Reliability and Robustness Analyses of the GPS Network of Oran City, Algeria.

Bachir GOURINE and Kamel BENAICHA, Algeria

Key-words: Statistical analysis, Reliability analysis, Robustness analysis, GPS network, Strain deformation.

SUMMARY

Nowadays, GPS geodetic networks are used for several types of surveys, such as geodetic and topographic surveys, geodynamic monitoring and structures auscultation, etc.

Generally, to analyse and control the quality of these GPS networks, the statistical and reliability aspects are studied. These analyses have been augmented with geometrical strength analysis using strain technique, resulting in the concept of the robustness analysis. This later is a combination of reliability and deformation of the network. The network robustness is quantified according to threshold values which are computed from errors confidence of adjusted points. The displacements of GPS points are compared to these threshold values, this permit to identify the weakness regions of the network in order to remedy them by changing the configuration or improvement of observations until acceptable robustness. The deformation of the network, due to measurements errors, can be expressed in terms of robustness in scale, in configuration and in twist.

Throw this work, a methodology of processing, reliability and robustness analyses of GPS networks has been established. The validation of this methodology was performed on a test network composed of 45 points of the Oran city GPS network. The obtained results show the powerful and the efficiency of the analysis methodology adopted.

Reliability and Robustness Analyses of the GPS Network of Oran City, Algeria.

Bachir GOURINE and Kamel BENAICHA, Algeria

1. INTRODUCTION

Actually, GPS geodetic networks are used in various types of surveys, such as topographic and geodetic, geodynamic monitoring surveys and auscultation of structures, etc. As conventional surveying, GPS networks are analyzed based on certain design criteria mainly the accuracy, reliability, robustness and cost. The accuracy refers to the quality of the network in terms of random errors. It is evaluated from the variance-covariance matrix of the estimated parameters by the least squares method. Reliability is the network's ability to detect and identify aberrations in the measurements, notably; it is to measure the maximum values of the magnitudes of the errors that cannot be detected by standard statistical tests, and the effect of these errors on the estimated parameters (Baarda reliability analysis). For outliers undetected by the approach of Baarda, the influence of these errors on the network can be evaluated using the approach developed by Vaníček. It concerns the augmentation of the reliability analysis by geometrical strength analysis, using the technique of the strain tensor, called robustness analysis. So, the objective of this work is to perform an analysis methodology based on the reliability and robustness concepts of the GPS networks.The application developed concerns the GPS network of Oran City, composed of 45points. The results obtained according to different analyses are presented and discussed.

2. RELIABILITY ANALYSIS

Generally, the reliability of a network is the network's ability to detect and resist against gross errors in the observations. In this regard, there are two aspects to the reliability of the network: internal and external. It depends on the network geometry and the accuracy of the observations [Seemkooei, 2001]. At this level it is desired to have an optimal network, in terms of reliability, which is to minimize the magnitude of errors undetectable by the stochastic analysis and therefore minimize the effect of these errors on the estimated parameters. Three main elements define reliability :

- Number of redundancy of an observation: measuring the absorption of an error r_i
- Internal reliability: ability of the network detect an error $\boldsymbol{\nabla_B}$
- External Reliability: effect of an undetectable error on network parameters ∇_x

2.1 Internal reliability

Internal reliability indicates the minimum bias (MDE: Minimum Detectable Error) or the maximum undetectable errors in the observations that can be detected by testing hypotheses. In calculating the MDE, the test power β is fixed at reference value of 80% [Moore et al., 2002]. The internal reliability of the i-th observation is expressed by [Vanicek et al., 2001] and [Gourine, 2004]:

$$\nabla_{bi} = \sigma_{bi} \cdot \frac{\delta}{\sqrt{r_i}} \qquad (1)$$

where,
σ_{bi}: RMS of the observation *bi*;
ri : number of redundancy of the observation *bi*;
δ : non-centrality parameter of alternative hypothesis as a function of selected probabilities (α, β).
The values of these numbers are between 0 and 1. A value of zero indicates that there is no

redundancy of observation considered and that error on observation cannot be identified. However, value 1 indicates that the observation is well controlled.

2.2. External reliability

The external reliability which is the influence of the effect of each MDE on the estimated coordinates of the network is represented by the following relation [Carosio et al., 1995]:

$$\nabla x = (A^T . P . A)^{-1} - A^T . P . \nabla_{bi} \qquad (2)$$

Baarda, [Han et al., 1999], proposed another measure of the external reliability, as relative external reliability, which is given by as:

$$\bar{\delta}^2 = \frac{\hat{\nabla}^T{}_x . (A^T . P . A)^{-1} . \hat{\nabla}_x}{{\sigma_0}^2} \qquad (3)$$

A, P are configuration matrix and observations weight matrix, respectively.

So to design a reliable geodetic network the following criteria should be fulfilled:

(a) gross errors should be detected and eliminated.

(b) marginal error in an observation, must be small in comparison with the standard deviation of the observation;

(c) effect of this error on the coordinates should be as small as possible.

3. Robustness analysis of 3D geodetic networks

The network robustness analysis is the combination of the reliability and the deformation of the network [Vanicek et al., 2001]. This is a more appropriate technique for the accurate evaluation of the effects of the observation network errors using the concept of strain tensor defining the gradient of the displacement field.

