Realistic Uncertainty Estimation of the Market Value Based on a Fuzzy-Bayesian Sales Comparison Approach

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RESULTS OF A DFG-FINANCED PROJECT

Motivation

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- In the property valuation, standardised methods are established in Germany.
- The less information is available, the more challenging is the valuation!
  - Experts decide about the market value only by their expertise and by (statistical) analysis of few existing data
  - Former approaches based on random variables: pure aleatory approaches

Scource: heimundhaus.net

Aims und Methodological Approach

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- The focus of this research:
  True and reality best approximating mathematical formulation of real relationships
- Aims of the evaluation process:
  - Quantifying uncertainties that arise in the estimation results for unknown model variables
  - Reducing and propagating errors
- Methodological approach:
  - Bayesian theory for aleatory uncertainties
  - Fuzzy theory for epistemic uncertainties

Scource: bettybrigade.com

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Agenda

- Motivation
- Fuzzy variables in Real Estate Valuation
- Approach of a Fuzzy Model
- Fuzzy-Bayes-Model for Real Estate Valuation
- First Results
- Conclusion and Further Work

Used Submarkets

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Spatial Submarket:

- Osnabrueck region of Germany
- Heterogeneous:
  - City of Osnabrueck is urban
  - Region in the north of the city is rural
- Transactions from the automated purchase price collection of the experts' committee.
  - 280 cases of purchase in 2007 (blue circles): prior information
  - 135 purchase of 2008: likelihood data (red circles)

Used Submarkets

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Functional submarket:

- Free-standing single and two-family homes
- Independent variable: Purchase price per living area
- Significant dependent variables:
  - Area of the lot,
  - Standard land value,
  - Age of the building,
  - Area of living space and
  - Equipment

Which uncertainty have these variables?

Difference between aleatory and epistemic variables

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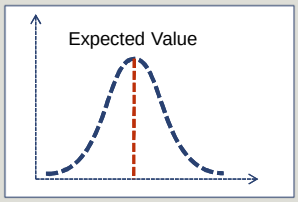
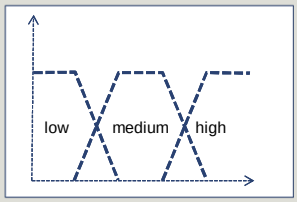
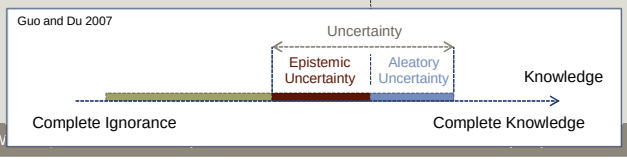
Aleatory Variable	Epistemic Variable
• Random Variable: Monte-Carlo	• Interval-mathematical: Fuzzy Variable







Fuzzy variables in Real Estate Valuation

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- Valuation is characterised by uncertainties
  - In Practice:
    Rounding of market value indicates the uncertainty
  - Normally, no specification of uncertainty in assessment

Aleatoric uncertainty
- Is caused by the random variability of the system
- Inherent random nature of the physical quantities

Epistemic uncertainty
- Results from the lack of knowledge about the system, simplifications or the limited availability of data

Fuzzy variables in Real Estate Valuation

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- With focus on target of market value:
  - **Aleatoric uncertainties** ⟶ e. g. area of a lot or area of living space
    - Also unknown, but are subject due to calculation method by a fundamental variability
    - Increasing the number of comparative purchases increases their accuracy
  - **Epistemic uncertainties** ⟶ e. g. location or remaining period of an economic usability of the building
    - Unknown, but in principle to accept as proof
    - Determination is influenced by experts' expertise

From Regression to a Bayes-Fuzzy Model

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Classical and Bayesian Regression

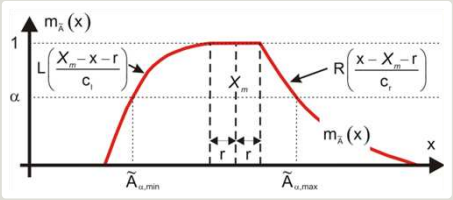
- Classical multiple linear regression
  - Established in valuation practice (sales comparison approach)

$$y = X\beta + \varepsilon$$

- Bayesian Regression
  - Developed and validated for use in valuation

$$\beta = (X'X + V^{-1})^{-1} (X'y + V^{-1}\beta)$$

Fuzzy Approach



- Fuzzy interval is uniquely defined by
  - Its membership function over a set of real numbers
  - With a membership degree between 0 and 1

