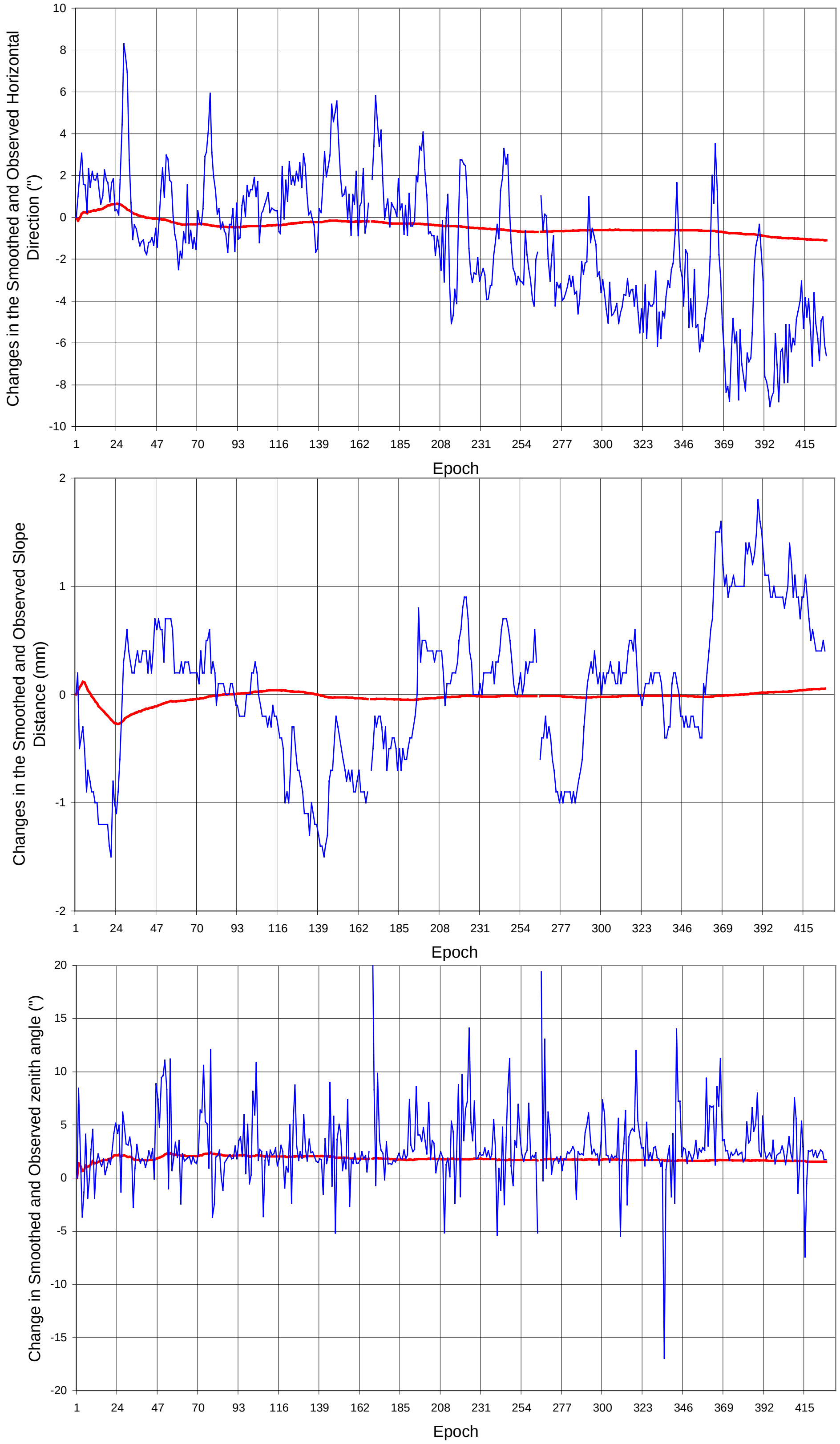


Figure 10. Kalman smoothed (in red) and the observed horizontal direction, slope distance and zenith angle of point G3

## 9. CONCLUSION

Using Kalman filtering and smoothing, changes in the X, Y and Z-coordinates of less than 1 mm had been obtained for a supposedly stable building in the NTU Test Site. In this study, Kalman filtering is proven feasible to be implemented in real-time to eliminate the high frequency noise due to wind and rain, diurnal changes in pillar rotation, changes in scale and changes in the refraction mainly due to the the changes in temperature. System noises were estimated to initiate the Kalman filtering and smoothing. Further testing and study are required to have a better understanding of the use of Kalman filtering and smoothing for deformation analysis.

## REFERENCES


Abraham, B. and Box, G.E.P. (1979) Bayesian analysis of some outlier problems in time series, Biometrika, 66, pp. 229-236.

De Alba, E. and Zartman D.L. (1980) Testing outliers in time series: an application to remotely sensed temperature in cattle, In: Anderson, O.D.(Ed) (1980) *Analysing Time Series*, North-Holland, Amsterdam, pp. 127-138.

Barrodale, I. and Roberts, F.D.K. (1974) Algorithm 478: Solution of an overdetermined system of equations in the $L_1$ norm, Comm. ACM, 17, p.319.

