CONSIDERATIONS REGARDING THE ACCURACY OF DETERMINATION OF SURFACES

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Abstract
In Romania, there is an acute problem of the cadastre execution on the entire territory of the country, starting, of course, from the cadastre on the cadastral sectors or on the entire Administrative Territorial Unit (ATU). The work is carried out by companies specialized in cadastre works, verified by specialists employed by the City Hall and validated by the County Cadaster and Land Registration Advertising Offices. The beneficiary of the work requires that the accuracy of determining the planimetric position of a mathematical point delimiting the buildings be ± 10 centimeters. The issue at hand is the accuracy of determining the area of a building. Owners, owners or holders do not understand why, after two measurements on the same building, two different surfaces may appear. For this reason, we try to explain through these articles which are the maximum and minimum differences that can occur after two measurements on the same building, and the accuracy of determination.
Keywords: cadastre, surface, perimeter, precision.

1. General considerations

In Romania, an extensive process of systematic cadastre of the Territorial Administrative Units (ATU) has been started, especially in rural areas, since 2010. The process is in full swing and involves: the executors, i.e., private companies, the City Halls together with the inhabitants of the ATUs as beneficiaries and verifiers and the county Cadaster and the Land Registration Advertising Offices (OCPI) which receive the final data and store them in the national database e-terra.

Before starting the process of systematic tabulation of properties, each owner had the possibility to be registered in the Land Book upon request. In fact, they were calling on a cadastre specialist, authorized by the National Agency for Cadaster and Land Registration Advertising (ANCPI). He performed the topographic measurements, drew up the cadastral plan of the immobile, took over the property documents and submitted them to OCPI for reception and approval. This documentation, received, was entered in the e-terra database of ANCPI and became public information.

At the start of the systematic cadastre works on the entire ATU or on cadastral sectors, it was imposed by the Technical Specifications the tabulation of each building and the verification of the already existing tabulations, i.e. those prepared upon request.

The advantage of this type of work is that, working on the entire ATU, each immobile must adhere to the neighboring immobile, so that there are no overlaps or gaps between them (topology). For this reason, some of the immobile registered in the Land Book on request have been repositioned or the surface modified, so as to respect the topology. Sure, the surface differences are not very large, but they do exist (Figure 1). [3]

The owners of these immobile, noticing the changes in surface between two records in the Land Book, become suspicious and ask for explanations.

Figure 1 – Surface and limits differences between an immobile in the cadaster on request and the systematic cadastre

Apart from this aspect, measurements are performed when registering in the Land Book. If the owners find differences between the surface in the property deeds and the surface resulting from the measurements, they ask themselves questions. For this reason, sometimes they turn to other specialists in the cadastre to perform another measurement. There will be some inherent differences between the three surfaces due to the technology used in the measurements, the operator's experience, the environmental conditions and the software used. In urban areas, where the price of a square meter is very high, problems can occur. If the area decreases, an owner can go to court to be compensated for the minus surface difference.
The question is how big these differences are. What is the accuracy of determining an area and what is the tolerance? How do we explain this to an owner? How can it be justified in court?

2. Possibilities to determine the position of the contour points of an immobile

Currently, to determine the position of a detail point, we have the following possibilities:

- Total station;
- GNSS Receivers;
- Georeferenced orthophotoplan, preferably resulting from UAV flight;
- Georeferenced LiDAR point cloud.

Each of these methods has advantages and disadvantages. Also, the accuracy of each method is different depending on certain factors.
At the total station, the accuracy of determining a point of detail depends on the accuracy of the points of the topographic network from which the detail points are measured and calculated.
For GNSS receivers the accuracy of the determination depends on the measurement method: static or RTK. In Romania there is a network of permanent stations that can transmit the corrections to satellites, so that in areas with a mobile phone signal, real-time positions can be determined with very good accuracy. Disadvantages are in areas with obstructions, where the sky is not clear or there are signal disturbances. [1]
The orthophotoplan is useful depending on the scale. The one obtained from UAVs is very good, but in areas with vegetation it cannot be used. Also, in these areas it is not possible to obtain altitudes. The planimetric accuracy is good.
The point cloud is useful, but requires special processing programs. The accuracy is superior.