Uncertainty Modelling Fuzzy

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- Uncertainty Modelling for the equation

$$y = f(z_1, z_2, ..., z_n) = f(\boldsymbol{z})$$

- Occurring uncertainties:
  - Aleatoric Errors → **Law of variance propagation / Monte Carlo approach**
  - Epistemic Errors → **Interval mathematics Fuzzy theory**

Different treatment of aleatoric and epistemic errors

Uncertainty Modelling (Fuzzy)

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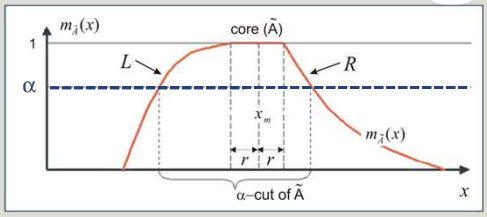
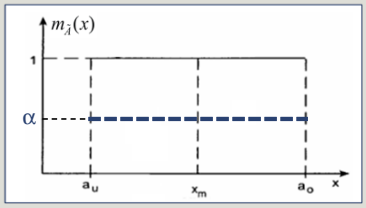
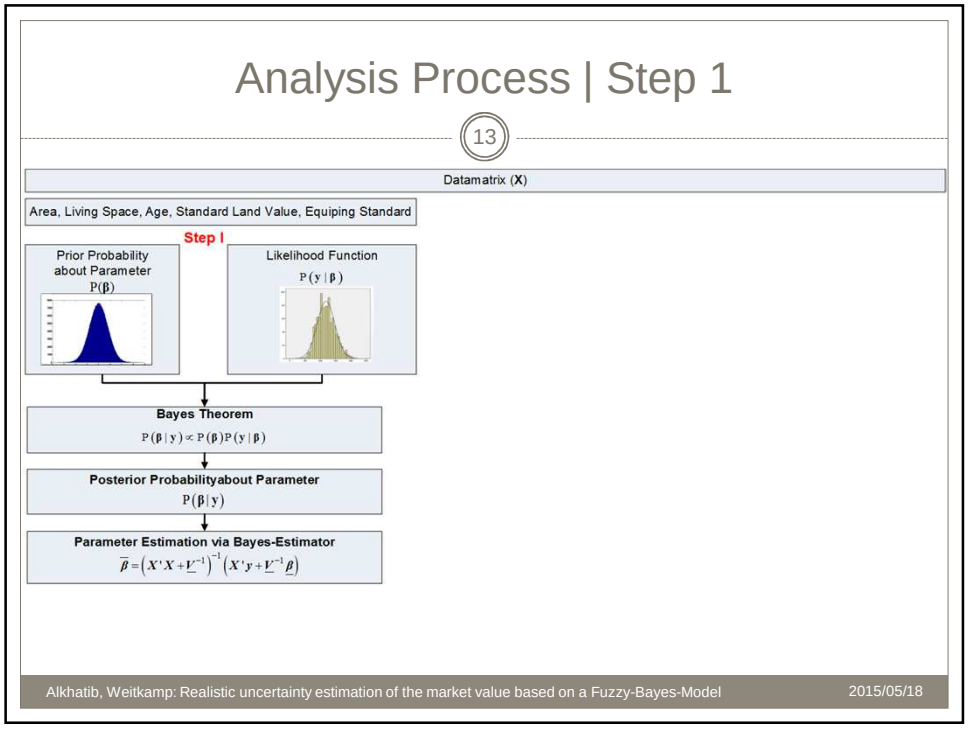


$\tilde{A}$	Fuzzy interval
x_m	Mean value
r	Radius
L (.)	Left reference fct.
R (.)	Right reference fct.

