FIG Working Week 2016

CHRISTCHURCH, NEW ZEALAND 2–6 MAY 2016

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Empirical Models of Vertical Crustal Motion in the Great Lakes Region

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- A new geoid-based reference surface (N) for physical heights (H) adopted in Canada in November 2013
  - Use GPS/GNSS ellipsoidal height (h) and N to obtain H
- The one centimetre-level geoid error requires the time variation of it be accounted for on a decadal time scale.
- The crustal motion should also be accounted for on a shorter time scale so that the equation $H = h - N$ holds both in space and time.
- Why the Great Lakes region?
  - Glacial isostatic adjustment of the crust and geoid
  - Line of zero motion, an important constraint for geodynamic modelling
  - Concentration of geodetic control stations (CGPS and EGPS data)

CONTENT

- Objectives
- Data sets
  - GRACE vertical motion rates
  - Filtering effects on GRACE rates
  - GPS vertical crustal velocities
  - Overview of the errors of the two data sets
- Least-squares adjustment model
- Analysis of the combined vertical motion surface
- Discussion

OBJECTIVES

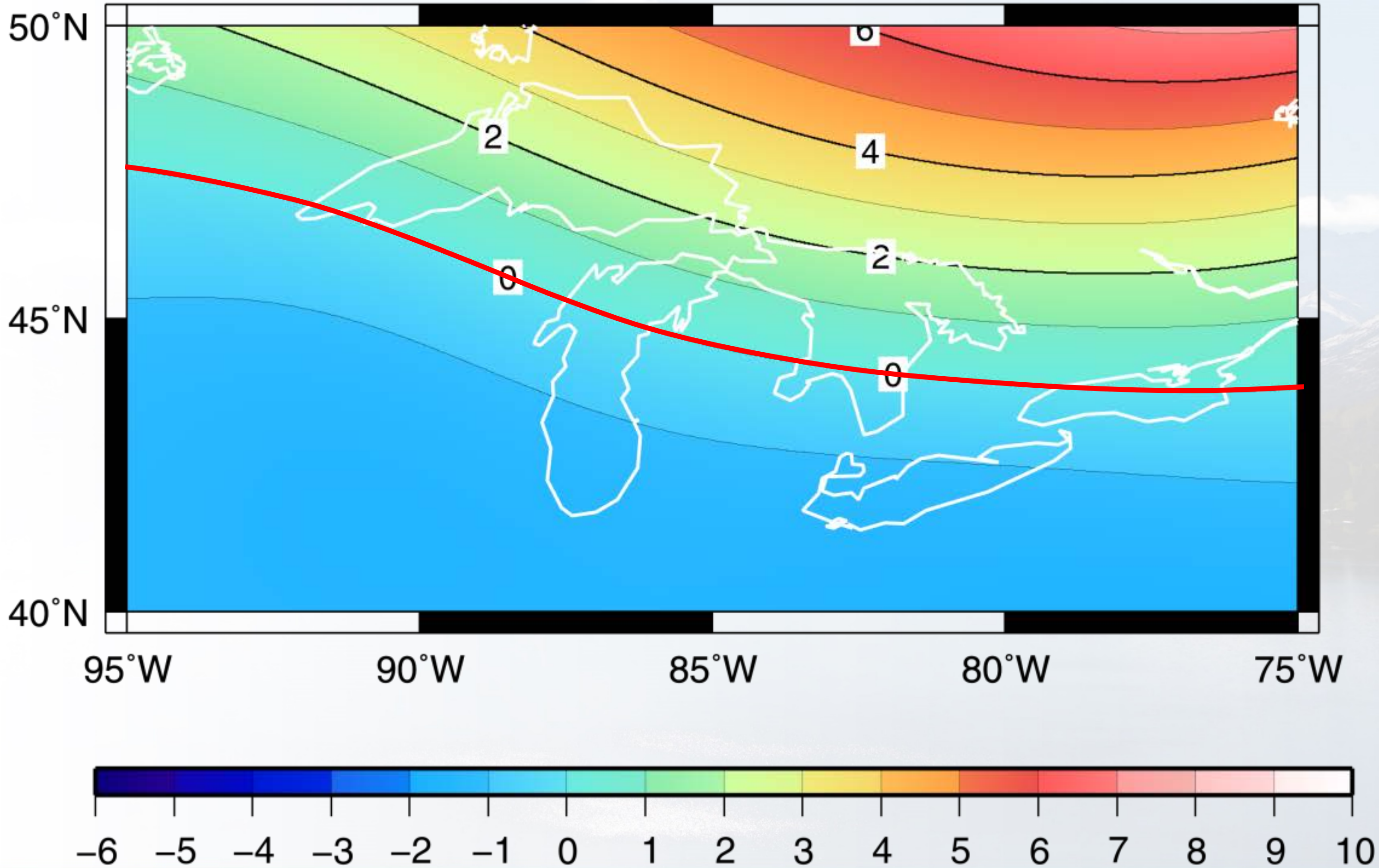
- Combine optimally the available heterogeneous vertical crustal motion data
- Calibrate data variance-covariance matrices
- Assess whether the GRACE and GPS vertical velocities converge