Given a three-dimensional displacement field U(x, y, z)= $(u, v, w)^T$, E strain tensor matrix is defined by [Seemkooei, 2001]; [Vanicek et al., 2001]:

$$E = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$$

Let to determine the displacement vector at each point Pi of the network as:

$$\Delta X_i = \begin{bmatrix} \Delta x_i \\ \Delta y_i \\ \Delta z_i \end{bmatrix} = \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \qquad (4)$$

and that

$$\Delta X = \nabla x = (A^T . P . A)^{-1} . A^T . P . \nabla_{bi} \qquad (5)$$

So, for each point P_i we have

$$E_i = \begin{bmatrix} \frac{\partial u_i}{\partial x} \frac{\partial u_i}{\partial y} \frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x} \frac{\partial v_i}{\partial y} \frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x} \frac{\partial w_i}{\partial y} \frac{\partial w_i}{\partial z} \end{bmatrix} \qquad (6)$$

For all j from 0 to t, where t is the number of point Pi connection with other points of the network, the components of the displacement vector satisfy the following relationships:

$$a_i + \frac{\partial u_i}{\partial x}(X_j - X_i) + \frac{\partial u_i}{\partial y}(Y_j - Y_i) + \frac{\partial u_i}{\partial z}(Z_j - Z_i) = u_j \quad (7)$$
$$b_i + \frac{\partial v_i}{\partial x}(X_j - X_i) + \frac{\partial v_i}{\partial y}(Y_j - Y_i) + \frac{\partial v_i}{\partial z}(Z_j - Z_i) = v_j \quad (8)$$
$$c_i + \frac{\partial w_i}{\partial x}(X_j - X_i) + \frac{\partial w_i}{\partial y}(Y_j - Y_i) + \frac{\partial w_i}{\partial z}(Z_j - Z_i) = w_j \quad (9)$$

where :
$a_i b_i$ et c_i are absolute terms,
(X_i, Y_i, Z_i) are coordinates of the point P_i
(X_j, Y_j, Z_j) are coordinates of the point P_j.

In matrix terms, the precedent relationships can be written as:

$$K_i . \begin{bmatrix} a_i \\ \frac{\partial u_i}{\partial x} \\ \frac{\partial u_i}{\partial y} \\ \frac{\partial u_i}{\partial z} \end{bmatrix} = u_i, K_i . \begin{bmatrix} b_i \\ \frac{\partial v_i}{\partial x} \\ \frac{\partial v_i}{\partial y} \\ \frac{\partial v_i}{\partial z} \end{bmatrix} = v_i, K_i . \begin{bmatrix} c_i \\ \frac{\partial w_i}{\partial x} \\ \frac{\partial w_i}{\partial y} \\ \frac{\partial w_i}{\partial z} \end{bmatrix} = u_i \quad (10)$$

We obtain :

$$\begin{bmatrix} a_i \\ \frac{\partial u_i}{\partial x} \\ \frac{\partial u_i}{\partial y} \\ \frac{\partial u_i}{\partial z} \end{bmatrix} = (K_i^T . K_i)^T . K_i^T . u_i, \begin{bmatrix} b_i \\ \frac{\partial v_i}{\partial x} \\ \frac{\partial v_i}{\partial y} \\ \frac{\partial v_i}{\partial z} \end{bmatrix} = (K_i^T . K_i)^T . K_i^T . v_i, \begin{bmatrix} c_i \\ \frac{\partial w_i}{\partial x} \\ \frac{\partial w_i}{\partial y} \\ \frac{\partial w_i}{\partial z} \end{bmatrix} = (K_i^T . K_i)^T . K_i^T . w_i \quad (11)$$

We put :

$$Q_i = \left(K_i^T . K_i\right)^{-1} . K_i^T \qquad (12)$$

The gathering of these equations in one hyper matrix, gives:

$$vec(E_i) = \begin{bmatrix} Q_i & 0 & 0 \\ 0 & Q_i & 0 \\ 0 & 0 & Q_i \end{bmatrix} . \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} \quad (13)$$

If T_i, is band hyper matrix obtained by elimination of the first raw of the Q_i, $vec(E_i)$ can be written as:

$$vec(E_i) = T_i . \Delta X_i \quad (14)$$

By consequence:

$$vec(E_i) = T_i.(A^T\,.P\,.A)^{-1}.A^T.P.\nabla_{bi} \quad (15)$$

To determine the displacements of the point P_i, we introduced the initial conditions (X_0, Y_0, Z_0). These are the coordinates obtained from the minimization of the standard of displacement vectors of all points of the considered network. Therefore, the components of the displacement vectors are estimated by the following system of equations:

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = E_i.\begin{bmatrix} X_i - X_0 \\ Y_i - Y_0 \\ Z_i - Z_0 \end{bmatrix} \quad (16)$$

The total displacement, for each Pi, is computed by:

$$d_i = \sqrt{{u_i}^2 + {v_i}^2 + {w_i}^2} \quad (17)$$

3.1. Computation of initial conditions

In order to calculate the initial conditions, the displacements caused by the maximum undetectable errors in the network, should be minimized. So we have

$$\min_{(X_0,Y_0,Z_0 \in \mathbb{R})} \sum_{i=1}^{n} \|\Delta\vec{r}\|_i = \min_{(X_0,Y_0,Z_0 \in \mathbb{R})} \sum_{i=1}^{n} {u_i}^2 + {v_i}^2 + {w_i}^2 \quad (18)$$

Which leads to :

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}^{-1} . \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \qquad (19)$$