Box, G.E.P. and Jenkins, G.M. (1970) Time series analysis, Forecasting and control. Holden Day, San Francisco, 575 pp.

Branham, Richard L. (1990) Scientific data analysis – an introduction to overdetermined systems, Springer-Verlag, New York, 237 pp.

Brown, R.G. and Hwang, Y.C. (1997) Introduction to Random Signals and Applied Kalman Filtering, 3rd Edition, John Wiley & Sons, New York, 484 pp.

Caspary, W.F., Borutta, H. (1987) Robust Estimation in Deformation Models, Survey Review, 29, 223: 29-45.

Chang, I. And Tiao, G.C. (1983) Estimation of time series parameters in the presence of outliers, Technical Report 8, University of Chicago, Statistics Research Center.

Chang, I., Tiao, G.C. and Cheng, C. (1988) Estimation of time series parameters in the presence of outliers, Technometrics, 30 (2): 193-204.

Chen, Y.Q. and Tang, C.(1993) Application of the theory of grey system in the analysis of deformation analysis, Proceedings of 7th International FIG Symposium on Deformation Measurement, Alberta 1993, pp. 303-309.

Cross, P.A. (1994) Advanced least squares applied to position fixing, Working Paper No. 6, Department of Land Surveying, University of East London, 205 pp.

Efron, B. and Tibshirani, R. (1986) Bootstrap measures for standard errors, confidence intervals, and other measures of statistical accuracy. Statistical Science, 1: 54-77.

Fox, A.J. (1972) Outliers in time series, J. Roy. Statist. Soc. Series B, 43: 350-363.

Gass, S.I. (1985) Linear Programming, methods and applications, 5th edition, International Thomson Publishing Inc., New York, 532 pp.

Marshall, J., Bethel, J.(1996), Basic Concepts of $L_1$ Norm Minimisation for Surveying Applications, Journal of Surveying Engineering, Vol. 122 No. 4, November 1996, pp 168-179.

Han, S. (1998) Digital signal processing techniques and their applications in Geomatic Engineering, GMAT9107 Course Notes, School of Geomatic Engineering, The University of New South Wales, 112 pp.
Harvey, B.R. (1993), Survey Network Adjustment by the L1 method, Australian Journal of Geodesy, Photogrammetry and Surveying, No.59, December 1993, pp. 39-52.
Kalman, R.E. (1960) A new approach to linear filtering and prediction problem, ASME, Journal of Basic Engineering, 82: 35-45.
Krakiwsky, E.J. (1975) A synthesis of recent advances in the method of least squares. Technical Report No. 42, Department of Geomatic Engineering, university of New Brunswick, 125 pp.
Marshall, J. and Bethel, J. (1996) Basic concepts of $L_1$ norm minimisation for surveying applications, Journal of Surveying Engineering, American Society of Civil Engineers, New York, 122(4), pp. 168-179.
Martin, R.D. (1980) Robust estimation of autoregressive models, in *Direction in Time Series* (Eds. D.R. Brillinger and G.C. Tiao), Institute of Mathematical Statistics, Hagwood, California, 254-288.
Ostram, C.W.Jr, (1990) Time series analysis – Regression Technique, 2$^{nd}$ ed. Sage Publication, Newbury Park, California, 95 pp.
Strang, G. and Borre, K. (1997) Linear algebra, geodesy and GPS, Wellesley-Cambridge Press, Wellesley, Massachusetts, 624 pp.
Sprent, P. (1998) Data Driven Statistical Methods, 1$^{st}$ Edition, Chapman & Hall, London; New York, 406pp.
Tor, Y.K., (1999), Study of Continuous Monitoring using Surveying Robot with Automatic Target Recognition, Proceeding of the 6th South East Asian Surveying Congress incorporating the 40th Australian Surveyors Congress, 1-6 November 1999, Fremantle,Western Australia, pp. 453-467.

## BIOGRAPHICAL NOTES

**Mr Tor** graduated from the School of Surveying (now School of Surveying and Spatial Information Science), University of New South Wales, Australia in 1976 with a Bachelor of Surveying degree and a Master of Surveying Science degree in 1984. He is currently a staff member of the School of Civil and Environmental Engineering of Nanyang Technological University, Singapore, and is currently pursuing his part-time Ph.D study in the same university.

Prior to joining the university in 1992, Mr Tor had 16 years of professional experience. His experience includes deformation monitoring survey of subway tunnel, reclaimed land and highrise building; major horizontal and vertical control for the development of New Town development in Singapore and software development in surveying applications.

Mr Tor obtained his registration with the Land Surveyors Board, Singapore, in 1987. Mr. Tor served as a member of Singapore Land Surveyors Board and Chairman of its Examination Committee in September 1994 for a period of two years. Professionally active, he was also the Honorary General Secretary of Singapore Institute of Surveyors and Valuers (SISV) in the 95/96 Council and Chairman of the Land Surveying Division of SISV in 1999/2000. He is a Member of Australian Institution of Surveyors since 1978 and Fellow of SISV since 1995.

The interest areas of Mr Tor include automation of surveying applications, precise engineering and industrial surveying, deformation measurement and analysis and engineering application of GPS.