Example: An interval describing a systematic error (error band)



Non-probabilistic interpretation of systematic errors


Analysis Process | Step 1
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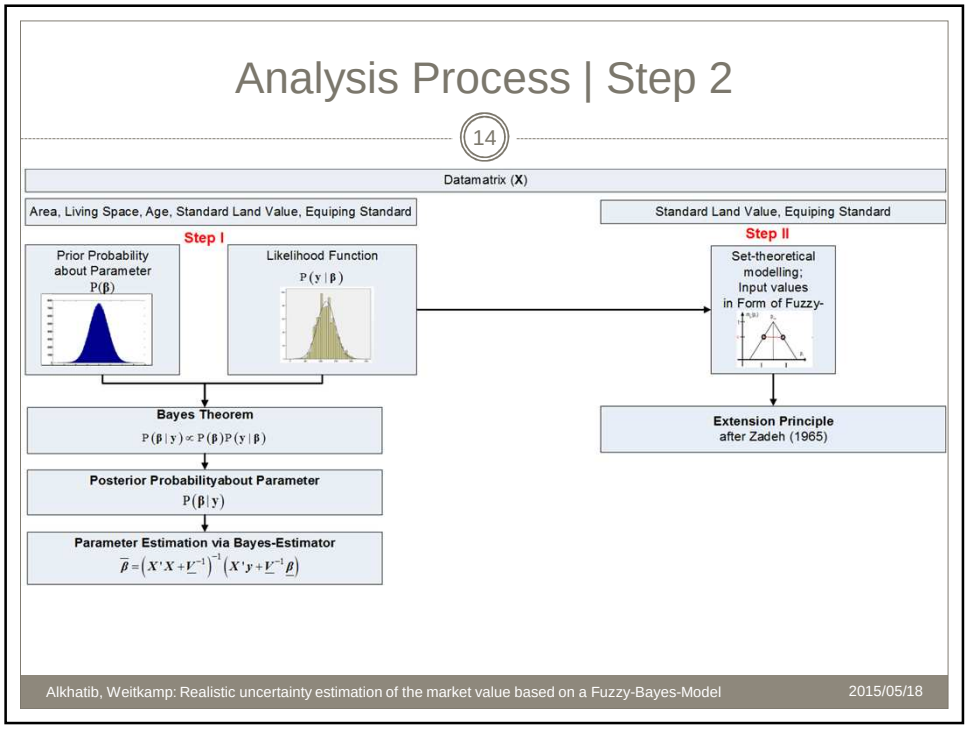
Datamatrix (X)
Area, Living Space, Age, Standard Land Value, Equiping Standard
Step I
Prior Probability about Parameter P(β)
Likelihood Function P(y | β)
Bayes Theorem
P(β | y) ∝ P(β)P(y | β)
Posterior Probabilityabout Parameter
P(β | y)
Parameter Estimation via Bayes-Estimator
Alkhatib, Weitkamp: Realistic uncertainty estimation of the market value based on a Fuzzy-Bayes-Model
2015/05/18


Analysis Process | Step 2
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Datamatrix (X)
Area, Living Space, Age, Standard Land Value, Equiping Standard
Step I
Prior Probability about Parameter P(β)
Likelihood Function P(y | β)
Standard Land Value, Equiping Standard
Step II
Set-theoretical modelling; Input values in Form of Fuzzy-
Bayes Theorem
P(β | y) ∝ P(β)P(y | β)
Posterior Probabilityabout Parameter
P(β | y)
Parameter Estimation via Bayes-Estimator
Extension Principle
after Zadeh (1965)
Alkhatib, Weitkamp: Realistic uncertainty estimation of the market value based on a Fuzzy-Bayes-Model
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Analysis Process | Fuzzy-Bayes-Step

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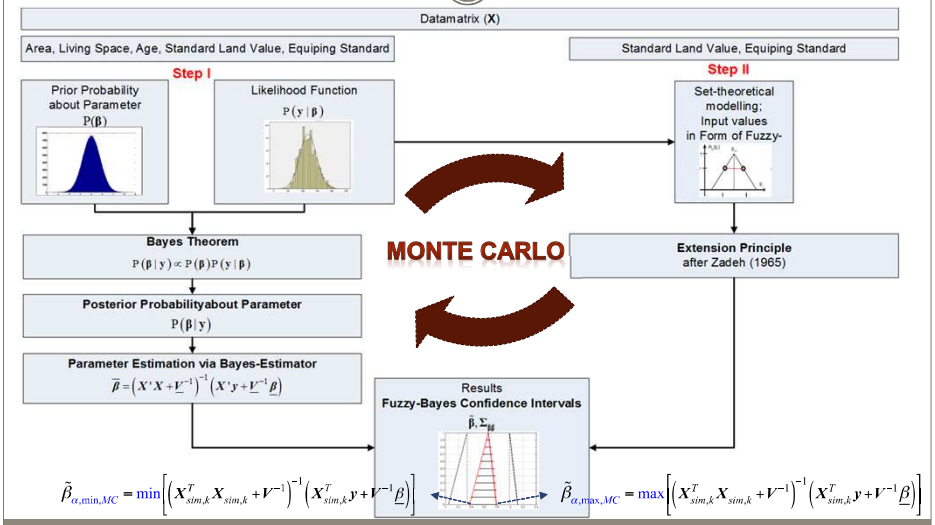