Where the coefficients of the equation (12) can be expressed by:

$$a_1 = \sum_{i=1}^{n} \left[\left(\frac{\partial u_i}{\partial x}\right)^2 + \left(\frac{\partial v_i}{\partial x}\right)^2 + \left(\frac{\partial w_i}{\partial x}\right)^2 \right]$$

$$b_1 = \sum_{i=1}^{n} \left[\frac{\partial u_i}{\partial x}.\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}.\frac{\partial v_i}{\partial y} + \frac{\partial w_i}{\partial x}.\frac{\partial w_i}{\partial y} \right]$$

$$c_1 = \sum_{i=1}^{n} \left[\frac{\partial u_i}{\partial x}.\frac{\partial u_i}{\partial z} + \frac{\partial v_i}{\partial x}.\frac{\partial v_i}{\partial z} + \frac{\partial w_i}{\partial x}.\frac{\partial w_i}{\partial z} \right]$$

$$d_1 = \sum_{i=1}^{n} \left[\begin{array}{c} \frac{\partial u_i}{\partial x}.\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial x}.\frac{\partial v_i}{\partial x} + \frac{\partial w_i}{\partial x}.\frac{\partial w_i}{\partial x} X_i + \frac{\partial u_i}{\partial x}.\frac{\partial u_i}{\partial y} + \frac{\partial v_i}{\partial x}.\frac{\partial v_i}{\partial y} + \frac{\partial w_i}{\partial x}.\frac{\partial w_i}{\partial y} Y_i \\ + \frac{\partial u_i}{\partial x}.\frac{\partial u_i}{\partial z} + \frac{\partial v_i}{\partial x}.\frac{\partial v_i}{\partial z} + \frac{\partial w_i}{\partial x}.\frac{\partial w_i}{\partial z} Z_i \end{array} \right]$$

With the same way, the other coefficients can be obtained. For more details about the developments of the equations, see [Berber et al., 2009].

3.2. Determination of threshold values

The threshold values enable to us to 'measure' the strength of any network. The formulas used to calculate these thresholds are proposed by [Berber, 2006].

After performing of the least squares method for adjustment of the network, the error ellipses of points are obtained from the variance covariance matrix of the parameters. The elements of these ellipses (semi-major axis and semi-minor axis) in the 2D case are determined by:

$$\sigma_{a_i} = \sqrt{(\sigma_{x_i}{}^2 + \sigma_{y_i}{}^2)/2 + q_i} \quad (20)$$

$$\sigma_{b_i} = \sqrt{(\sigma_{x_i}{}^2 + \sigma_{y_i}{}^2)/2 - q_i} \quad (21)$$

where :

$$q_i = \sqrt{(\sigma_{x_i}{}^2 + \sigma_{y_i}{}^2)/4 + \sigma_{xy_i}} \quad (22)$$

$\sigma_{x_i}{}^2$: Variance of x of point P_i ;
$\sigma_{y_i}{}^2$: Variance of y of point P_i ;
$\sigma_{xy_i}{}^2$: Covariance of point P_i .

In this work, two vectors are compared: d_i calculated from equation (17) and δ_i which is determined by,

$$\delta_i = \sqrt{\sigma_{a_{95i}}{}^2 + \sigma_{b_{95i}}{}^2 + \sigma_{h_{95i}}{}^2} \quad (23)$$

where,

$$\sigma_{a_{95i}} = 2.45.\sigma_{a_i} \quad (24)$$
$$\sigma_{b_{95i}} = 2.45.\sigma_{b_i} \quad (25)$$
$$\sigma_{h_{95i}} = 1.96.\sigma_{h_i} \quad (26)$$

$\sigma_{a_{95i}}$: semi major axis of confidence ellipsis for probability of 95% ,
$\sigma_{b_{95i}}$: semi minor axis of confidence ellipsis for probability of 95%,
$\sigma_{h_{95i}}$: confidence interval of ellipsoidal altitude for probability of 95%.
σ_{h_i} : RMS of ellipsoidal altitude of point P_i.

The confidence interval of 95%, of ellipsoidal altitude is obtained by multiplying σ_{h_i} by an expansion factor of 1.96, for an univariante probability distribution. In equation (23), the semi axes $(\sigma_{a_{95i}}, \sigma_{b_{95i}})$ should be reduced by a factor of (2.795/2.447) and vertical interval by (2.795/1.960), in order to replace the expansion factor of 2D case and 1D case by 3D one, before the formation of 3D limits (approximation of the ellipsoids confidence).

3.3. Comparaison entre les vecteurs *d* et *δ*

The decision on robustness of network is made by comparing the vectors d_i and δ_i, according to the following rules:

- If $d_i < \delta_i$: the network is robust at level of the required probability ;
- If $d_i > \delta_i$: the network is weak, i.e., some points do not respect the required level of robustness.

3.4 Strain deformation of 3D networks

This matrix gathers most of the information of displacement field behavior. Let a local field of 3D displacement,

$$\Delta X(x,y,z) = \begin{bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{bmatrix} \quad (24)$$

around a point with coordinates (*x*, *y*, *z*), the strain matrix E(*x*, *y*, *z*) is determined by gradient of displacement as :

$$E(x,y,z) = \begin{bmatrix} \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \frac{\partial w}{\partial z} \end{bmatrix}_{(x,y,z)} = \begin{bmatrix} \varepsilon_{ux}\varepsilon_{uy}\varepsilon_{uz} \\ \varepsilon_{vx}\ \varepsilon_{vy}\varepsilon_{vz} \\ \varepsilon_{wx}\varepsilon_{wy}\varepsilon_{wz} \end{bmatrix}_{(x,y,z)} \quad (25)$$

The gradient of the local displacement field is evaluated component by component. For each coordinate of this field, the partial derivatives according to each coordinate axis are evaluated to give the components of the deformation matrix. However, the interpretation of such tensor is not obvious, but its decomposition may help extracting some characteristic quantities known as deformation primitives. These later represent the deformation, in a more meaningful way.