When carrying out the systematic cadastre, there are provisions regarding the precision of determining the detail points that make up the outline of the immobile. Thus, at the last auction on this topic we have the following clarifications:
Acquisition of systematic registration services in the Integrated Cadastre and Land Registry System of the immobile from 261 ATUs, within the major project "Increasing the coverage and inclusion of the property registration system in rural areas in Romania"

SECTION II - TASK BOOK
1.3.2.2 Identifying the boundaries of immobile
Art. 27 (1) The systematic registration works are performed by using the graphical representation of the ATU boundaries and the urban boundaries held by OCPI.
(2) The precisions to be provided are those corresponding to the 1:5000 scale plan for the extraurban and the 1:2000 scale plan for the intraurban areas.

As we have specified, in cadastral works there are often problems related to the accuracy of the determination of the surfaces and the way in which this precision was calculated.

The Equation that gives us the accuracy of reading a point on plans and maps is:

$$\frac{e}{P_p} = \frac{1}{N} \tag{2.1}$$

namely:

$$P_p = e\,N \tag{2.2}$$

or:

$$P_p = e\,N\,10^{-3} \tag{2.3}$$

In which:
- P_p - Graphic reading accuracy on plans and maps;
- the number 10-3 is the factor for transforming accuracy into meters;
- N - the denominator of the scale of the plan or map on which the reading is made;
- e - the approximation (accuracy) with which we can read with the ruler on plans and maps of 0.1 millimeters.

For the 1:2000 maps, i.e. intraurban areas:

$$P_p = 0.1 * 2{,}000 * 10^{-3} \ = 0.2\, meters \tag{2.4}$$

Or: $P_p = \pm 0.10$ meters.
Like in Malaysia. [4]

3 Theoretical considerations.

3.1 Tolerance

The tolerance is the maximum limit set for an error of a measurement, provided in the technical instructions, for the acceptance of the result of this measurement.

The immobile can have different shapes on the land, depending on the geographical area in which they are located, or the mentality of the inhabitants of that area. For example, in the countries of northern and western Europe, the shape of the land is usually geometric, and can be easily assimilated with geometric figures. Figure 2.

There are also many objects on the field that have curved line boundaries, which can be described by points as often as possible determined on the curved line.

Figure 2 - Immobile outline in Denmark and immobile outline in Romania

To establish a Equation for calculating the tolerance for determining surfaces, we considered an area of 1 hectare in three forms: square, rectangle and circle. Figure 3.

