Reference Frames, Transformations, and GIS

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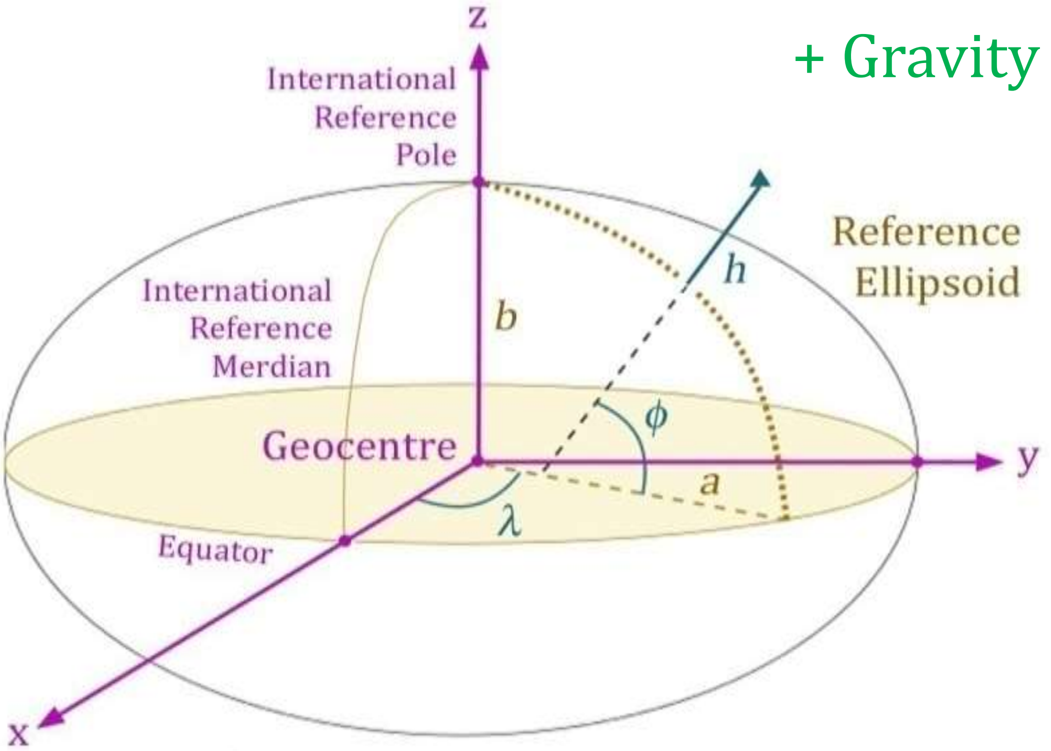
Terrestrial Reference System