In this study, we selected three components among the various primitives deformation, which are invariants of strain in the 3D case:

$$Dilatation\ \sigma = \frac{1}{3}\left(\varepsilon_{ux} + \varepsilon_{vy} + \varepsilon_{wz}\right) \quad (26)$$

$$Maximum\ Shear:\ \gamma_{max} = \max(\lambda_i) - \min(\lambda_i)\ , i = 1,3 \quad (27)$$

Where λ_i is the Eigen values of the E matrix.

$$\textit{Global twist:}\quad \Omega = \sqrt{\omega_{xy}^2 + \omega_{xz}^2 + \omega_{yz}^2} \quad (28)$$

With : $\omega_{xy} = \frac{1}{2}\left(\varepsilon_{vz} - \varepsilon_{wy}\right)$, $\omega_{xz} = \frac{1}{2}(\varepsilon_{uz} - \varepsilon_{wx})$ and $\omega_{yz} = \frac{1}{2}\left(\varepsilon_{uy} - \varepsilon_{vx}\right)$

This choice is not arbitrary; it was based on studies of [Vanicek et al., 1995] and [Berber, 2006] about the deformation invariants, i.e., the components of the strain tensor which remain unchanged with changing Datum (network reference frame). According to [Vanicek et al. 1995], the values of the three primitives of deformation can be compared to the precision required, with respect to the order of the network. It would be interesting to investigate the deformation thresholds to monitor and assess the robustness of the network in terms of configuration, scale and orientation. Previous studies on this subject have been made in the case of 2D networks, such as in [Gourine, 2004] and [Michel, 2001].

4. APPLICATION ON GPS NETWORK OF ORAN CITY

4.1. Description of the study zone

The Oran city, 2nd town in Algeria, have knew since decade an unprecedented urban extension which required the establishment of a geodetic reference network for all works of Surveying, Civil Engineering (infrastructure, bridges, roads, ..), Mapping and geodynamics monitoring of the region. In this context, a GPS network project of Oran city was undertaken in 2009, by the Division of Space Geodesy (DGS) of Space Techniques Centre (CTS), for the benefit of the Direction of public works of Oran (DTPO).

This network covers an area limited by [0° 21 '10 "0 ° 17' 43"] West in longitude and [35° 21 '05 "; 35

°27' 14"] North in latitude, Figure (1). In our case, the test network, bounded by a red circle in the figure (1), is composed of 45 points.

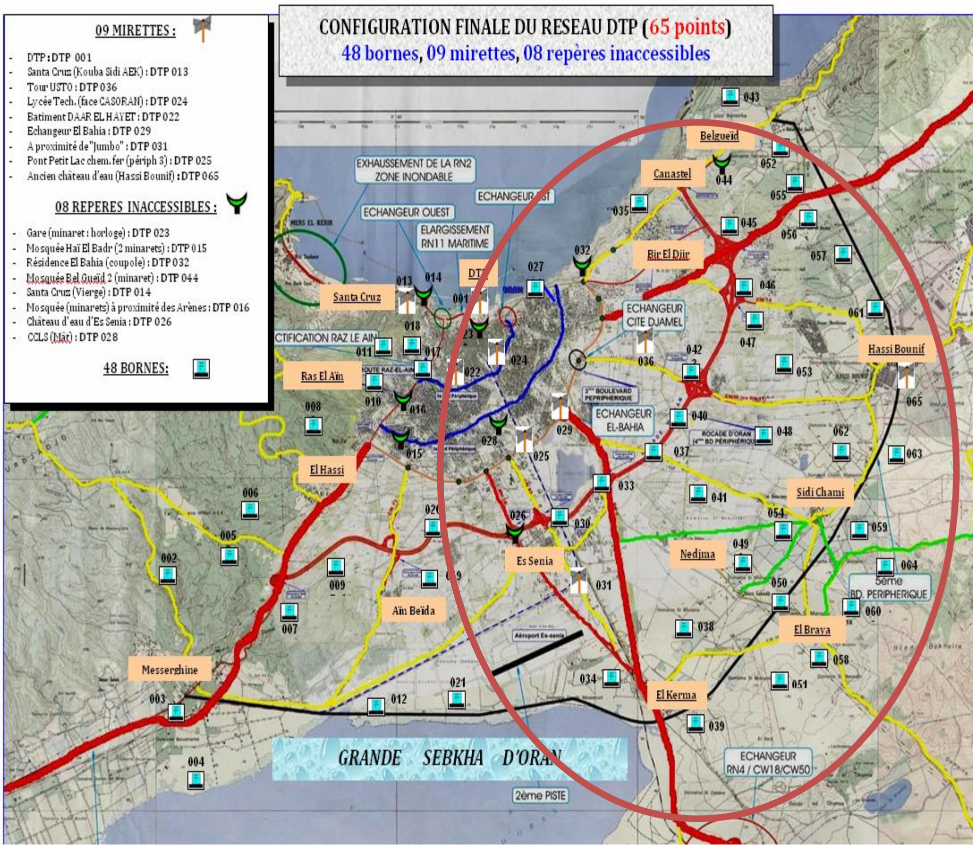

Figure (1): Configuration of the GPS network of Oran city [DGS, 2009]

Figure (2) illustrates some kind of benchmarks of the GPS network.

Figure (2): Photos of benchmarks of the GPS network of Oran city

4.2. Processing and Results

- ***Adjustment of the network***

At stage of pre-processing, the GPS data of the test network were processed using WINPRISM software, in order to get the GPS baselines. 28 sessions have been considered. The RMS obtained on the baselines is about 4.3 mm (3.2 mm, 1.5 mm and 2.3 mm, according to ΔX, ΔY, ΔZ components, respectively). Then the adjustment by least squares method, was performed, and leads to the following results:

				Error Ellipse	
	σ_E(m)	σ_N (m)	σ_H (m)	a (m) semi-major axis	b (m) semi-minor axis
Average	0.008	0.004	0.009	0.008	0.004
RMS	0.002	0.001	0.002	0.002	0.001
Minimum	0.005	0.003	0.006	0.005 (pts:33,40)	0.003 (pts:26,40,42,49)
Maximum	0.013	0.007	0.014	0.013 (pts : 15)	0.007 (pts: 35)

Table(1) : Precisions of adjusted coordinates and error ellipses

Table (2) shows that the estimated accuracy on position is about of 9 mm. The accuracy in the northern *N* component is significantly better than the other two coordinates. However, RMS on the vertical component remains the largest at centimeter level. Given the scale of the test network, such precision are acceptable.