GRACE VERTICAL MOTION RATES

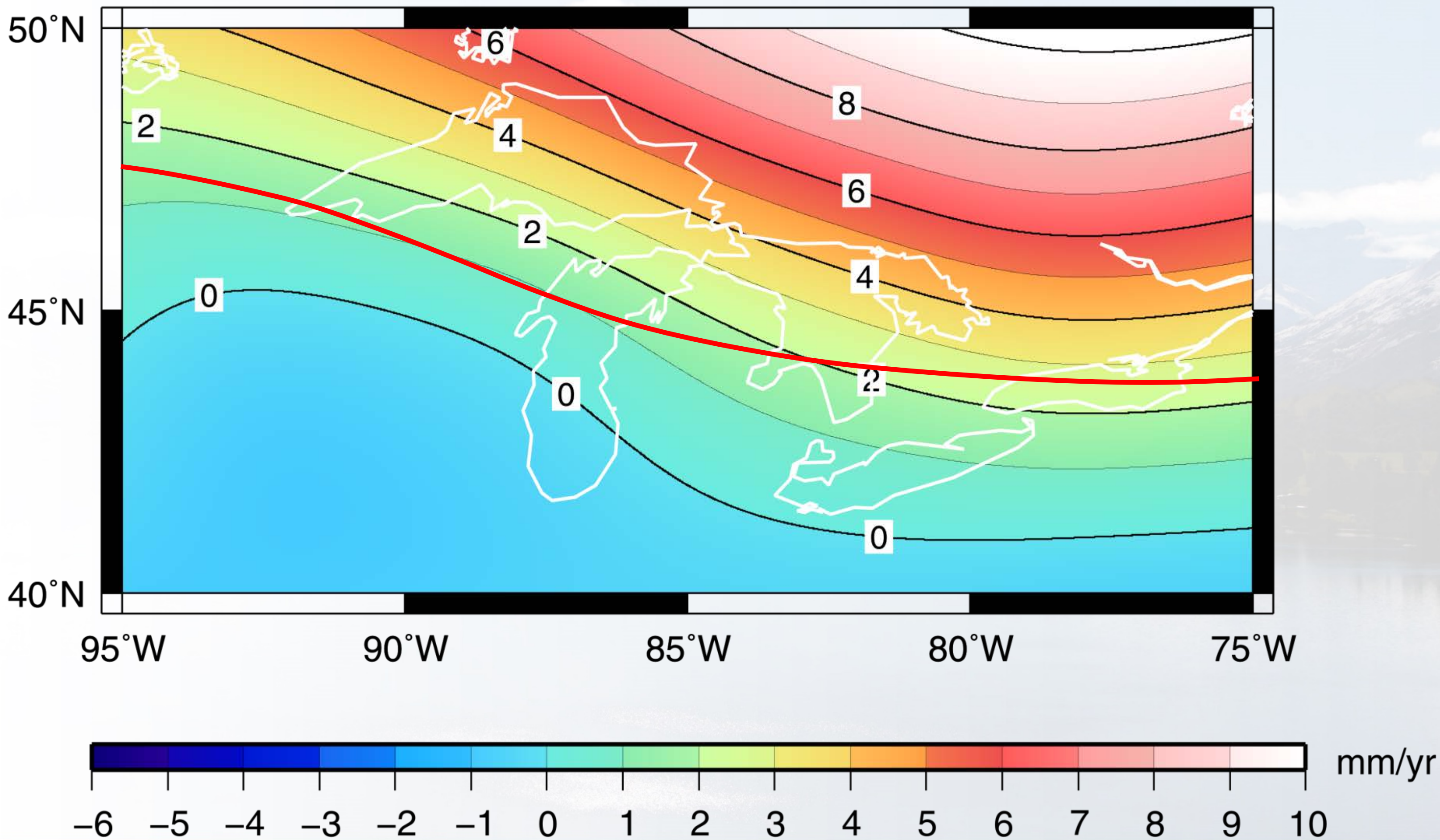
- 144 months of CRS RL05 GRACE data (April 2002 to August 2015)
- The mean field is subtracted from the time series
- GLDAS hydrology model correction
- De-striping filter and isotropic smoothing (a 400 km filter radius)
- Vertical rates of crustal motion calculated by a second isotropic filter