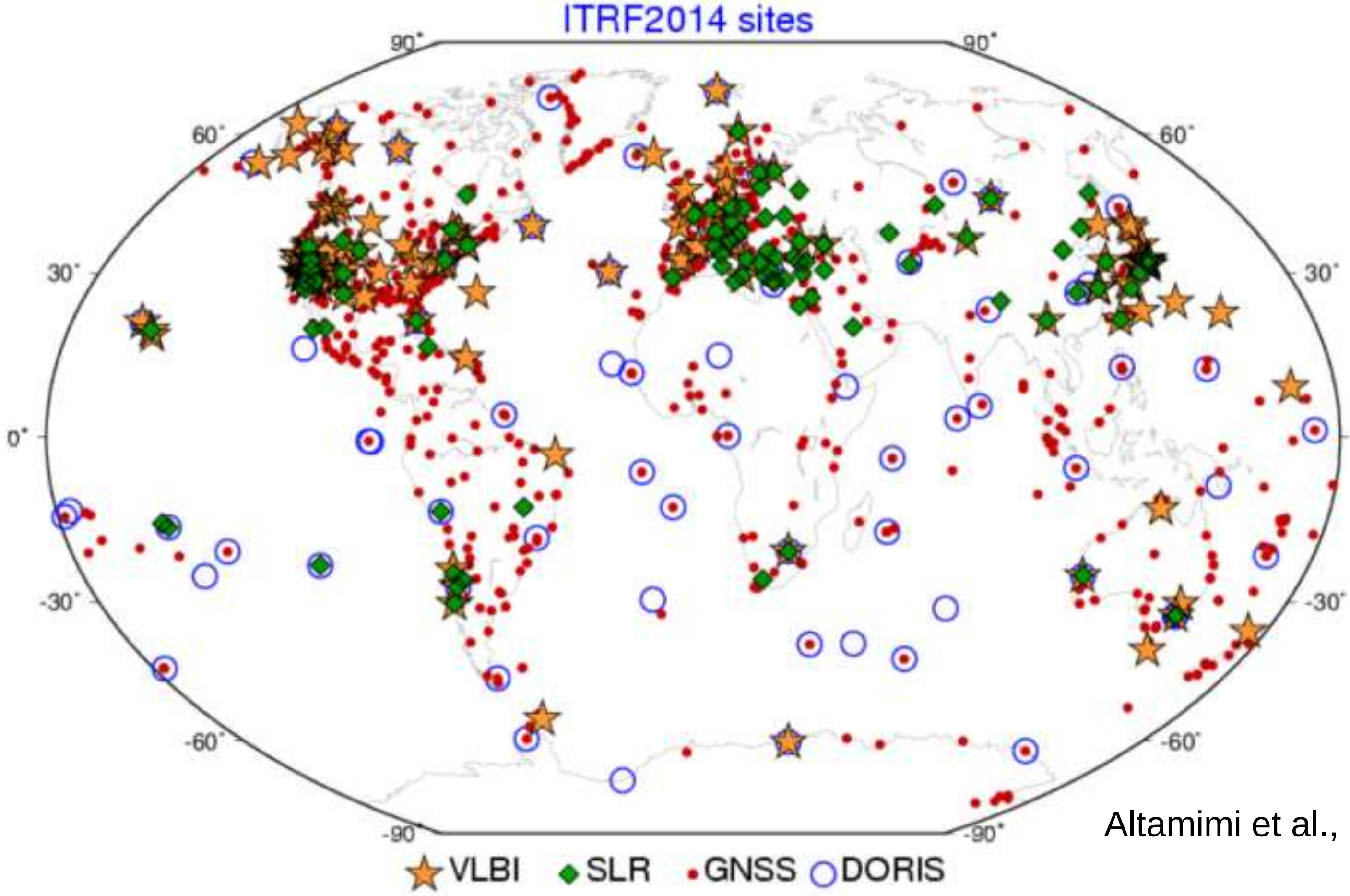


+ Gravity model

In principle a TRS should be invariant with time

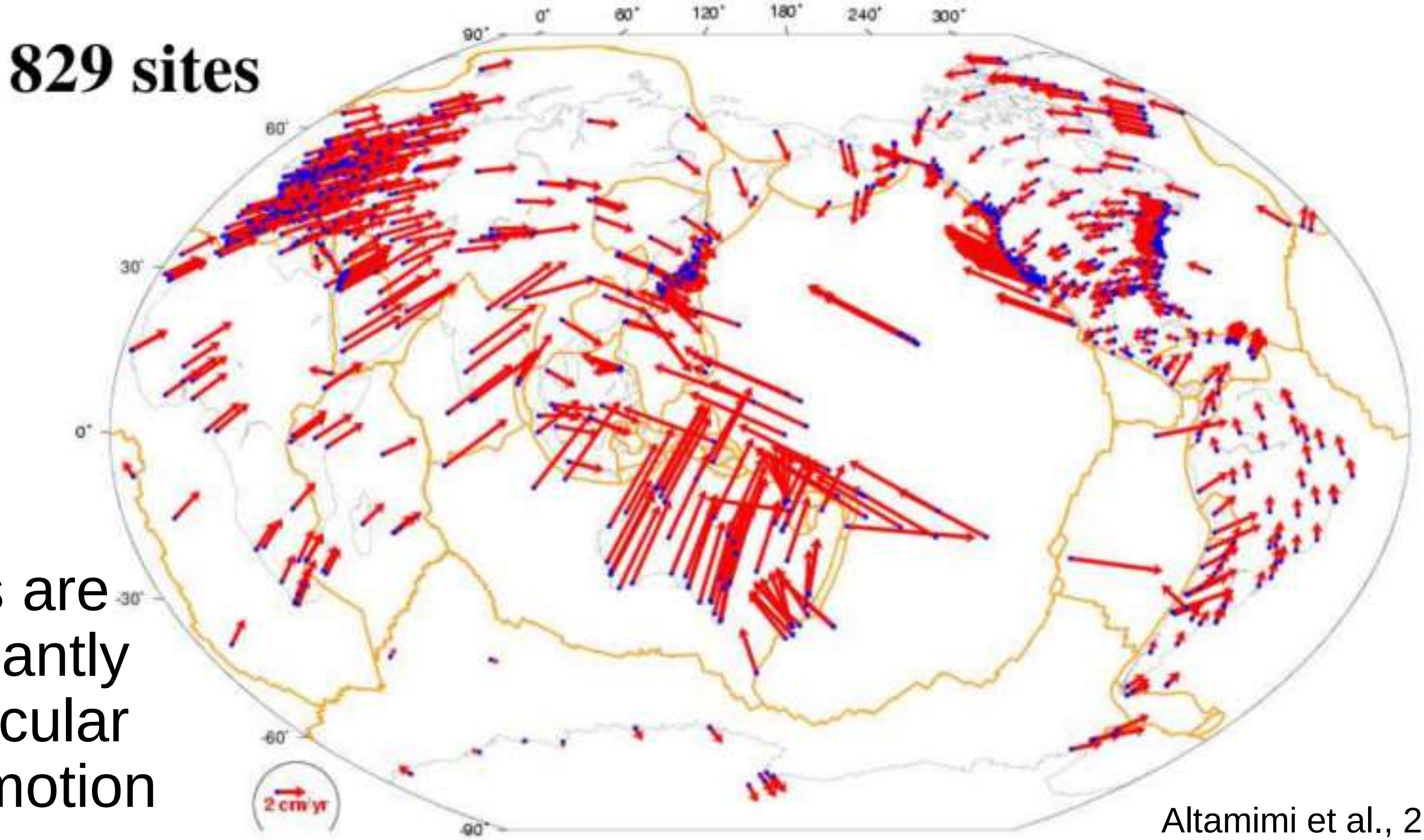
example:Geocentric ITRS

International Terrestrial Reference Frame



Altamimi et al., 2015

ITRF Kinematics – NNR site velocities

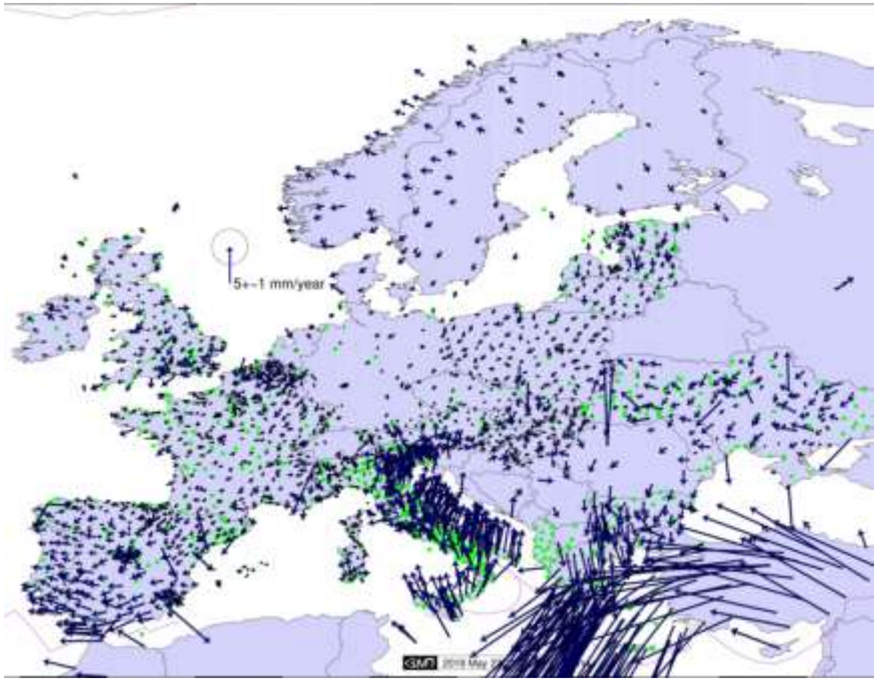


Altamimi et al., 2015

Velocities are predominantly due to secular tectonic motion and GIA

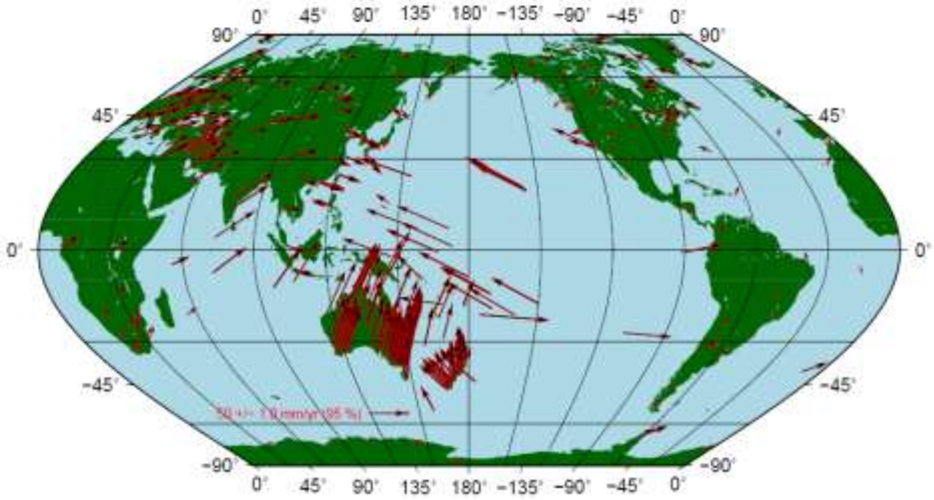
Regional Reference Frames

Plate fixed



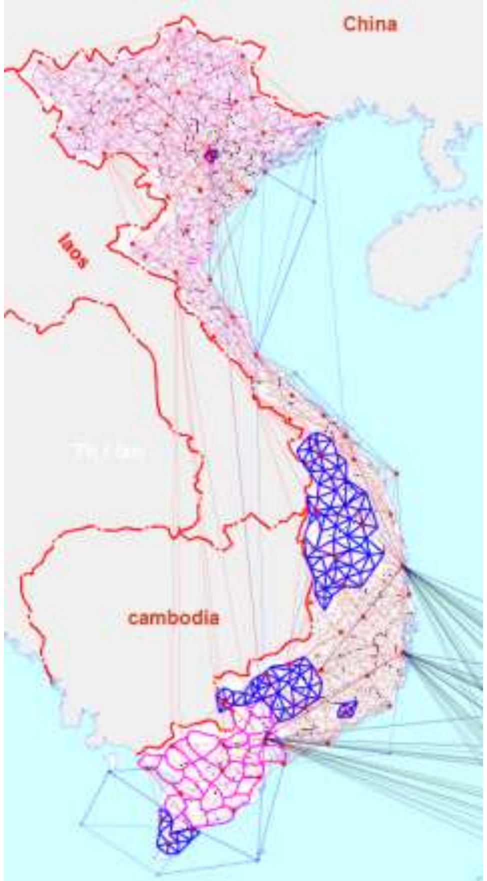
e.g. ETRF – velocities minimised for most of Eurasian plate
(figure: EUREF, 2018)

NNR (No-Net-Rotation)



e.g. APREF – no dominant plate within frame coverage so NNR model is used (figure: Hu, 2014)

National & Local Reference Frames



Characteristics:

Now usually a fixed epoch of ITRF using GNSS (recent geocentric frames)

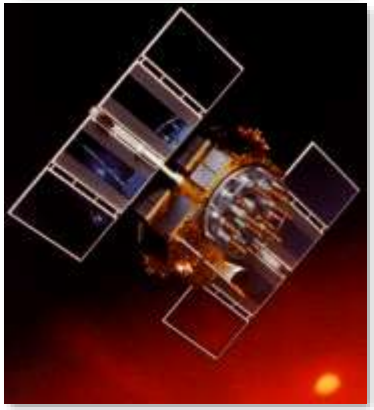
e.g. VN2000 for mainland Vietnam
(figure: Vietnam Dept. of Surveying Mapping, 2016)

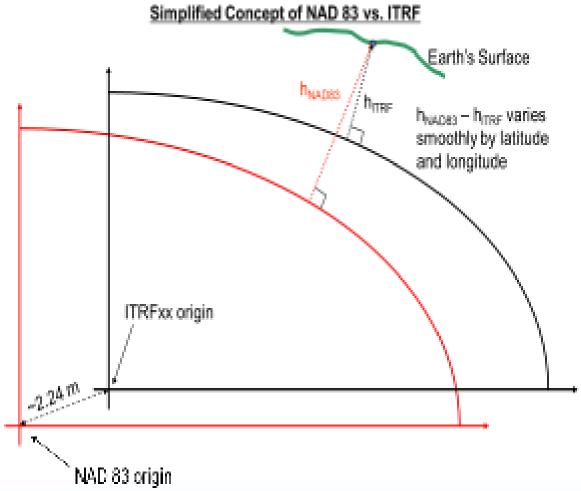
or astronomical determination of frame origin
(pre space geodetic era)
(frame often not geocentric)

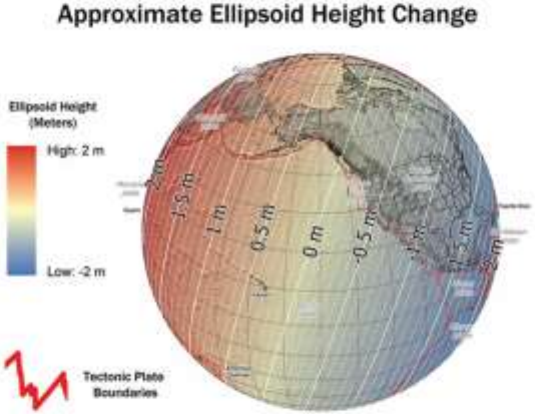
e.g. Australian Geodetic Datum 1966 (AGD66)
(figure: Paul Wise, 2016)