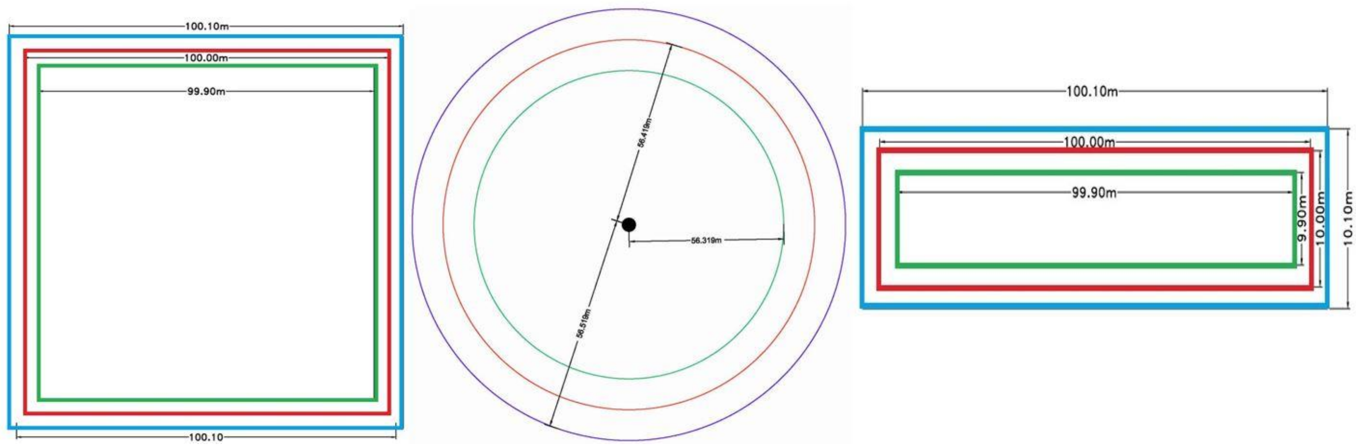


Figure 3 - Simple geometric figures with an area of 1 hectare

3.1.1. Considering the square, each side is 100 meters. The accuracy of determining each of the 4 points that define the shape is ± 10 centimeters, i.e. ± 0.1 meters. The application of precision on each point results in three squares (Figure 3). The blue square will have sides of 100.20 meters, the red square of 100 meters, and the green square 99.80 meters. The surfaces of the three squares, according to the known Equation (Equation 3.1): the square side, results from 10040.04, 10000 and 9960.40 square meters.

$$S = L^2 \tag{3.1}$$

For the same figure, we will calculate the surface difference due to the measurement error by using the perimeter. Thus, the perimeter of the initial square is 400 meters. Multiplying the perimeter by the measurement error, results in 40 square meters (Equation 3.2). In Figure 3 it is the shaded area on either side of the original square.

$$D = P\,P_p = 400 * 0.1 = 40\,square\ meters \tag{3.2}$$

In which:

- D – the surface difference;
- P – the perimeter;
- P_p – precision of determination of the contour elements.

This value means the tolerance of the surface to be calculated.
Adding this area to the area of the square, results in 1040 square meters, i.e. the area of the outer square. Subtracting the same area of 40 square meters from the area of the square, it results in 9960 square meters, i.e. the area of the inner square.
In conclusion, for a square surface, the measurement error of ± 0.1 meters gives us a maximum value of 0.4% error in the calculation of the surface.

3.1.2. Considering the rectangle, the most unfavorable case is a length of 1000 meters and a width of 10 meters. Applying the precision, as in the previous case, three rectangles result (Figure 3). The blue rectangle has a length of 1000.20 meters and a width of 10.20 meters, the red one 100 meters by 10 meters, and the green one 999.80 meters by 9.80 meters. Applying the known formula (Equation 3.3), the length multiplied by the width, the areas of 10202.04, 10000 and 9798.04 square meters are obtained.

$$S = L\,l \qquad (3.3)$$

The perimeter of the initial square is 2020 meters. Multiplying the perimeter by the measurement error, results in 202 square meters (Equation 3.2).

This value represents the tolerance of the surface to be calculated.

Adding this area to the area of the square, results in 10202 square meters, i.e. the area of the outer rectangle. Subtracting the same surface of 202 square meters from the area of the square, results in 9798 square meters, i.e. the surface of the inner square.

In conclusion, on a rectangular surface the measurement error of ± 0.1 meters gives us a maximum value of 2% error in the calculation of the surface.

3.1.3. Considering the circle, at an area of 10,000 square meters results a radius of about 56.419 meters and the circumference of 354.491 meters (Equation 3.4 and 3.5).

$$S = \pi R^2 \;\Rightarrow\; R = \sqrt{\frac{S}{\pi}} \qquad (3.4)$$

$$C = 2\,\pi\,R \qquad (3.5)$$

Following the same principle as above, three circles result. The blue circle has a radius of 56.519 meters and a circumference of 355.12 meters, the red one has a radius of 56.419 meters and a circumference of 354.491 meters and the green one has a radius of 56.319 meters and a circumference of 353.863 meters (Figure 3). Applying the Equation for calculating the surface of the circle, the values are: 10035.50, 10000.01 and 9964.60 square meters.

Calculating the difference in surface by multiplying the circumference with the precision, it results in 35.45 square meters (Equation 3.2). The circumference of the circle is actually a perimeter.

This value represents the tolerance of the surface to be calculated.

Applying this difference, it results in a maximum area of 10035.46 and a minimum of 9964.57 square meters.

In conclusion, for a rectangular area, the measurement error of ± 0.1 meters gives us a maximum value of 0,35% error in the calculation of the area.

3.1.4. For any polygon, the tolerance in which the calculation of an area can be included is also calculated with the Equation 3.2, respectively by multiplying the perimeter with the precision of determining the contour points. The perimeter results from a simple command in the case of CAD type programs.