$$\dot{h}(\varphi,\lambda) = R\sum_{l=2}^{50}\frac{2l+1}{2}W_l\sum_{m=0}^{l}P_{lm}(\sin\varphi)\times\left[\dot{C}_{lm}\cos(m\lambda)+\dot{S}_{lm}\sin(m\lambda)\right]$$

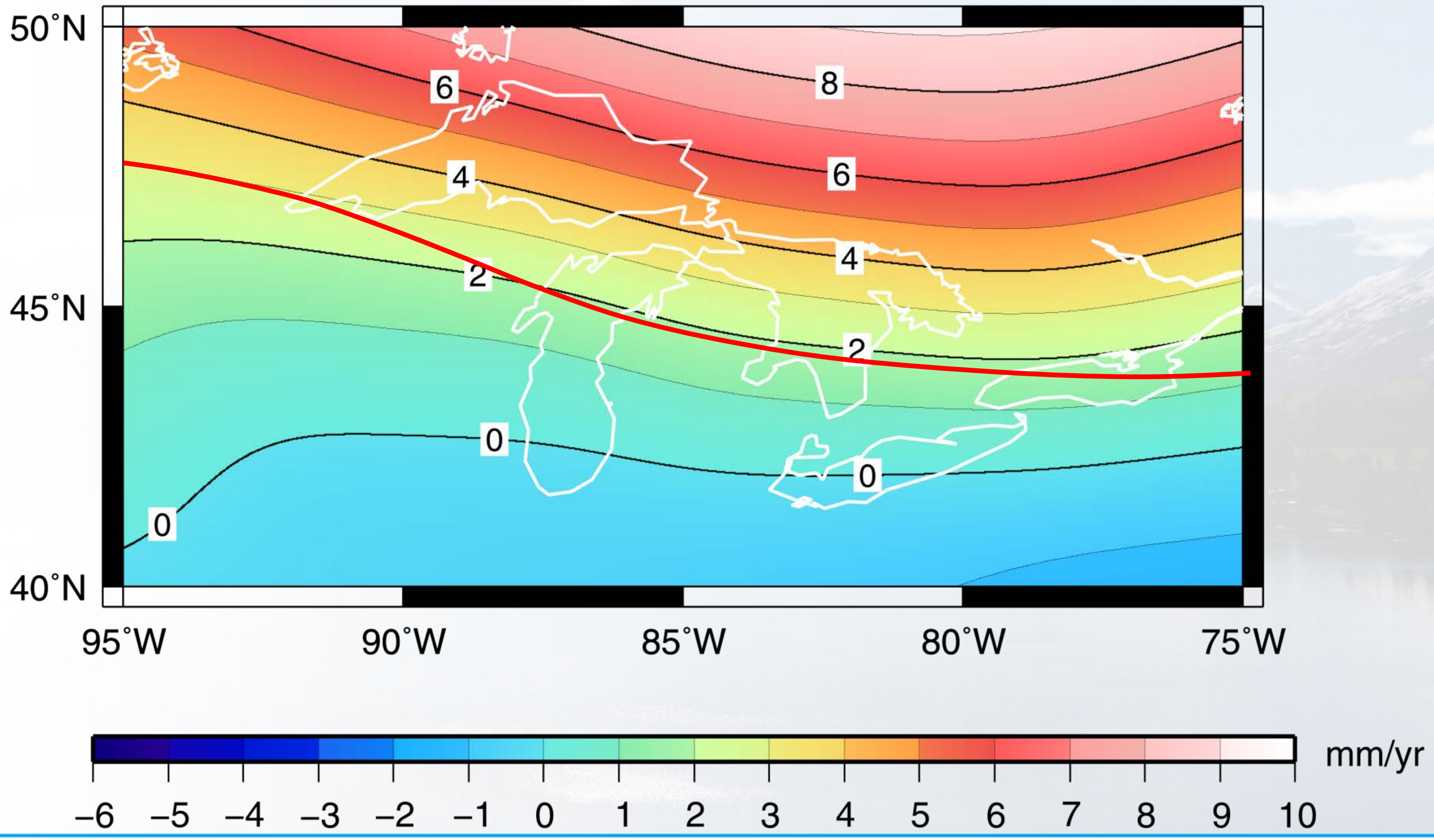
GIA model: ICE-6G_C (VM5a) by Peltier et al.