NAD 83

- Original realization completed in 1986
  - Consisted (almost) entirely of classical (optical) observations
- Various re-adjustments
- ***New realization: NAD 83(2011) epoch 2010.00***







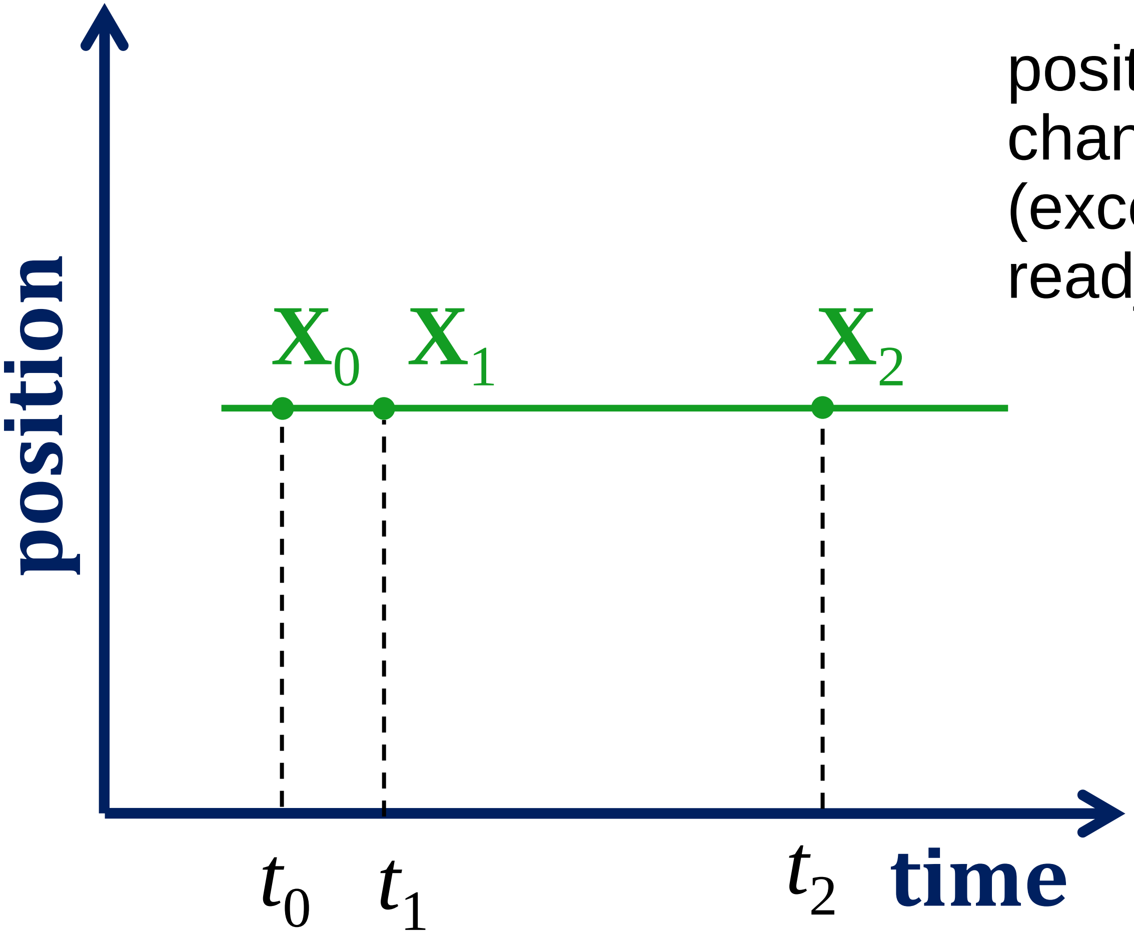
Static and time-dependent frames

Static (time-invariant) – no displacement is assumed

Kinematic (time dependent)
- includes all displacements wrt. ITRS
  (e.g., tectonic motion, glacial isostatic adjustment, coseismic deformation)

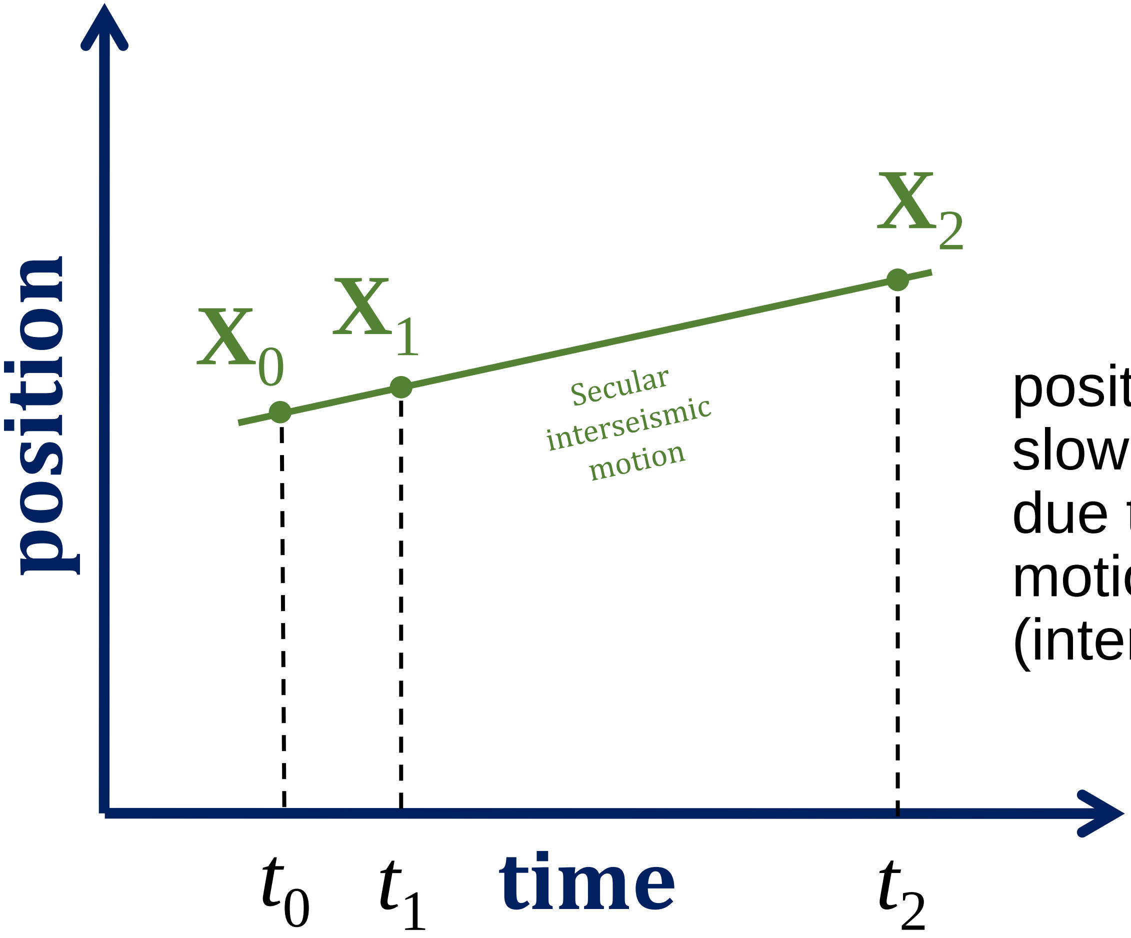
Semi-kinematic (time dependent)
- secular tectonic motion is modelled out
- coseismic displacement models applied
  after earthquakes