Figure (3) shows the error domains of the estimation of the unknown parameters. These domains are represented by error ellipses, for the horizontal components (E, N) and by the error intervals for the vertical component (h).

The absolute errors ellipses of GPS points have an average size of about of 8 mm. Error intervals for the vertical component are about of 9 mm. The points lying within the boundary of the network have substantially large ellipses that those within the GPS network. The size of the error ellipse of points 21 and 35 reaches values of 11 mm, the point 15 has a maximum of 13 mm. The same observation is made for the error ranges for the vertical component, where maximum value of 14 mm is reached at point 15.

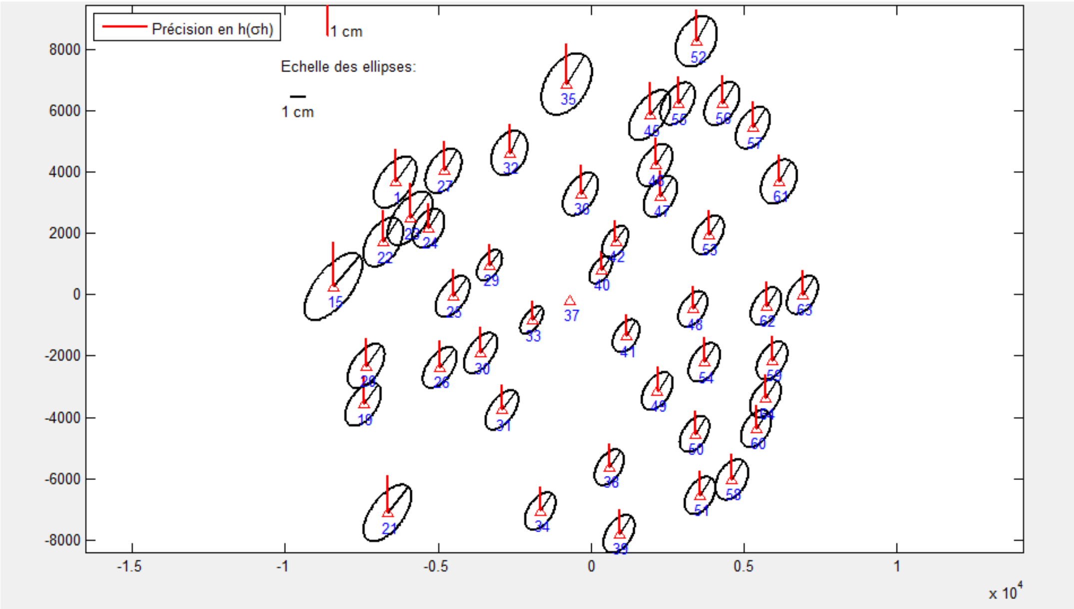

Figure (3): Absolute Error ellipses and Interval errors of the GPS points

- ***Reliability analysis of the network***

The presence of small residuals is not, necessarily, an indication of a good fit. So, the reliability evaluation phase of the network is useful to detect any biased observations and assess their influences on network parameters.

The redundancy numbers of GPS observations, which express the contribution of each individual observation in the freedom degree of the network, are between 15% and 98%. As the number of redundancy is large (i.e., close to 1), the absorption level of the error will be large, and therefore, this is an indication that the corresponding observation is reliable.

Table (2) gives the results of the reliability of the network. Internal reliability is showed by figure (4). Errors on all observations are at level of 6 mm, 3mm and 4 mm, according to the three components of the GPS baselines, respectively, or 7.8 mm in 3D. The maximum error is reached at the base 64 -54, while it is lowest for the baseline 62-59. Generally, the internal reliability mainly depends on the accuracy of the observations and a priori the network geometry (redundancy).

Given these results, we can conclude that the reliability of the observations is good. The external reliability shows that the effects of blunders on the network parameters do not exceed several millimetres. Therefore, the network is considered reliable. However, the maximum (algebraic) external reliability, with respect to local geodetic coordinates, reaches 15 mm, at point 15, which have the biggest error ellipse.

	Fi(ΔX)	Fi(ΔY)	Fi(ΔZ)	Fe(dE)	Fe(dN)	Fe(dh)
Average (m)	0.006	0.003	0.004	-0.004	-0.001	-0.001
RMS (m)	0.003	0.001	0.003	0.004	0.002	0.003
Minimum (m)	0.002	0.001	0.002	-0.015	-0.006	-0.008
Maximum (m)	0.016	0.010	0.012	0.006	0.004	0.007

Table (2): Results of internal and external reliabilities. Fi: Internal reliability; Fe: External reliability

- ***Robustness analysis of the network***

For a better interpretation of the results on the analysis of geodetic networks, we have introduced in this paper, the concept of network robustness analysis. First, we examine the optimal displacements of the GPS points. It is known that the value of the displacement of a point, in the network, must be smaller as possible. This is considered as a robustness criterion, such as: max (di) → min.

From Table (3), the average 3D displacement is at order of 3 mm, only. The highest values are about of 3 cm. To qualify the robustness of the network, the threshold values were calculated on the basis of error domains parameters (error ellipses and error intervals). These thresholds are at level of 24 mm. The comparison between the displacements and thresholds permits to identify weak (not robust) GPS points, as 20 and 58, Figures (4).

Figure (5) shows the displacements of the GPS points. It is clear that the network is robust. However, to overcome the weakness of network areas, two solutions to be considered:

- Eliminate bad GPS baselines

- Increase network configuration with new GPS bases at weak points.