GRACE model 1 (UofC)

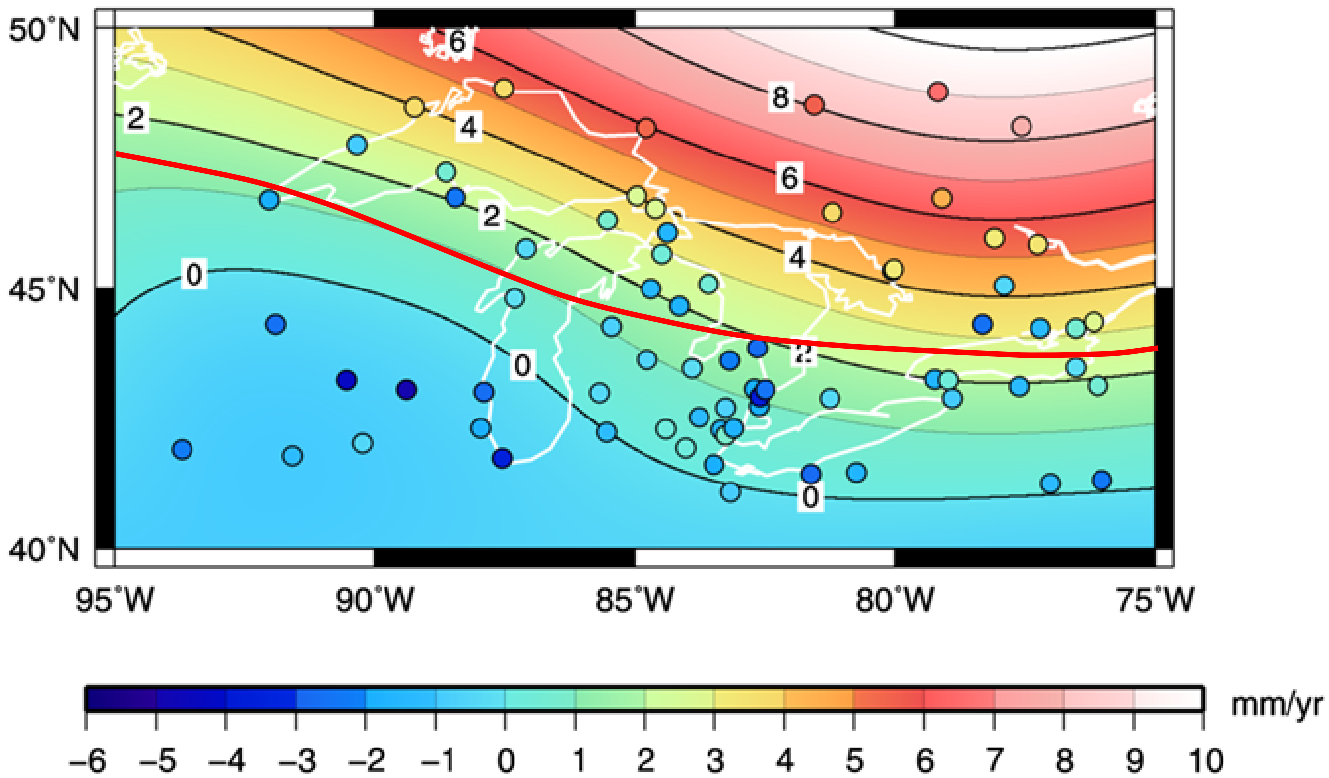


GRACE model 2 (UofC)



GPS VERTICAL VELOCITIES

- 71 GPS points (IGb00) in both Canada and USA
  - Sella et al., (2007): 57 CGPS and 14 EGPS stations (Canada)



OVERVIEW OF DATA ERRORS

- Long-wavelength errors in the GRACE data
  - Geophysical signals leakage and hydrology model errors
- Distortions of the GRACE-derived vertical motion surface
  - introduced by the de-striping and smoothing filters
- Different reference epochs and time span of data series
- Scale factors of variance-covariance (VC) matrices not known
- The GRACE vertical motion VC matrix is fully populated