Static reference frame



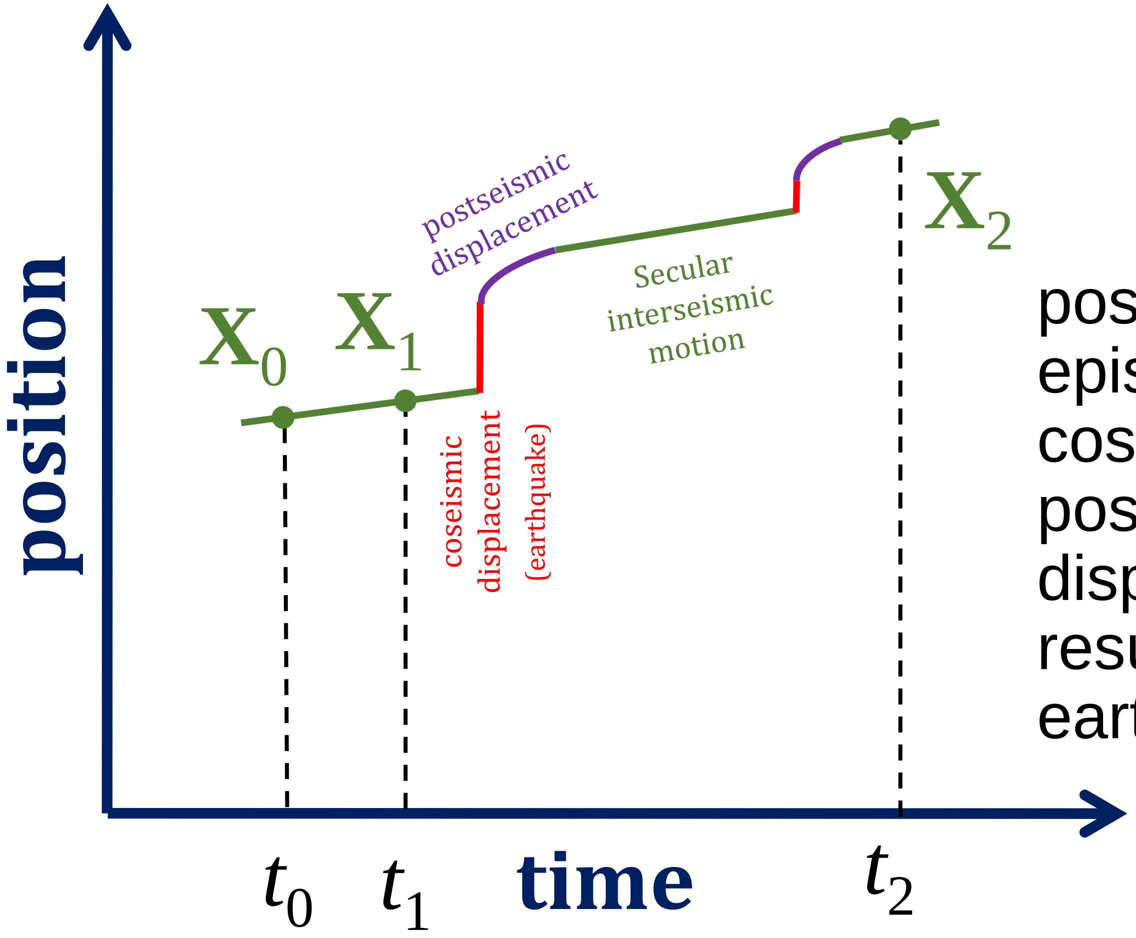
position does not change with time (except perhaps for readjustments)

Kinematic frame – stable plate regions



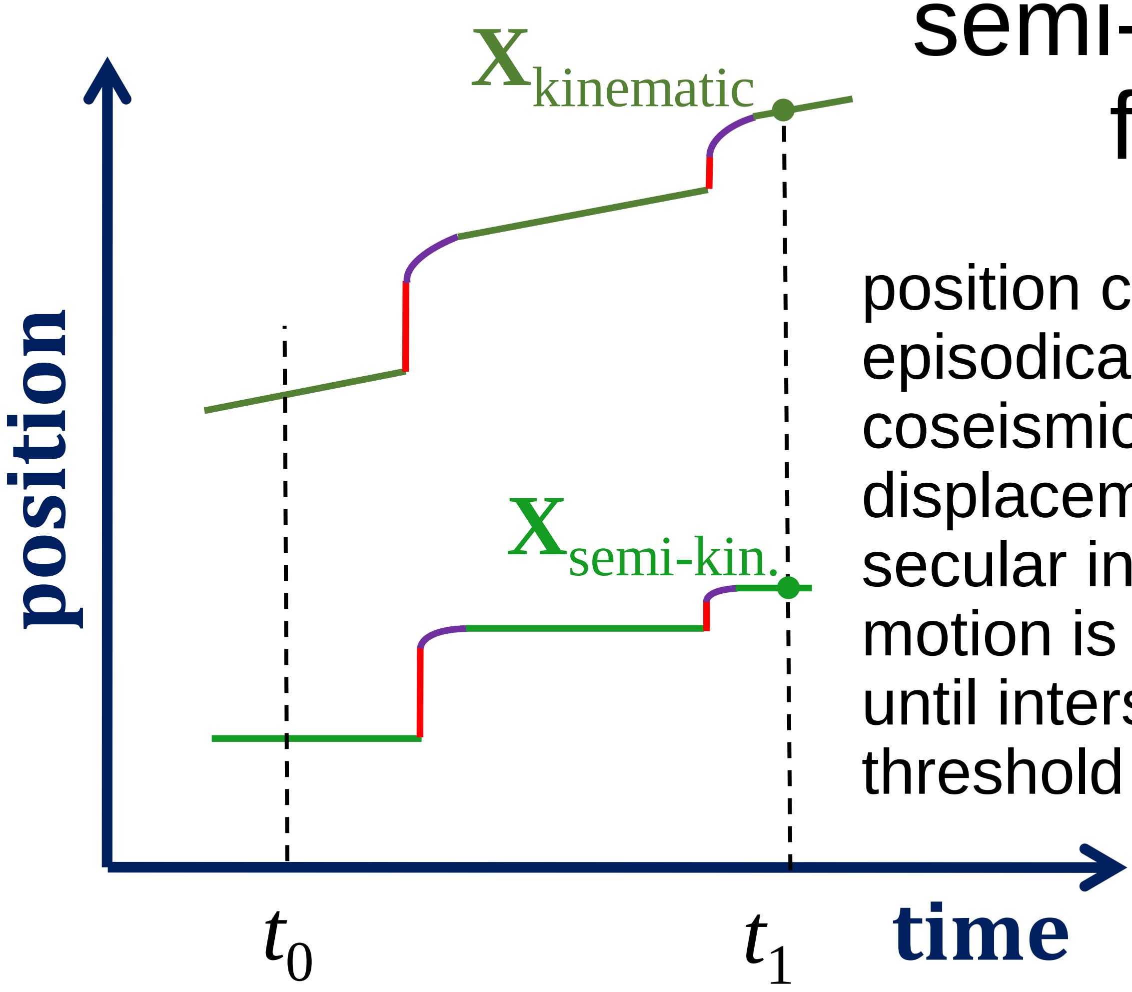
position changes slowly (< 8 cm yr^{-1}) due to secular plate motion and GIA (interseismic motion)

Kinematic frame – plate boundary zones



position changes episodically with coseismic and postseismic displacements resulting from earthquakes

semi-kinematic frame

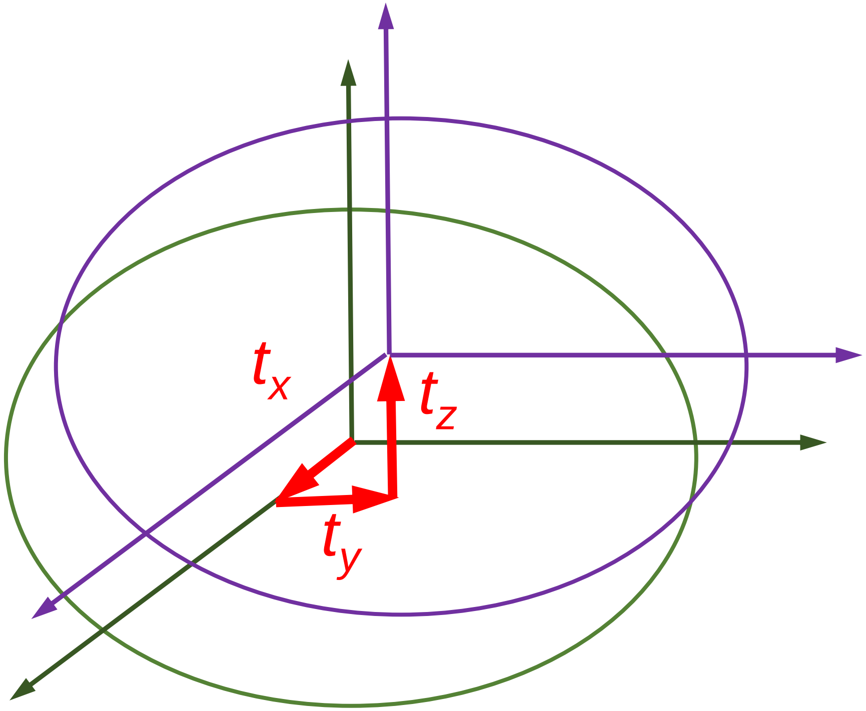


position changes episodically with coseismic displacements however secular interseismic motion is modelled out until interseismic strain threshold is reached

Transformations – frame to frame

	Static frame	time-dependent frame
Conformal Parametric	3 parameter 7 parameter	14 parameter (7 parameters at reference epoch + rates)
Grid model	displacement grid distortion grid	interseismic velocity model + coseismic displacement grid + postseismic decay term grid