	Displ. (d) (m)	threshold (δ) (m)	Dil (σ) (ppm)	Max Shear (λ) (ppm)	Rot (Ω) (ppm)
Average	0.003	0.024	-9.0	61.0	27.0
RMS	0.007	0.060	400.0	400.0	200.0.
Minimum	0.0	0.0	0.0	0.0	0.0
Maximum	0.030	0.040	400.0	4200.0	3200.0

Table (3): Displacements and primitive deformations

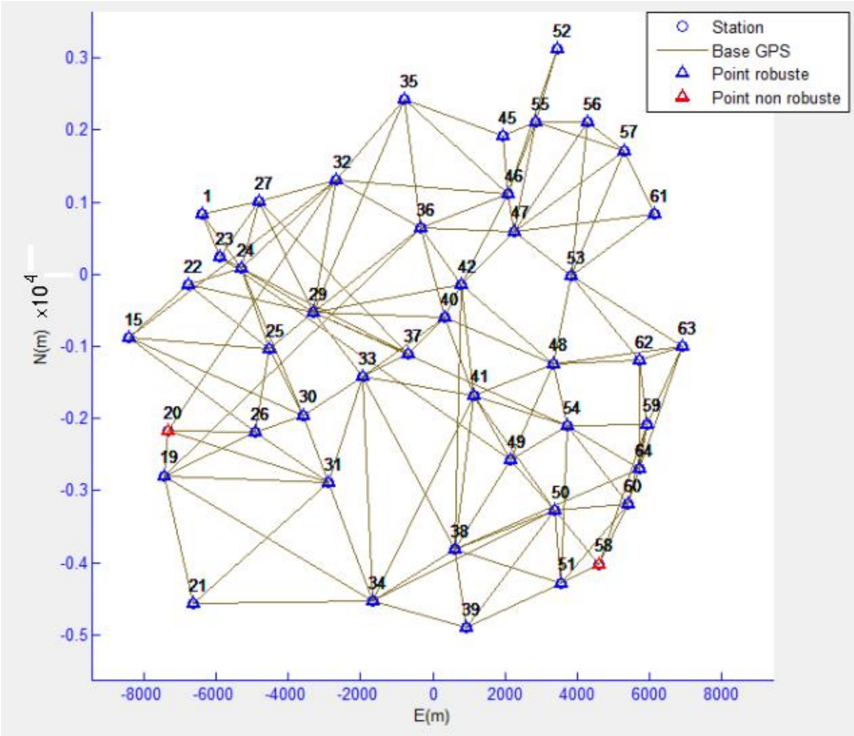

Figure (4): Robust and weak GPS points

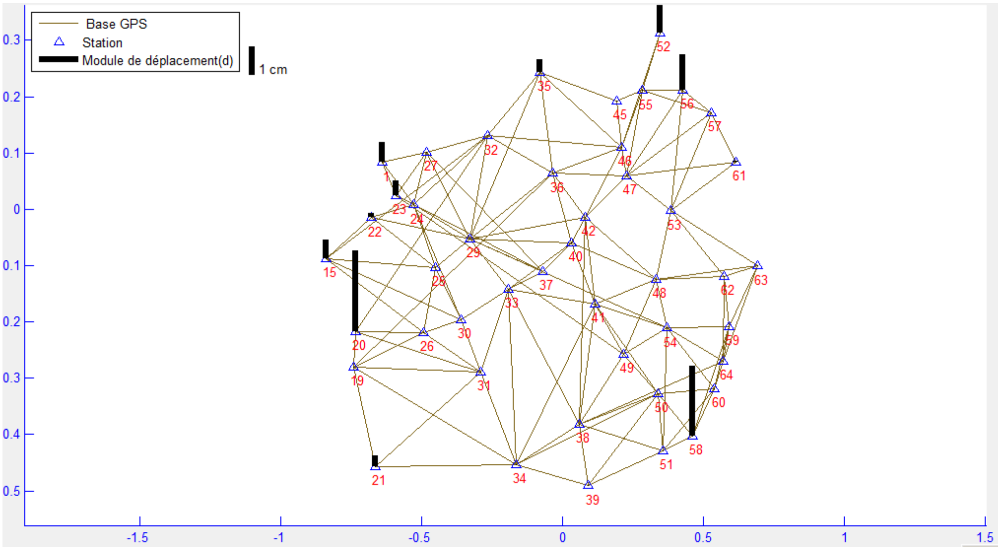

Figure (5) : Displacements vectors (d) of GPS points

Another way to represent the strength of the network is by using deformation primitives of the network. We saw earlier how the effects of undetected errors on the network, are expressed by displacements of GPS points. These can lead, in turn, to deformations, not in physical sense but in sense of the effect of errors on the network parameters, according to strain tensor concept. It has been shown by [Vanicek et al., 1995] and [Berber, 2006] that the strain tensor can be interpreted easily by invariant primitive strains, as dilation, maximum shear and total differential rotation (twist). In other words, they represent the network robustness in terms of scale, configuration and orientation, respectively. The figures (6, 7 and 8) depict the different deformations. It is clear that the most deformed points are those located on the edge of the network, for example: points 61, 56, 45.

Statistics of deformation primitives are summarized in the table (3). The network has undergone considerable deformation in dilatation, shear and differential rotation, in a few points. However, the network is less distorted (stable) in other points.

According to scale robustness, figure (6), the dots having a higher or equivalent strain to 10 ppm are in the edges of the network, such as at points (61, 56, 52, 23, 15, 20). Similarly, for robustness in configuration and orientation, some common points have undergone significant deformation, like points (61, 56, 45). However, some points, previously identified as weak points (20 and 58), appear less deformed as other points estimated as robust. At this level, we must pay attention to two things. First, is that the two aspects of robustness analysis discussed, are complementary, and second is that statistical significance of the deformation primitives should be taken into account in judging the deformation.