LS Adjustment Model

$$\min\|\mathbf{l}-\mathbf{A}\mathbf{x}\|^2, \quad \mathbf{C}_l$$

$$\mathbf{l}=\mathbf{A}\mathbf{x}+\mathbf{v}$$

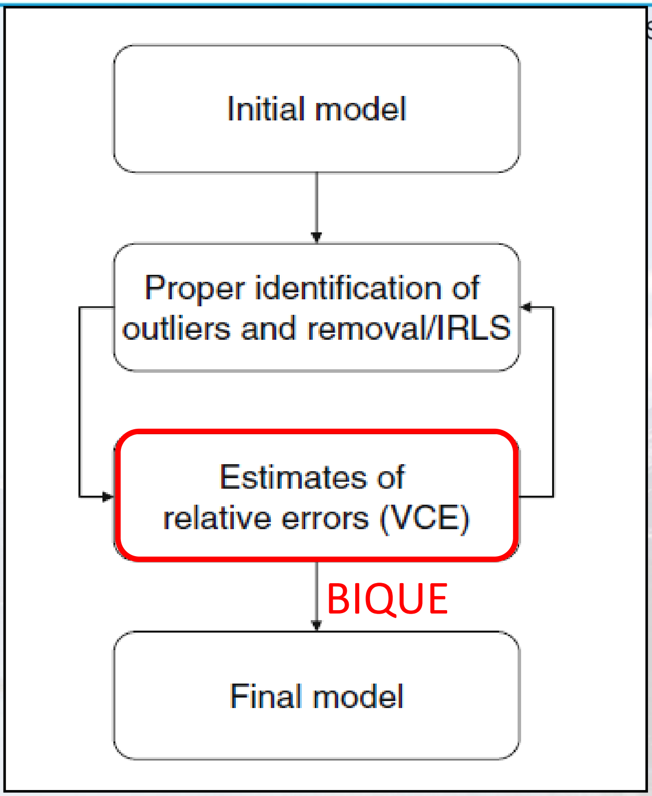
$$\mathbf{l}=\left[\mathbf{l}_{GRACE}^T \quad \mathbf{l}_{GPS}^T\right]^T$$

$$\mathbf{l}_{GRACE}=\left[\dot{h}_1^{GRACE}\,\dot{h}_1^{GRACE}\ldots\dot{h}_n^{GRACE}\right]^T, \quad (n\times 1)$$

$$\mathbf{l}_{GPS}=\left[\dot{h}_1^{GPS}\,\dot{h}_1^{GPS}\ldots\dot{h}_n^{GPS}\right]^T, \quad (n\times 1)$$

Stochastic Model

$$\mathbf{C}_l=\begin{bmatrix}(\sigma^2\mathbf{Q})_{GRACE} & 0\\ 0 & (\sigma^2\mathbf{Q})_{GPS}\end{bmatrix}$$



Parametric Model

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}_0 & \mathbf{A}_{GRACE}\\ 0 & \mathbf{A}_{GPS}\end{bmatrix}$$

$$\mathbf{A}_{GRACE,GPS}=[\Phi(r_{ij})],\ i=1,\ldots,n,\ j=1,\ldots,m$$

$$(\mathbf{a}_0)_i=[1 \quad \varphi_i-\bar{\varphi} \quad \lambda_i-\bar{\lambda}], \quad i=1,\ldots,n$$

Vector of unknown bias&tilt and weights: $\mathbf{x}=[\mathbf{x}_o \quad \mathbf{x}']^T$

Velocity of a new grid point: $\dot{h}_p=\mathbf{a}_p^T\hat{\mathbf{x}}'=\sum_{j=1}^{m}\Phi_{pj}\hat{x}_j$

Inverse mutliquadric base function: $\Phi(r_{ij})=(r_{ij}^2+c^2)^{-0.5}$

Iterative LS procedure

$$\hat{\mathbf{x}}^{(k+1)} = (\mathbf{A}^T \overline{\mathbf{W}}^{(k)} \mathbf{A})^{-1} \mathbf{A}^T \overline{\mathbf{W}}^{(k)} \mathbf{l},\ \hat{\mathbf{v}}^{(k)} = \mathbf{l} - \mathbf{A}\hat{\mathbf{x}}^{(k)}$$

