Realistic Uncertainty Estimation of the Market Value Based on a Fuzzy-Bayesian Sales Comparison Approach

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Keywords: Sales Comparison Approach, Uncertainty, Regression Analysis, Bayesian Approach, Fuzzy Theory

SUMMARY

The real estate and finance crisis has shown the importance of real estate valuation: The market value has to satisfy high objective quality requirements. Besides, the German jurisdiction demands a maximum dispersion of ± 20 % of the market value. The sales comparison approach as one of the valuation methods is from a mathematical-statistical point of view based on a multiple linear regression analysis. Since decades, it has been considered as a standard procedure for analysing the real estate market and to determine the current market value. The estimated comparative value is in particular depending on the number and the type of influencing variables which are considered within the regression model. Nevertheless, the uncertainty estimation of this approach has not been extended since its introduction. The uncertainty here results from the inherent inaccuracy of the observations on the one hand, on the other hand from the selected model as imperfect realisation of the reality. The aim of this research is to develop and enhance the uncertainty estimation in the used regression analysis by dividing the uncertainty budget in epistemic and aleatoric parts. While the aleatoric components describe random variability, which can be modelled by means of Bayesian inferences, the epistemic components characterise systematic and/or deterministic influences which result from unsatisfactory knowledge, assumptions, simplifications and linguistic formulations. Epistemic components can be modelled by selected models from fuzzy theory.

This paper introduces a Fuzzy-Bayesian approach, which is able to consider the uncertainty of the value affected by the above described characteristics and thus to quantify its impact on the market value. As starting point for this investigation, the data basis is prepared: The market value affecting attributes, which have a significant influence on the valuation approaches, were listed and categorised for showcase samples of different spatial and objective partial markets. The methodology is tested on a real data set. The establishment of the advanced mathematical approach should allow predicting any real estate values for objects within the selected spatial and objective submarket. It can be concluded, that this approach should provide more precise and appropriate uncertainty estimations of predicted values.

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1. MOTIVATION

In the property valuation, standardised methods are established in Germany. These methods reach their limits when only a few or no current comparative prices are available, as for example in inner cities or in rural areas, which can be denoted, by areas with few purchases. The less information is available, the more challenging is the valuation. Often, the experts have to decide about the market value only by their expertise and by statistical analysis of few existing data. Alkhatib & Weitkamp (2012) and Weitkamp & Alkhatib (2012a) had demonstrated that the Bayesian regression can be recommended in areas with few data by using prior information. In Weitkamp & Alkhatib (2012b) areas with small samples had been simulated. The Bayesian regression using a-prior information collected by experts solved the functional relationship better than a classical regression. In case of prediction for unknown cases by means of the classic regression function, the method will fail and the results were unreliable and skewed. Contrary, the Bayesian regression has a clear advantage by using prior data. This foreknowledge compensates the missing data and leads to a sufficient solution. The former paper uses samples with few data but still more data than there is in areas with few purchases. In Weitkamp & Alkhatib (2012a) and Weitkamp & Alkhatib (2012b), the size of the data-set was sufficiently large. It must be mentioned, that also with the Bayesian approach numerical problems often lead to unsolvable equations, so Alkhatib & Weitkamp, (2013) and Weitkamp & Alkhatib (2014) present an approach for a robust Bayesian solution.

All former approaches are based on random variables: as pure aleatoric approaches. But in generally, the uncertainty definition has two core elements – aleatoric and epistemic parts. Using only aleatoric variables does not lead consequently to a goal-oriented methodology for an accurate derivation of uncertainties in real estate valuation. In addition, the need of specifying an appropriated span exists, in which the real purchase price and the market value, respectively, lies with high probability (issue of uncertainty). The estimation of the market value makes multivariate statistical methods crucial. It can be stated, that innovative research approaches has to question if an expanded uncertainty modelling can significantly enhance current methods. The approaches need to fulfil both: the requirements of real estate-specific data and the enlarged definition of uncertainty needs for the estimation of market value. Particularly in view of the expanded uncertainty definition, a high potential for testing new approaches can be seen by the possibilities shown by the mathematical treatment: The focus of this research is a true and reality best approximating mathematical formulation of real relationships with the aim of the evaluation process to quantify uncertainties arise in the estimation results for unknown model variables and targets to reduce and propagate errors. As methodological approaches the Bayesian theory for aleatoric uncertainties and the fuzzy theory for epistemic uncertainties will be used.