3 parameter transformation



$$\mathbf{X}_\mathrm{B} = \mathbf{X}_\mathrm{A} + \mathbf{T}_\mathbf{AB}$$

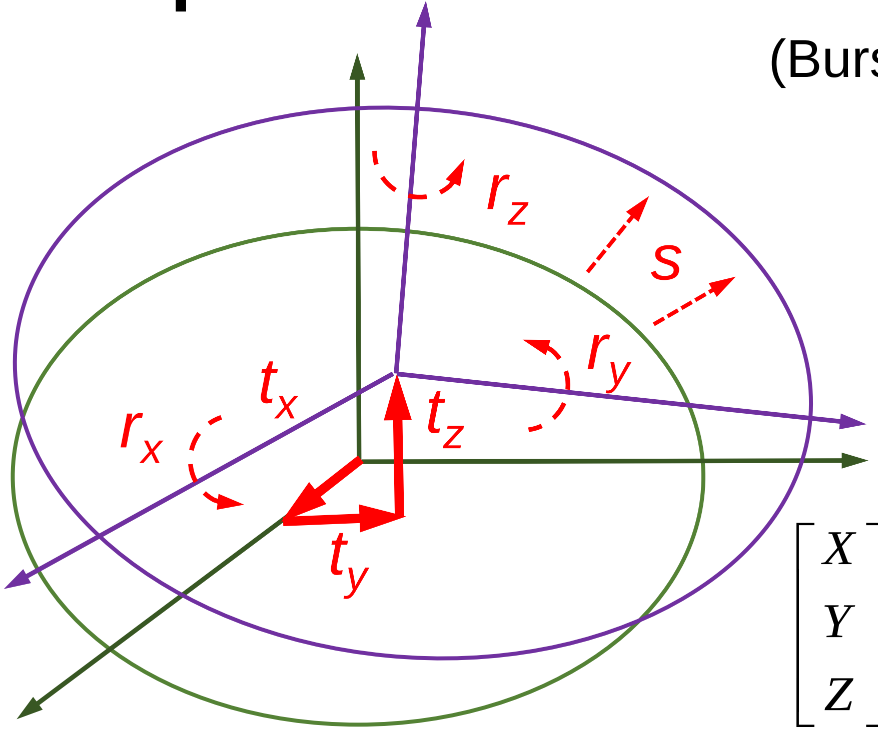
or

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_\mathrm{B} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_\mathrm{A} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}_\mathrm{AB}$$

usually sufficient for most transformations between homogeneous geocentric RF (no rotation or scale difference)

7 parameter transformation

(Bursa-Wolf simplification)



$$\mathbf{X}_{\mathrm{B}} = \mathbf{T}_{\mathbf{AB}} + (1+s)\mathbf{R}_{\mathrm{AB}}\mathbf{X}_{\mathrm{A}}$$

or

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{B}} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}_{\mathrm{AB}} + (1+s)\begin{bmatrix} 1 & r_z & -r_y \\ -r_z & 1 & r_x \\ r_y & -r_x & 1 \end{bmatrix}_{\mathrm{AB}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{A}}$$

used for most classical RF transformations (e.g., between a geocentric and astronomical RF). Bursa-Wolf method assumes small rotations (<10”).
Coordinate Frame (CF) convention illustrated.
Position Vector (PV) rotations are defined in opposite sense

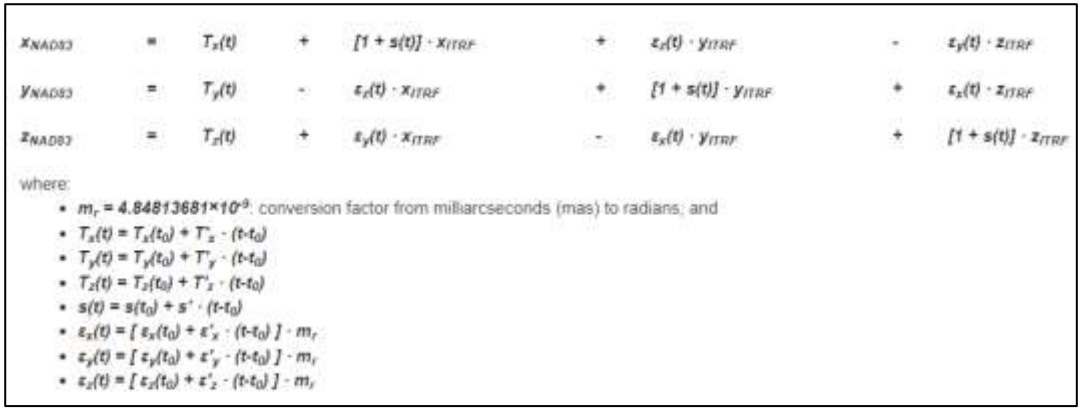
14 parameter transformation

7-parameter transformation at reference epoch t_0

+ parameter rates to estimate 7-parameters at different epochs

Used for static-kinematic or kinematic-kinematic RF transformations

model is reasonably good within stable tectonic plates



$$\begin{aligned}
x_{NAD83} &= T_x(t) + [1 + s(t)] \cdot x_{ITRF} + \varepsilon_z(t) \cdot y_{ITRF} - \varepsilon_y(t) \cdot z_{ITRF} \\
y_{NAD83} &= T_y(t) - \varepsilon_z(t) \cdot x_{ITRF} + [1 + s(t)] \cdot y_{ITRF} + \varepsilon_x(t) \cdot z_{ITRF} \\
z_{NAD83} &= T_z(t) + \varepsilon_y(t) \cdot x_{ITRF} - \varepsilon_x(t) \cdot y_{ITRF} + [1 + s(t)] \cdot z_{ITRF}
\end{aligned}$$

where:

- $m_r = 4.84813681 \times 10^{-9}$: conversion factor from milliarcseconds (mas) to radians; and
- $T_x(t) = T_x(t_0) + T'_x \cdot (t-t_0)$
- $T_y(t) = T_y(t_0) + T'_y \cdot (t-t_0)$
- $T_z(t) = T_z(t_0) + T'_z \cdot (t-t_0)$
- $s(t) = s(t_0) + s' \cdot (t-t_0)$
- $\varepsilon_x(t) = [\ \varepsilon_x(t_0) + \varepsilon'_x \cdot (t-t_0)\ ] \cdot m_r$
- $\varepsilon_y(t) = [\ \varepsilon_y(t_0) + \varepsilon'_y \cdot (t-t_0)\ ] \cdot m_r$
- $\varepsilon_z(t) = [\ \varepsilon_z(t_0) + \varepsilon'_z \cdot (t-t_0)\ ] \cdot m_r$

[Helmert Transformation Parameters Used at NGS - National Geodetic Survey (noaa.gov)]

Plate Motion Model

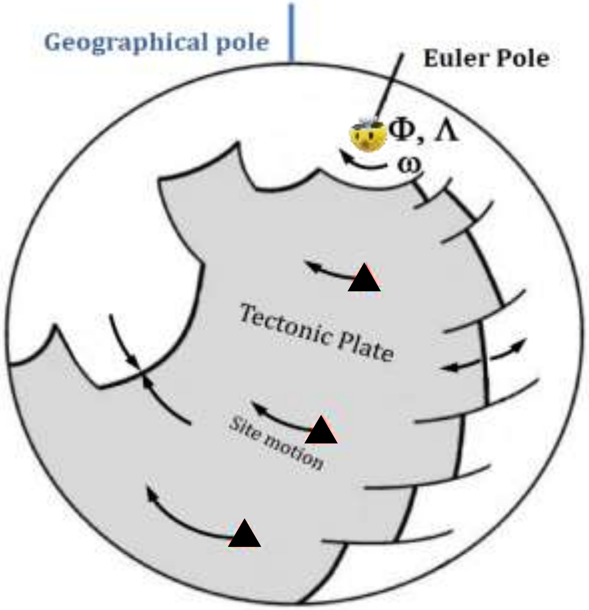


Plate	Euler pole of rotation			Equivalent Cartesian angular velocity		
	Φ (°)	Λ (°)	ω (° Ma^{-1})	Ω_x (Rad Ma^{-1})	Ω_y (Rad Ma^{-1})	Ω_z (Rad Ma^{-1})
Antarctic	58.8	-127.4	0.219	-0.001202	-0.001571	0.003272
Arabian	51.2	-6.7	0.515	0.005595	-0.000659	0.007001
Australian	32.4	38.1	0.631	0.007321	0.005730	0.005890
Eurasian	55.1	-99.1	0.261	-0.000412	-0.002574	0.003733
Indian	51.6	-0.2	0.516	0.005595	-0.000024	0.007049
Nazca	45.8	-102.2	0.629	-0.001614	-0.007486	0.007869
North American	-5.2	-88.0	0.194	0.000116	-0.003365	-0.000305
Nubian	49.7	-80.8	0.267	0.000480	-0.002977	0.003554
Pacific	-62.6	111.3	0.679	-0.001983	0.005076	-0.010516
South American	-19.1	-131.9	0.119	-0.001309	-0.001459	-0.000679
Somalian	47.7	-98.7	0.332	-0.000587	-0.003849	0.004286

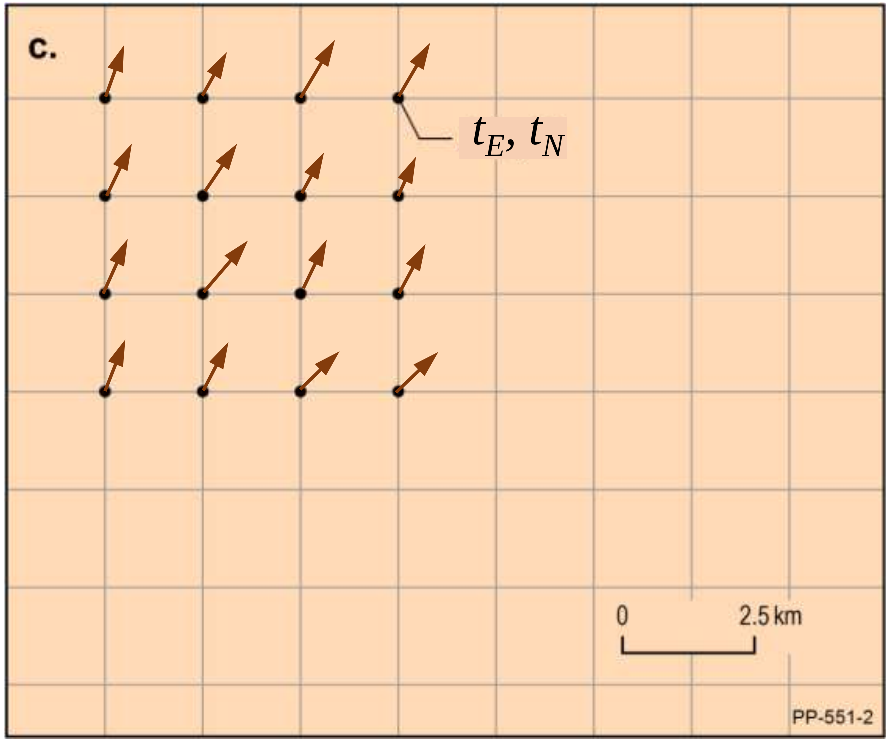
ITRF2014 Plate Motion Model (from Altamimi *et al.*, 2017)