$$\overline{\mathbf{W}}^{(k)} = \mathbf{C}_l^{-1} \mathbf{W}^{(k)}$$

$$\mathbf{W}^{(k)} = diag(w_1^{(k)},\ldots,w_i^{(k)},\ldots,w_n^{(k)})$$

$$\mathbf{C}_l = diag(\sigma_1^2,\ldots,\sigma_i^2,\ldots,\sigma_n^2)$$

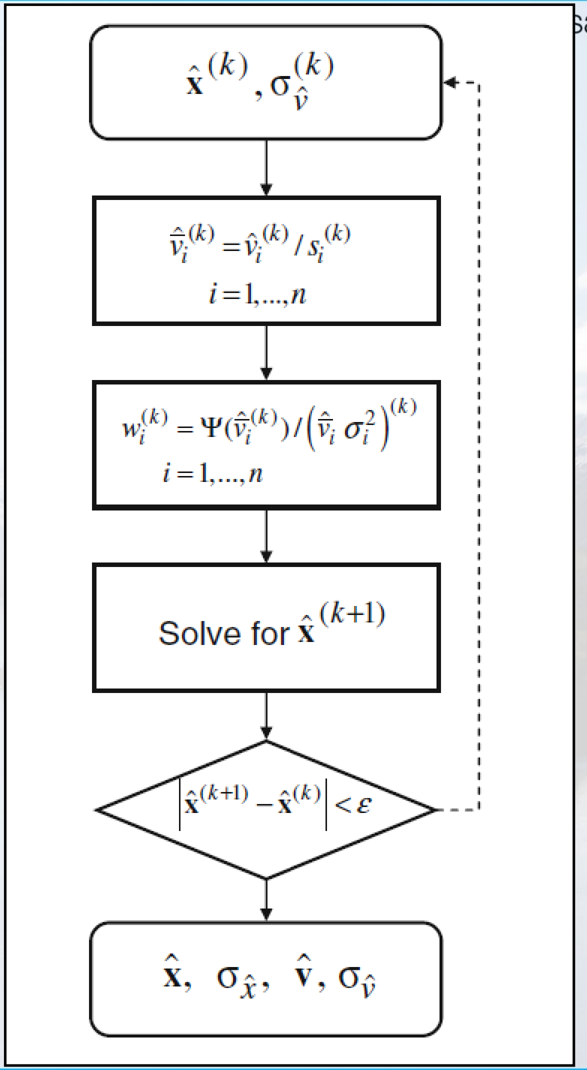
Computation of weights

$$w_i^{(k)} = \Psi\left(\hat{\bar{v}}_i^{(k)}\right) / \hat{\bar{v}}_i^{(k)} \qquad \hat{\bar{v}}_i^{(k)} = \hat{v}_i^{(k)} / s$$

Fair influence function

$$\Psi\left(\hat{\bar{v}}_i^{(k)}\right) = \hat{\bar{v}}_i^{(k)} / \left(1 + \left|\hat{\bar{v}}_i^{(k)}\right| / F\right), \quad F = 1.4$$