2. FUZZY VARIABLES IN REAL ESTATE VALUATION

2.1 State of the Art

Uncertainties in the input variables affect the valuation. The reason for this lies in the fuzzy market. Many experts are leaving out an accuracy estimation of the market value. On the one hand, a goal-oriented methodology for the determination of uncertainties is missing; on the other hand, they are afraid of providing large uncertainties in the assessments (Jester & Roesch, 2006). Instead of deriving a statistically based uncertainty of the market value, in the fuzzy case, the market value is presented as a range of values. As a result, it reflects the fact that a precise determination of market value is not possible (Engel, 2008; Jester & Roesch, 2006; Metzger, 2010; Petersen, 2007; Streich, 2003). The market value is rounded without specifying cent to prevent the fake a seemingly accuracy like cent-accuracy (Kleiber, Fischer, & Simon, 2010). The experts call for the indication of the uncertainties (Engel, 2008; Jester & Roesch, 2006; Metzger, 2010; Petersen, 2007) to consider quality parameters in the valuation process.

With a combined Fuzzy-Bayesian model, uncertainties of variables could be differentiated by random (aleatoric) and deterministic (epistemic) parts in one approach. This classifying subdivision of uncertainties can be found in Möller & Beer (2008) and in Der Kiureghian & Ditlevsen (2009).

The aleatoric uncertainty is caused by the random variability of the system (inherent random nature of the physical quantities and will therefore usually modelled by random variables). The epistemic uncertainty, however, results from the lack of knowledge about the system, simplifications or the limited availability of data. In contrast to the epistemic, the aleatoric uncertainty components are in generally not reducible.

Applying this subdivision to the field of determining the market value, we can see epistemic uncertainties e.g. in the input variables of the location or the remaining period of an economic usability of the building. Although these are unknown, but in principle to accept as proof. Their determination is influenced by experts' expertise. Input variables such as area of a lot or living space are influenced by aleatoric uncertainties. These are also unknown, but are subject due to the calculation method by a fundamental variability. Increasing the number of comparative purchases increases their accuracy.

This subdivision suggests that probability-theoretical approaches (such as Bayesian methods) are used for modelling the random uncertainties and that non-stochastic methods (such as interval and fuzzy theory) are used for modelling the deterministic parts.

In the Bayes theory, it is possible to process knowledge about the input variables and their uncertainties in the form of complete probability density functions (PDF) for the evaluation and analysis (Koch, 2007). For the approximation of the probability distribution, so-called Sigma Points are generated as central moments. For each point, a non-linear function is used. With this function, the central moments could be estimated. If the functional relationship of input variables is linear and no prior information exists (non-informative prior density), the results of the classic approach and the Bayesian one are equal. This evidence is integrated in an international standard calls "Guide to the Expression of Uncertainty in Measurement (GUM)" (ISO-IEC guide, 2009).

An adequate alternative for modelling the uncertainty that is not based on the narrower field of stochastic can be derived from the interval mathematics and fuzzy theory. The uncertainty ranges of the input variables are not perceived in the form of prior densities (as in the

Bayesian approach), but on the basis of intervals or fuzzy sets or fuzzy numbers (Bandemer & Näther, 1992; Viertl & Hareter, 2006). In these approaches, information about the uncertainty with the aid of models from the set theory is considered. A key point is here the construction of the membership functions.
Beneath the own preparatory work, different approaches of engineering disciplines are presented. Beer (2006) and Möller & Beer (2004) show different approaches and methods to deal with different reasons for uncertainty. The work of Wälder et al. (2008) has the same background. In fuzzy theory, it is possible to incorporate expert knowledge by modelling of Nested Sets; while the views of experts is summarized within fuzzy membership functions (Nguyen & Kreinovich, 1996).
Recent work on modelling uncertainties mainly engaged in a combination of the above probabilistic and set-theoretic approaches. Here, the Bayesian theory is ideally suited for the description of the random variability and interval mathematics and fuzzy theory for modelling uncertainty (e.g. Viertl, 2008; Viertl & Hareter, 2006). An overview on common modelling of random variability and fuzzy theory could be found within several studies (Bandemer, 2005; Ferson et al. 2002; Wälder et al., 2009).
The expected advantages of presented approach are realistic uncertainties and a better modelling of systematic/deterministic uncertainty parts.

2.2 Approach of a Fuzzy Model

The integration of epistemic uncertainties in the Bayesian approach is done by the fuzzy theory based on fuzzy sets. Besides the identification of their origin to be assessed, as epistemic factors appropriate membership functions must be determined which realistically represent the fuzzy input variables.
The modelling of epistemic data was limited to exemplary factors. Approaches to expert surveys and their integration in Bayesian approaches can be found in addition to the research project of Alkhatib & Weitkamp (2012) and Weitkamp & Alkhatib (2012a).
Under the described approach, the purchase price (target of the model) is basically not considered as fuzzy. The purchase price of a single object is the result of negotiations between buyers and sellers and is the price paid by mutual consent. Fuzziness in the result can therefore not be assumed. The uncertainty is limited to the input variables that are considered as value-relevant for the valuation object. A lot of experts' expertise flows in the determination of the market value. Many variables are not directly measurable or represent only an estimate. Beneath the linguistic fuzziness also those variables can be considered as fuzzy, which are given numerically, but are not exactly appropriate to the individual valuation case.
As an example, the land value can be stated. It represents itself as the average “location value” within a land value zone. Deviations from this average are not considered, e.g. by conversion factors, but are used in its original form. Discrepancies between characteristics of the benchmark land of the land value zone and the various real properties with the real characteristics can be expected. Appraisers usually apply the increases and reductions by their expertise. Another relevant example of a classical epistemic variable is the equipment standard: A categorical classification contains usually the levels "good", "medium" and "bad". While generally different forms of membership functions exist, triangular functions were selected as suitable in the present approach. The interval of the standard land value was