Euler pole can be estimated by LS inversion of interseismic site velocities within a stable plate

$$\Omega_x = \cos(\Phi)\cos(\Lambda)\omega$$
$$\Omega_y = \cos(\Phi)\sin(\Lambda)\omega$$
$$\Omega_z = \sin(\Phi)\omega$$

Rotation rates can be used in a 14 parameter model with zeroes for other parameters
(now used for GDA2020 to ITRF2014 transformations)

Displacement and Distortion Grids



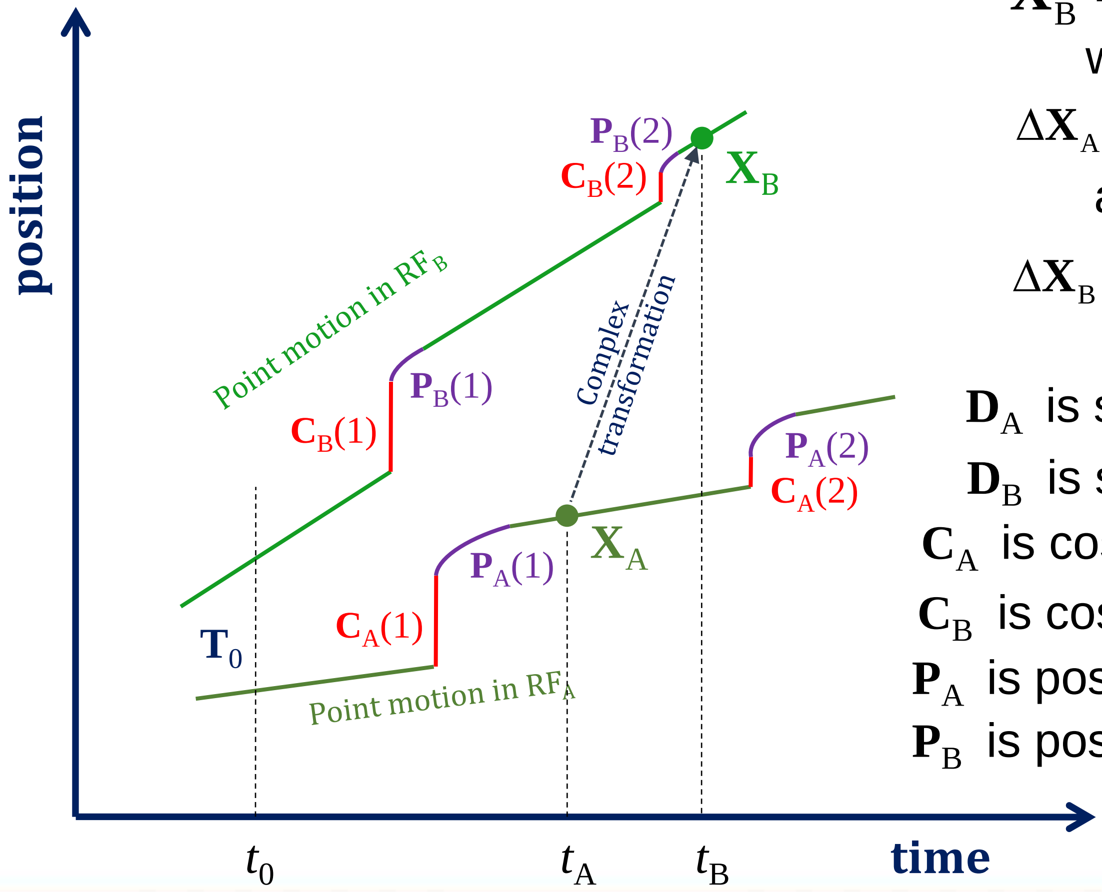
adapted from GDA2020 Technical Manual, ICSM, 2018

$$\begin{bmatrix} E \\ N \end{bmatrix}_B = \begin{bmatrix} E \\ N \end{bmatrix}_A + \begin{bmatrix} t_E \\ t_N \end{bmatrix}$$

Topocentric shifts estimated by bilinear interpolation of grid model (e.g., in NTv2 format)

Ideal for heterogeneous RF transformations and handling local distortions

Complex time-dependent transformations



$$\mathbf{X}_B = \mathbf{X}_A - \Delta\mathbf{X}_A + \mathbf{T}_0 + \Delta\mathbf{X}_B$$

where

$$\Delta\mathbf{X}_A = \mathbf{D}_A + \sum_{t_0 \geq \mathbf{C} > t_A}^{n} \mathbf{C}_A + \sum_{t_0 \geq \mathbf{P} > t_A}^{n} \mathbf{P}_A$$

and

$$\Delta\mathbf{X}_B = \mathbf{D}_B + \sum_{t_0 \geq \mathbf{C} > t_B}^{n} \mathbf{C}_B + \sum_{t_0 \geq \mathbf{P} > t_B}^{n} \mathbf{P}_B$$

$\mathbf{D}_A$ is secular displacement in RF_A

$\mathbf{D}_B$ is secular displacement in RF_B

$\mathbf{C}_A$ is coseismic displacement in RF_A

$\mathbf{C}_B$ is coseismic displacement in RF_B

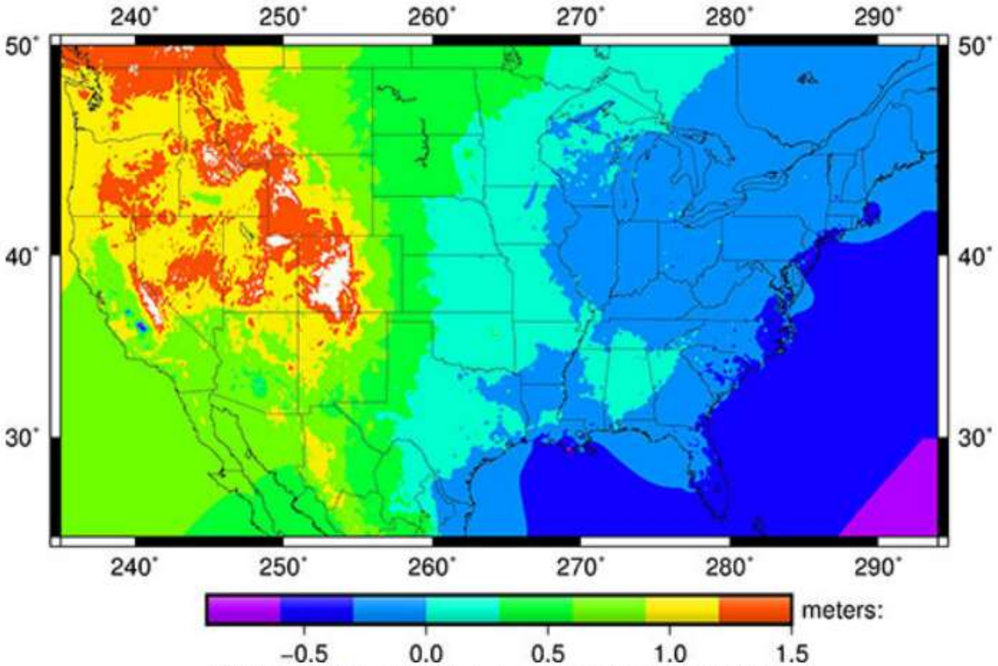
$\mathbf{P}_A$ is postseismic displacement in RF_A

$\mathbf{P}_B$ is postseismic displacement in RF_B

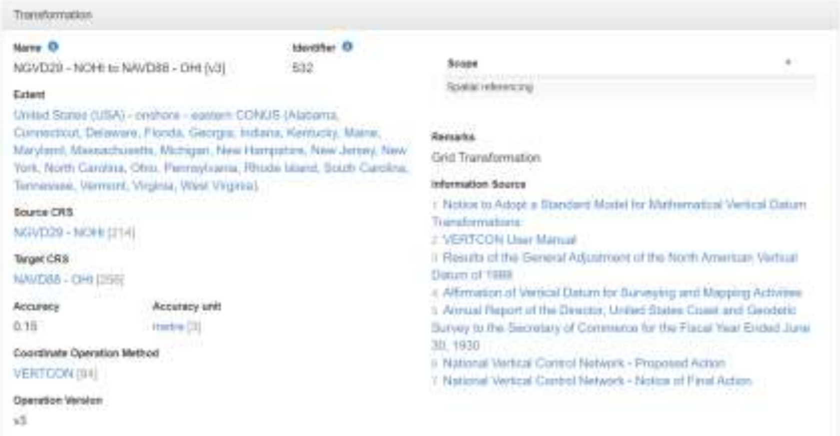
$\mathbf{T}_0$ is interframe translation

Vertical Grid Transformation Example:

VERTCON 3.0



VERTCON 3.0 height shifts between NGVD 29 and NAVD 88



Transformation

Name
NGVD29 - NOHI to NAVD88 - OHI [v3]

Identifier
532

Scope
Spatial referencing

Extent
United States (USA) - onshore - eastern CONUS (Alabama, Connecticut, Delaware, Florida, Georgia, Indiana, Kentucky, Maine, Maryland, Massachusetts, Michigan, New Hampshire, New Jersey, New York, North Carolina, Ohio, Pennsylvania, Rhode Island, South Carolina, Tennessee, Vermont, Virginia, West Virginia).