MAD estimator $s = med\{|\,\hat{\mathbf{v}} - med\{\hat{\mathbf{v}}\}\,|\}.$



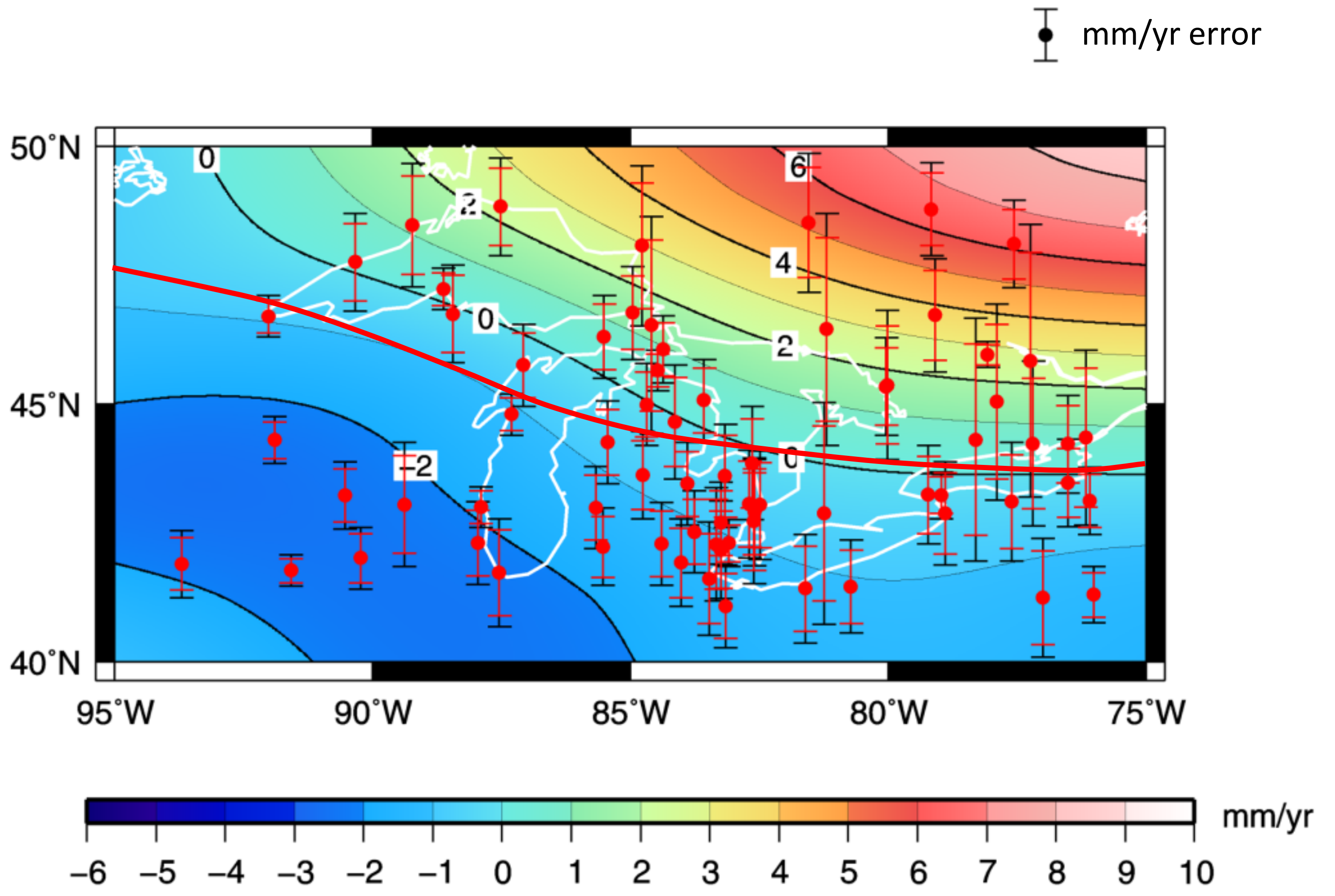
Estimated GRACE data bias in mm/yr and tilt in mm/yr/deg

Method	Bias	NS tilt	EW tilt
Least-squares adjustment	2.10 ± 0.10	0.12 ± 0.07	0.04 ± 0.04
Iterative re-weighting least-squares	2.06 ± 0.15	0.09 ± 0.07	0.04 ± 0.04