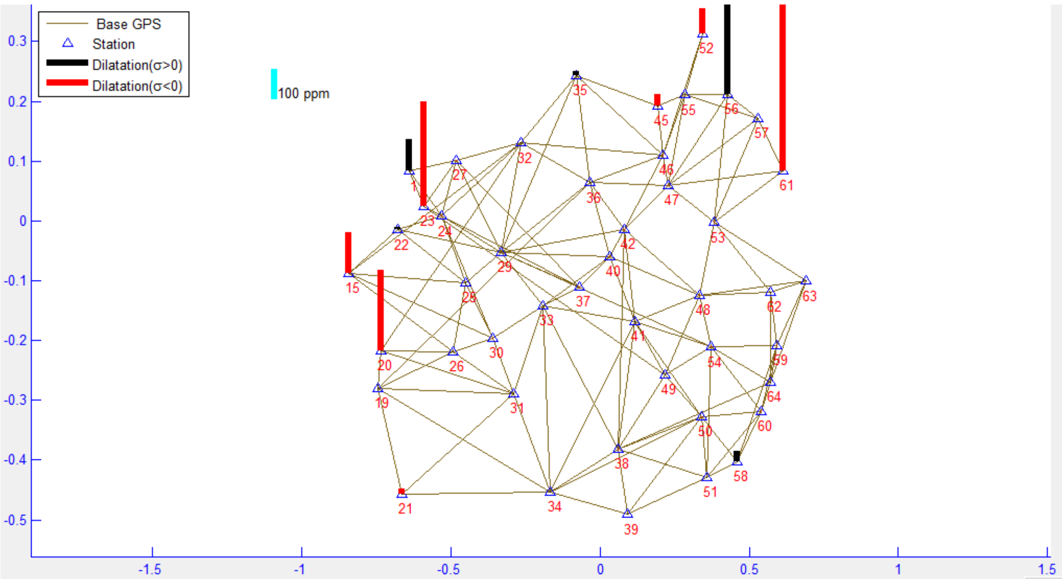

Figure (6) : Robustness in scale (dilatation).

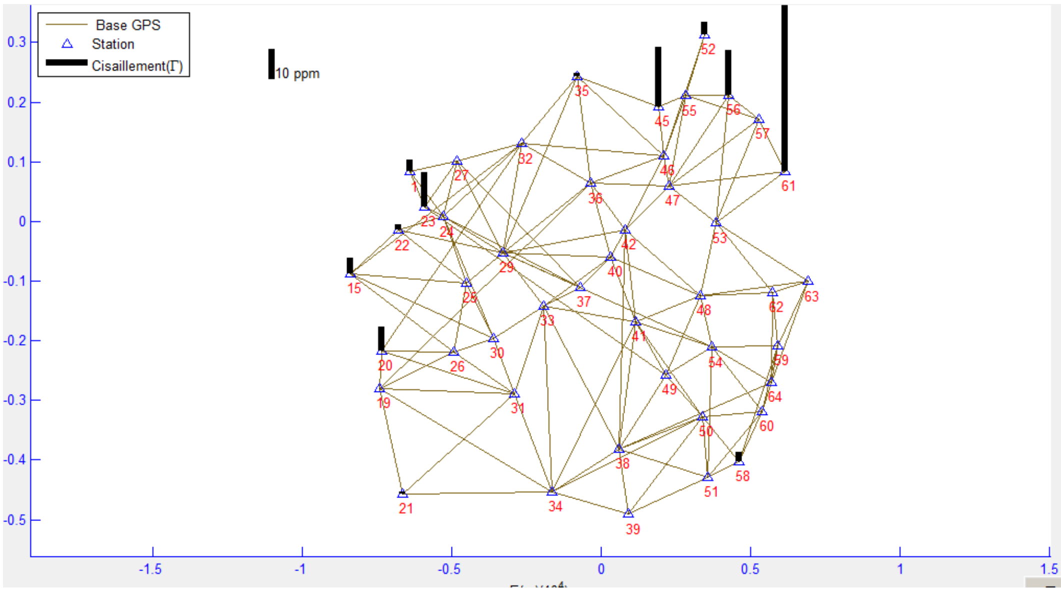

Figure (7) : Robustness in configuration (shear)

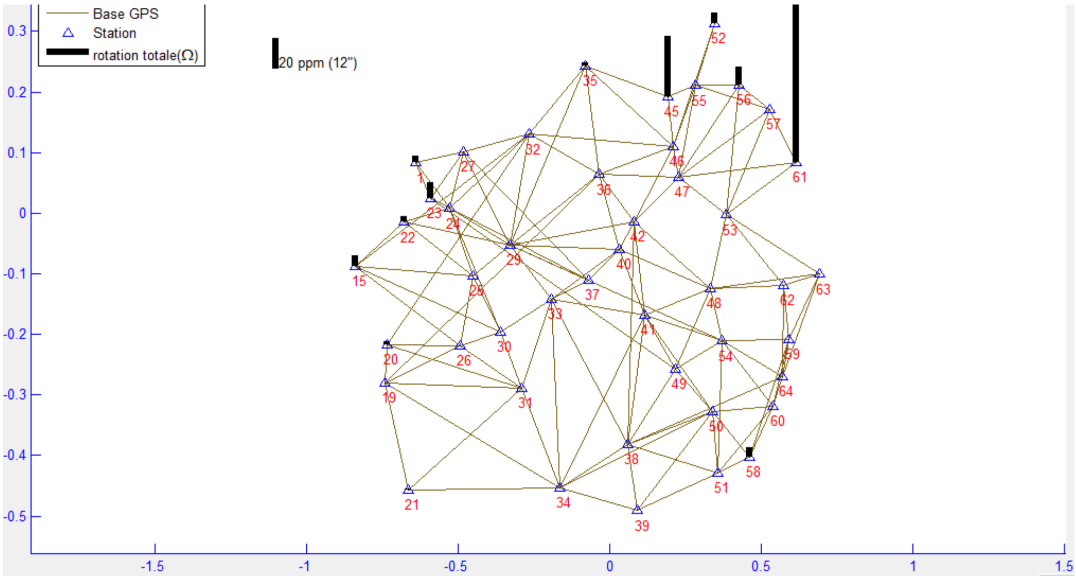

Figure (8): Robustness in twist (differential rotation)

5. CONCLUSION

Through this paper, the aspects of analyses of reliability and robustness of 3D geodetic networks, notably GPS networks, were discussed.