Remarks
Grid Transformation

Source CRS
NGVD29 - NOHI [214]

Target CRS
NAVD88 - OHI [255]

Information Source
1. Notice to Adopt a Standard Model for Mathematical Vertical Datum Transformations
2. VERTCON User Manual
3. Results of the General Adjustment of the North American Vertical Datum of 1988
4. Affirmation of Vertical Datum for Surveying and Mapping Activities
5. Annual Report of the Director, United States Coast and Geodetic Survey to the Secretary of Commerce for the Fiscal Year Ended June 30, 1930
6. National Vertical Control Network - Proposed Action
7. National Vertical Control Network - Notice of Final Action

Accuracy
0.15

Accuracy unit
metre [3]

Coordinate Operation Method
VERTCON [94]

Operation Version
v3

ISOTC211 Geodetic Registry: NGVD29 to NAVD88

GIS transformations

Main consideration is alignment of precise data defined in different RF and projections.

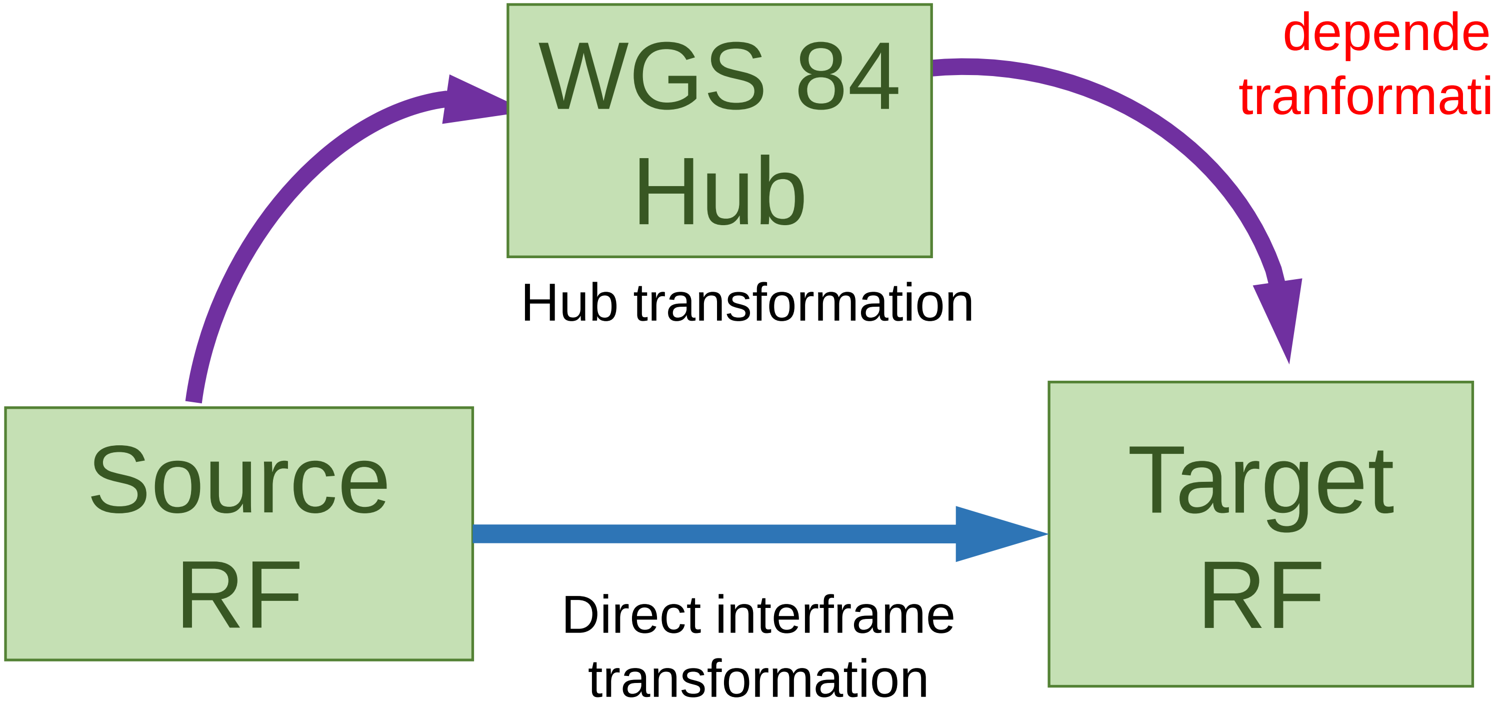
Geodetic Registries (EPSG and newly minted ISO TC211 Registry) standardise definitions and transformation workflows for use in GIS, however there are still limitatations with complex time-dependent RF.

e.g. No standard for coseismic and postseismic displacement grids and no standardised epoch of WGS 84 when used as a hub transformation.

Direct interframe transformations are the preferred option.

GIS transformations

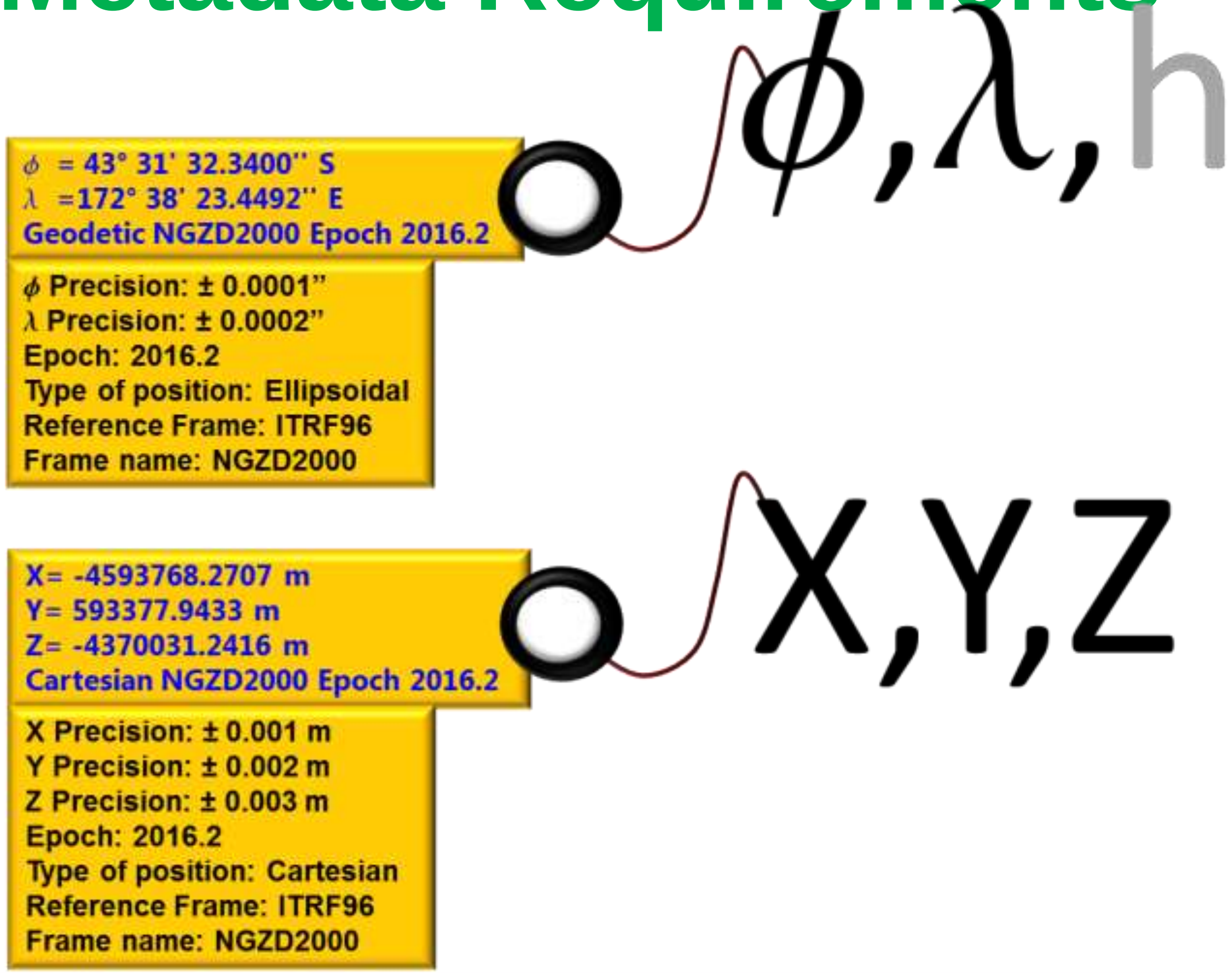
Hub method not recommended for time-dependent tranformations