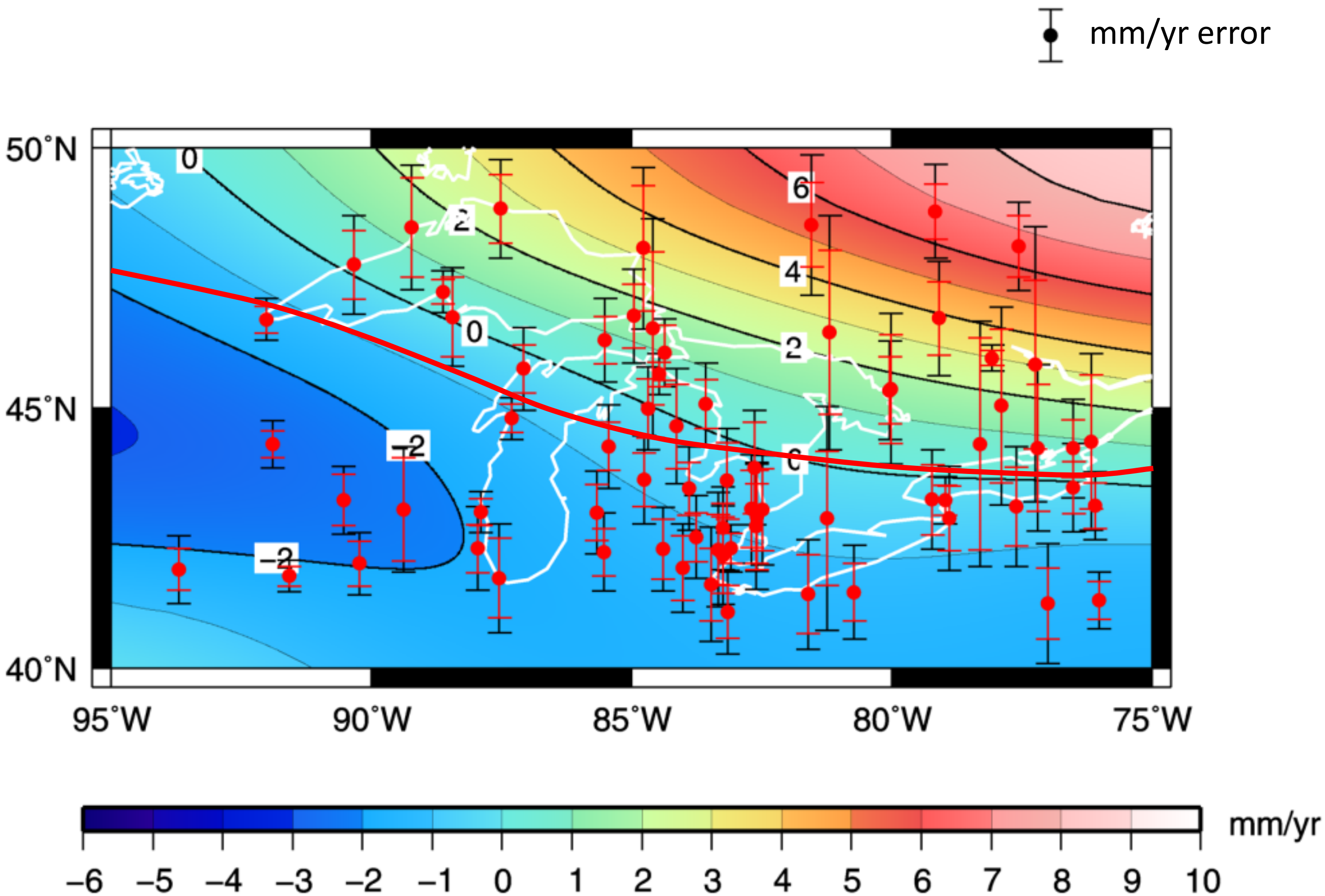
Statistics of the a-posteriori errors of GRACE and GPS velocities in mm/yr

Data set	Min	Max	Mean
A priori errors			
GRACE	0.6	0.6	0.6
GPS	0.5	5.3	2.0
Least-squares adjustment			
GRACE	0.2	0.2	0.2
GPS	0.4	4.2	1.7
Iterative re-weighting least-squares			
GRACE	0.5	0.8	0.6
GPS	0.3	4.1	1.3

The LSA vertical motion surface with the a-priori (black) and a-posteriori (red) errors of the GPS vertical velocities.



The IRLS vertical motion surface with the a-priori (black) and a-posteriori (red) errors of the GPS vertical velocities.



DISCUSSION

- The line of zero motion in the lakes area is well constrained by the geodetic observations.
- If outliers are present in the data, these data points are down-weighted and preserved in the optimal combination
  - Baarda's data snooping can test a good observation as an outlier or may fail to detect a single outlier in peripheral areas with less data constraints.
  - The pattern of vertical motion surface could change globally because base functions are global.
  - IRLS keeps more data constraints in the peripheral areas.

DISCUSSION

- The increased time span of the GRACE mission has lead to vertical motion rates that converge to GPS velocities
  - The estimated GRACE bias has decreased by 2 mm/yr due to the additional 5 years of data since the previous study.
  - GRACE tilt became less significant
    - NW tilt: -0.21±0.08 (8 years of data)
    - NW tilt: 0.12 ± 0.07 (13 years of data)
  - The spread of residuals decreased
    - GRACE: from ±0.4 mm/yr to ±0.2 mm/yr
    - GPS: from ±1.5 mm/yr to ±1.3 mm/yr

Peltier, W.R., Argus, D.F., and Drummond, R., 2015, Space geodesy constrains ice age terminal deglaciation: The global ICE-6G_C (VM5a) model. J. Geophys. Res. Solid Earth 120, 2014JB011176.

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Sella, G.F., Stein, S., Dixon, T. H., Craymer, M., James, T.S., Mazzotti, S., Dokka, R.K., 2007, Observation of glacial isostatic adjustment in “stable” North America with GPS, Geophys. Res. Lett., 34:L02306, doi:10.1029/2006GL027081.

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THANK YOU!

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