The application was carried out on a test network with 45 points of the GPS network of Oran city. This network was established by the Division of Space Geodesy (CTS, Algeria) for the benefit of the Direction of Public Works of Oran, in 2009. The analysis methodology of the network is based on the following: (i) adjustment statistical analysis: analysis of quality of GPS baselines and quality analysis of the estimated parameters (coordinates of the GPS points), (ii) reliability analysis of GPS observations and network parameters and (iii) robustness analysis: identification of weak and strong points of the network by calculating the optimal displacement and thresholds; and also punctual representation of network deformation according to scale, configuration and orientation.

The results obtained showed that the standard deviations on the GPS baselines are about of ± 4 mm. The estimated accuracy on local geodetic coordinates of GPS points is at level of ± 9 mm in position and height. The domain errors of estimated parameters (ellipses and intervals of errors) are greater in points on the network edge.

The errors detected by the internal reliability of the network are of order of 8 mm. The maximum effect of these errors on estimated parameters is of 4 mm and 1mm, respectively, in horizontal and vertical components. Given these results, the network is considered reliable.

Regarding analysis robustness, the results showed that the network is robust globally, except in a few points on the perimeter where the displacements and deformations are important due to low number of connections of these points with others. The optimal displacements are of approximately of 3 mm with thresholds of 30 mm. The rate of deformation, in terms of, robustness in scale, in configuration and in orientation remains relatively low considering the required accuracy about 20 ppm.

Finally, some refinement of the reliability and robustness analyses should be considered, as:

- Investigation on significance levels used in the univariate and multivariate tests;
- Computation of tolerance limits of primitives of strain tensor, based on Monte Carlo method.
- Performing the methodology analysis on huge networks.

REFERENCES

- **Berber M., 2006 :** *Robustness analysis of geodetic networks.* PhD thesis. Department of Geodesy and Geomatics Engineering University of New Brunswick.
- **Berber M., Vanicek P., Dare P., 2009 :** *Robustness analysis of 3D networks.* Journal of Geodynamics, Elsevier, Vol.47, No.1, January 2009.
- **Carosio A., Reis O., 1995:** *Méthodes géodésiques et modèles mathématiques pour l'implantation des nouvelles transversales alpines*, Revue XYZ -n° 62- 1er trimestre 1995.
- **DGS, 2009 :** *Mise en place par GPS du réseau géodésique de la ville d'Oran*. Technical report of the GPS network of Oran City project of Division of Space Geodesy (DGS), CTS.
- **Han S., Rizos C. 1999 :** *Network adjustment issues using mixed GPS surveying techniques.* 40th & 6th S.E. Asian surveyors congress, Fremantle, Australia, pp. 68-76, 30 October – 05 November 1999.
- **Gourine., 2004:** *Ajustement avec contraintes du réseau géodésique national. Application: réseau de base.* Magister thesis, at CNTS, Arzew – Algeria.
- **Michel V., 2001**: *Utilisation des tenseurs pour la représentation des déformations et de leur degré de signification*, rapport de stage de 3e année du cycle I.T.G.C.E, Ecole Nationale des Sciences géographiques (ENSG), octobre 2001.
- **Moore M., Rizos C., Wang J., 2002:** *Quality control issues relating to an attitude determination system using a multi-antenna GPS array.* School of surveying and spatial information systems, University of New South Wales, Sydney, Australia.

- **Seemkooei A.A., 2001:** *Comparison of reliability and geometrical strength criteria in geodetic networks.* Springer Verlag, V.75, No.4, July 2001.
- **Vanicek P., Craymer M.R., Krakiwsky E.J., 2001:** *Robustness analysis of geodetic horizontal networks.* Springer Verlag, volume 75, n° 4, pp199-209, 2001.
- **Vanicek P., Krakiwsky E.J., 1995:** *Geodesy: The concepts.* Edition Elsevier Science B. V., second edition, 697p, 1995. ISBN. 0-444-87777-0.

BIOGRAPHICAL NOTES

Bachir GOURINE received an Engineer Diploma in *Geodesy* in 1994 at the National Centre of spatial Techniques (CNTS, Arzew/Algeria) and obtained his Magister degree in *Space Techniques and Applications* at the Centre of Spatial Techniques (CTS), in 2004. During 2004, he is a permanent researcher at the Space Geodesy Division of CTS. On January 2011, he is PhD graduated in *Telecommunication*, Faculty of Electrical Genius, Department of Electronics - University Sciences and Technology Mohamed Boudiaf (USTO, Oran/Algeria).
His research fields include: Space Geodesy, Geosciences Time series Analysis, Combination of space positioning systems data, Deformation modelling by neural networks, Adjustment of geodetic networks...

Kamel BENAICHA obtained his Engineer Diploma in Geodesy on 2012 at Centre of Spatial Techniques (CTS, Arzew/Algeria). He worked on reliability and robustness analyses of 3D geodetic networks.

CONTACTS

Centre des Techniques Spatiales (CTS)
Division de Géodésie Spatiale.
Arzew
ALGÉRIE
Tél. +213 41 792142
Fax. +213 41 792176
Email: bachirgourine@yahoo.com