Hub transformation

Direct interframe transformation

GIS Metadata Requirements



$\phi, \lambda,$ h

ϕ = 43° 31' 32.3400'' S
λ =172° 38' 23.4492'' E
Geodetic NGZD2000 Epoch 2016.2

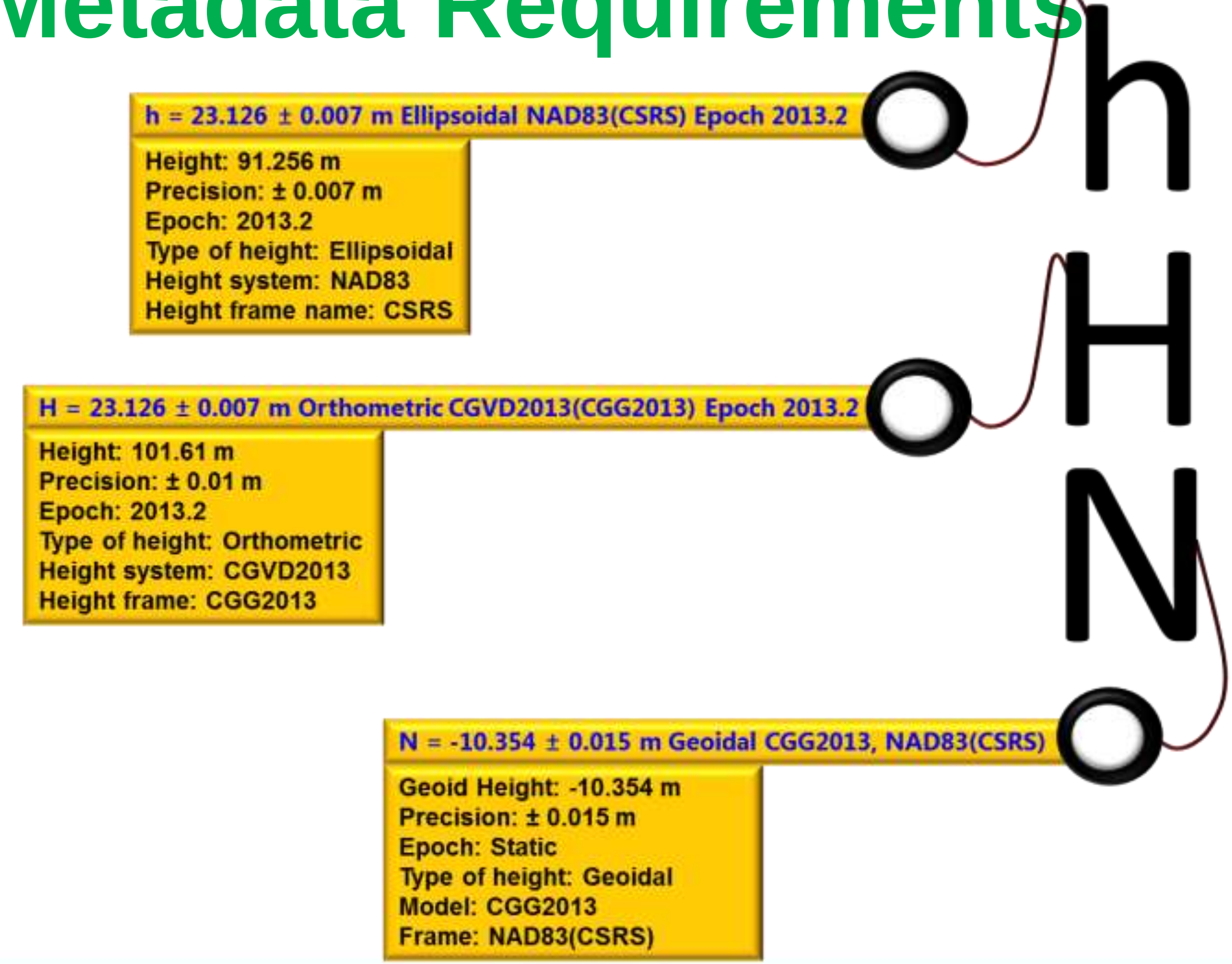
ϕ Precision: ± 0.0001"
λ Precision: ± 0.0002"
Epoch: 2016.2
Type of position: Ellipsoidal
Reference Frame: ITRF96
Frame name: NGZD2000

X,Y,Z

X= -4593768.2707 m
Y= 593377.9433 m
Z= -4370031.2416 m
Cartesian NGZD2000 Epoch 2016.2

X Precision: ± 0.001 m
Y Precision: ± 0.002 m
Z Precision: ± 0.003 m
Epoch: 2016.2
Type of position: Cartesian
Reference Frame: ITRF96
Frame name: NGZD2000

GIS Metadata Requirements



h = 23.126 ± 0.007 m Ellipsoidal NAD83(CSRS) Epoch 2013.2

Height: 91.256 m
Precision: ± 0.007 m
Epoch: 2013.2
Type of height: Ellipsoidal
Height system: NAD83
Height frame name: CSRS

H = 23.126 ± 0.007 m Orthometric CGVD2013(CGG2013) Epoch 2013.2

Height: 101.61 m
Precision: ± 0.01 m
Epoch: 2013.2
Type of height: Orthometric
Height system: CGVD2013
Height frame: CGG2013

N = -10.354 ± 0.015 m Geoidal CGG2013, NAD83(CSRS)

Geoid Height: -10.354 m
Precision: ± 0.015 m
Epoch: Static
Type of height: Geoidal
Model: CGG2013
Frame: NAD83(CSRS)

Questions

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Links:

ITRF: https://itrf.ign.fr/en/homepage

ISO TC211 Geodetic Registry: https://geodetic.isotc211.org/

PROJ: https://proj.org/

Topocentric frame

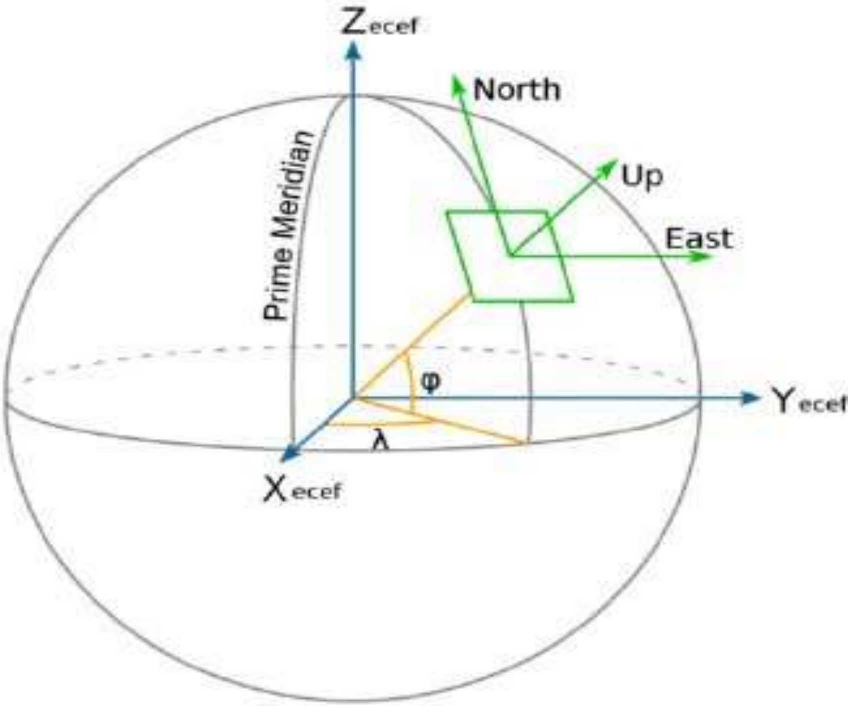

Image: Haasdyk and Janssen, 2011

Converting geocentric translation to topocentric (rate or shift)

$$\begin{bmatrix}\dot{E}\\ \dot{N}\\ \dot{U}\end{bmatrix}=\begin{bmatrix}-\sin\lambda & \cos\lambda & 0\\ -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi\\ \cos\phi\cos\lambda & \cos\phi\sin\lambda & \sin\phi\end{bmatrix}\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix}$$

Converting topocentric translation to geocentric (rate or shift)

$$\begin{bmatrix}\dot{X}\\ \dot{Y}\\ \dot{Z}\end{bmatrix}=\begin{bmatrix}-\sin\lambda & -\sin\phi\cos\lambda & \cos\phi\cos\lambda\\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda\\ 0 & \cos\phi & \sin\phi\end{bmatrix}\begin{bmatrix}\dot{E}\\ \dot{N}\\ \dot{U}\end{bmatrix}$$

A topocentric projection canvas is useful for complex transformation computations involving displacement grids