3.2 Precision

Accuracy is a value that indicates the degree of confidence of a direct or indirect measurement. The accuracy must be within the calculated tolerance, depending on the accuracy of the instrument, the operator's experience, the environmental conditions or the software used.

In the case of the determination of polygonal areas, the formulas for the case of indirect measurements shall apply. Indirect measurements are known to come from direct measurements by applying calculation formulas (Equation 3.6). Their accuracies depend on the directly measured quantities and the accuracies with which they were determined. [2]

$$F = F(m_1, m_2, \dots \dots \dots m_i) \tag{3.6}$$

In which:

- F – is the function by which the indirect measurement results;
- m_i – is the size measured directly.

The accuracy of this function, in relation to the probable errors of the directly measured quantities is:

$$s_F^2 = \left(\frac{\partial F}{\partial m_1}\right)^2 s_{m_1}^2 + \left(\frac{\partial F}{\partial m_2}\right)^2 s_{m_2}^2 + \cdots \dots \dots .. \left(\frac{\partial F}{\partial m_i}\right)^2 s_{m_i}^2 \tag{3.7}$$

In which:

- F – it is the function for which its accuracy is calculated;
- s_F – it is the function for which its accuracy is calculated;
- s_{m_i}– directly determined size error.

3.2.1. The case of the square with the side of 100/100 meters.

Equation 3.1 is the formula for determining the surface of a square (indirect function):

Accuracy, calculated by Equation 3.7:

$$m_S^2 = m_L^2 \left(\frac{\partial S}{\partial L}\right)^2 = m_L^2 (2L)^2 \tag{3.8}$$

$$m_s = \pm 20\ square\ meters$$

The surface of the square is: 10000 ± 20 square meters.

Tolerance: ± 40 square meters.

3,2,2. The case of the rectangle with the dimensions 1000/10 meters.

Equation 3.3 is the formula for determining the surface of the square (indirect function):

Accuracy, calculated by Equation 3.7:

$$m_S^2 = m_L^2 \left(\frac{\partial S}{\partial L}\right)^2 + m_l^2 \left(\frac{\partial S}{\partial l}\right)^2 = m_L^2 (l)^2 + m_l^2 (L)^2 \tag{3.9}$$

$m_s = \pm 100.005\ square\ meters$

The surface of the rectangle is: 10000 ± 100.005 square meters.
Tolerance: ±202 square meters.

3.2.3. The case of the circle with dimensions: radius 56.419 meters and circumference of 354.491 meters.
The formula for determining the surface of the circle (indirect function) is Equation 3.4. Accuracy, calculated by Equation 3.7:

$$m_S^2 = m_R^2 \left(\frac{\partial S}{\partial R}\right)^2 = m_R^2 (\pi 2R)^2 \tag{3.10}$$

$m_s = \pm 35.45\ square\ meters$

The surface of the circle is: 10000 ± 35.45 square meters.
Tolerance: ± 35.45 square meters.

3.2.4. The case of any polygon.
The formula for calculating the surface of any polygon divides that polygon into triangles and it results the Equation:

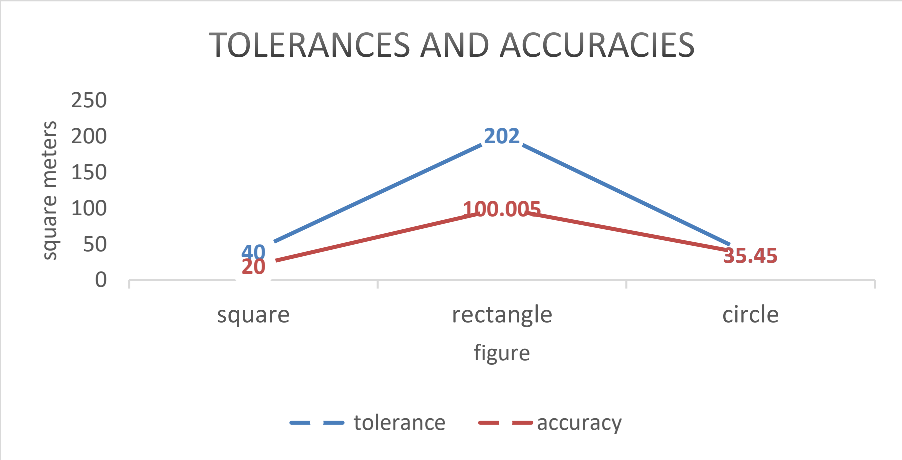
$$2S = \sum_1^n x_i\,(y_{i+1} - y_{i-1}) \tag{3.11}$$