$$\tilde{\beta}_{\alpha,\min,MC} = \min\left[\left(X_{sim,k}^T X_{sim,k} + V^{-1}\right)^{-1}\left(X_{sim,k}^T y + V^{-1}\underline{\beta}\right)\right] \quad \tilde{\beta}_{\alpha,\max,MC} = \max\left[\left(X_{sim,k}^T X_{sim,k} + V^{-1}\right)^{-1}\left(X_{sim,k}^T y + V^{-1}\underline{\beta}\right)\right]$$

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First Results

- Design matrix is fuzzified for two rows:
  - standard land value and
  - equipment standard
- # Simulation = 250.000
- Hold equation is a strongly non-linear function with fuzzy input variables
- Search over the hold range of values of each exceeds the computing capacity

Findings on coefficients

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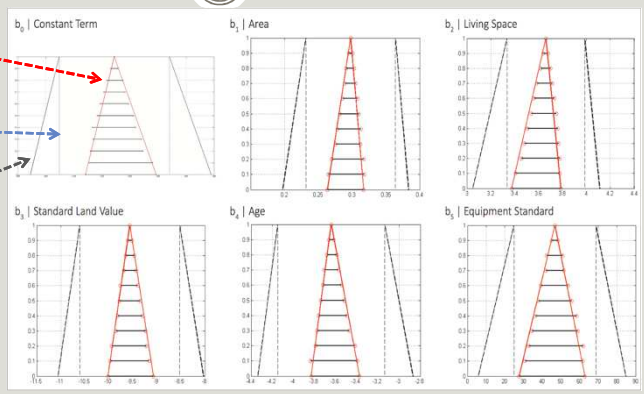
Fuzzy membership function

Random confidence interval

Extended confidence interval



- Basic equation: $\tilde{\beta}_{\alpha,\min,MC} = \min\left[\left(X_{sim,k}^T X_{sim,k} + V^{-1}\right)^{-1}\left(X_{sim,k}^T y + V^{-1}\beta\right)\right]$
- Non-linear optimization problem leads to concave / convex membership functions
- Extended confidence interval due to superposition of fuzzy and random uncertainty

Findings on Dependent Variable

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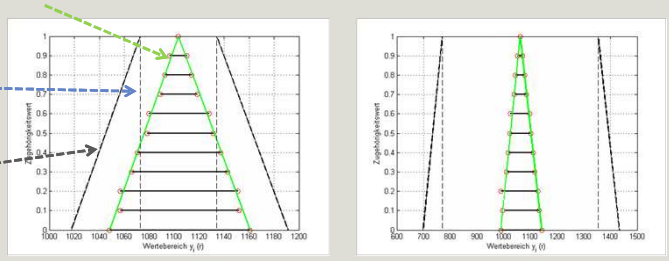
Fuzzy membership function of dependent variable

Random confidence interval

Extended confidence interval



- Basic equation: $\tilde{y}_{\alpha,\min,MC} = \min\left[X_{sim,k}^{pT} \square \hat{\beta}_\alpha\right]$
- Here linear optimization problem leads to linear fuzzy membership functions
- Strong influence of the epistemic uncertainties → consideration worthwhile!

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Conclusions and Further Work

- Fuzzy-Bayes-Approach:
  - In the design matrix, the input values of standard land value and equipping standard are extended by means of fuzzy sets.
  - The difference to a standard Bayesian Approach lies only in the treatment of the uncertainties and not in the functional model of the estimation of the target values.
- Advantage of Fuzzification:
  - Leads to reasonable results.
  - A more realistic uncertainty budget.
  - Uncertainty of experts opinions can be represented
- The gain of more realistic and reliable uncertainty measures for real estate examples leads to the conclusion of a need for further work with fuzzy variables:
  - Should be done in areas with few transactions.
  - A comparison with a pure Bayesian approach worthwhile.
  - **Next step:**

    Fuzzification, which is knowledge based from several experts instead of only one expert.

Contact of Presenters

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