Where x_i and y_i are the planimetric coordinates of each point on the contour of the polygon (Figure 3). Basically, the polygon is divided into triangles with the tip at a point on the contour. If we used this formula to calculate the accuracies, the accuracy of each triangle should be calculated. This would not be a correct solution because of each triangle only the side that defines the polygon is real, the other two being built to be able to define the triangle. For this reason, we considered the polygon reduced to a circle in which the perimeter is the circumference, and the radius results from Equation 3.4.

4 Conclusions

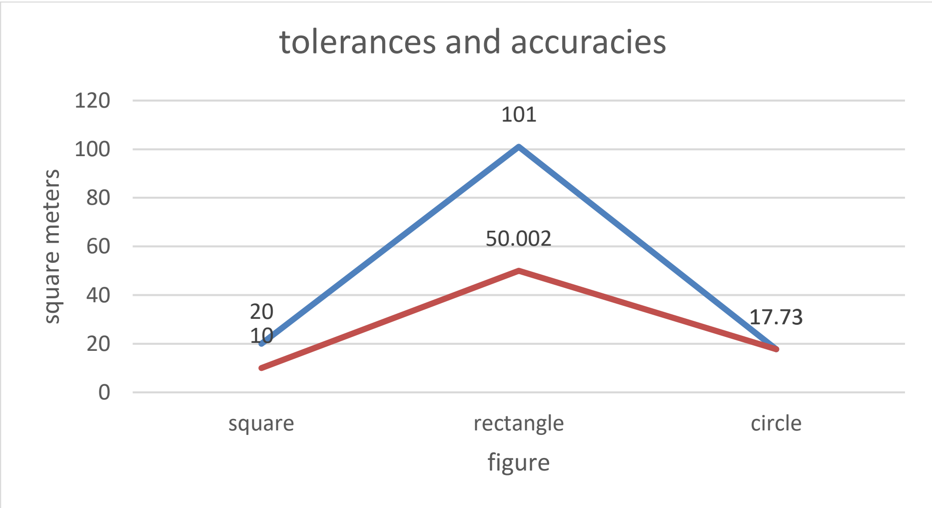
It is easy to notice that, having as precision the determination of the contour points the value of ± 10 centimeters, each type of resulting geometric figure has a different tolerance and precision. Thus, the optimal configuration is the circle, and the unfavorable configuration is a square with a very small side relative to the other side.
From graph 1 it can be seen that between accuracy and tolerance there is a similarity in absolute values.

Graph 1, Tolerances and accuracies if the accuracy of determining a point on the contour is ± 10 centimeters



For irregular polygons, the proposed solution is the transformation into a circle, in which the perimeter is the circumference and the radius results from Equation 3.4.
With the help of the formulas set out above, it is possible to prove, both to the beneficiaries and in court, what is the degree of trust we place in the calculation of a surface when there are differences between two or even three determinations of the same surface.
Of course, the formulas can be used with other values for the degree of accuracy with which the contour points are measured. In our case, the ANCPI requirement of ± 10 centimeters was taken into account. For more accurate measurements, made with GNSS receivers, total stations, pixels resulting from photogrammetric flight or cloud of LiDAR points, errors of a few centimeters can be used. In this case, both the tolerance and the accuracy of the surface determination decrease (Graph 2). As an example, in graph 2 we considered the measurement accuracy ± 5 centimeters.

Graph 2 - Tolerances and accuracies if the accuracy of determining a contour point is ± 5 centimeters.



The conclusion is that the tolerance and accuracy of determining the surfaces is related to the accuracy of determining the position of the contour points. The smaller its value, the more precisely the surfaces are determined and the smaller the differences between two measurements.

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