chosen according to the expert recognized rule that single land values in a land value zone do not differ more than 20% from the mean. Thus, there is a real interval of ± 20% of the observed land benchmark. For the equipment categories, the linguistically formulated classifications are divided into categories 1 to 7, where an interval of ± 0.5 can be assigned, so there are smooth transitions between the stages.
As a result, the market value gets a fuzzy extension. Whereas the purchase price cannot be fuzzified, the market value can be interpreted as a "mean purchase price" of normal purchases without outliers. As such, the market value is classified as fuzzy.

3. FUZZY-BAYES-MODEL FOR REAL ESTATE VALUATION

Bayesian regression analysis has been developed an adopted to real estate valuation in Alkhatib & Weitkamp (2012), Weitkamp & Alkhatib (2012a, 2014), Zaddach & Alkhatib (2013a) and Zaddach & Alkhatib (2013b). Fuzzy linear regression was introduced by Tanaka et al. (1981). In this paper the general form of Bayesian regression equations is combined by means of fuzzy intervals.

3.1 Classical Multiple Regression Analysis

The classical regression model (Fahrmeir et al., 2009), is based on the idea of explaining variations of observations (e. g. purchase price per square meter) by the variability of m predictor variables $x_1, \dots, x_m$:

$$y_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_m x_{im} + \varepsilon_i = \beta_0 + \sum_{j=1}^{m} \beta_j x_{ij} + \varepsilon_i, \qquad [1]$$

where $i = 1, \dots, n$ as the number of data sets and $\beta_0, \dots, \beta_m$ indicate the m regression coefficients. The general trend model is superposed with random noise, since the relationship between the response variable and the predictor variables are often known only vaguely and knowledge about an exact function is not given. The residual ε_i is therefore introduced to overcome the inconsistency in the mathematical equation. The use of matrix-vector notation allows rewriting the n equations in Eq. 1 to

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \qquad [2]$$

where $\boldsymbol{\beta}$ is the unknown vector of regression coefficients and $\boldsymbol{X}$ indicates the design matrix.

3.2 Bayesian Regression Analysis

Concerning the Bayesian regression approach, the main interest lies in learning about the unknown, stochastic regression coefficients $\boldsymbol{\beta}$ based on the data $\boldsymbol{y}$. In case of estimating the regression coefficients, a normal distribution of the dependent variable as well as linearity of the whole model has to be assumed in order to receive an analytically solvable solution. Under these conditions, Koch (2007) derives the mathematical procedure for estimating the coefficients as it can be taken from Table 1. The presented approach is based on the analytical solution:

$$\overline{\boldsymbol{\beta}} = \left(\boldsymbol{X}'\boldsymbol{X} + \underline{\boldsymbol{V}}^{-1}\right)^{-1}\left(\boldsymbol{X}'\boldsymbol{y} + \underline{\boldsymbol{V}}^{-1}\underline{\boldsymbol{\beta}}\right) \quad [3]$$

In these equations, prior information is marked per underline, resulting posterior information per overbar. As a result from the posterior density the regression coefficients can be derived more precisely. In addition, the uncertainty (cofactor matrix) and confidence intervals (Highest Posterior Density Intervals, HPDI) are determined. The way of gaining prior knowledge is not strictly specified. For this purpose, experts' opinion can be used as well as the results of former analysis. The first mentioned approach has been applied successfully to real estate valuation in Alkhatib & Weitkamp (2012), Weitkamp & Alkhatib (2012a, 2012b), and Zaddach & Alkhatib (2013).

3.3 Fuzzy-Bayes Regression Analysis

In this fuzzy approach we will recall the basic form and the sequence of steps of constructing fuzzy regression models. A fuzzy interval $\tilde{A}$ is uniquely defined by its membership function $m_{\tilde{A}}(x)$ over the set $\mathbb{R}$ of real numbers with a membership degree between 0 and 1:

$$\tilde{A} := \{(x, m_{\tilde{A}}(x)) \,|\, x \in \mathbb{R}\} \quad [4]$$

with $m_{\tilde{A}} : \mathbb{R} \to [0,1]$. The membership function of a fuzzy interval can be described by its left (L) and right (R) reference function (see also Fig. 1):

$$m_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{x_m - x - r}{c_l}\right), & \text{for} \quad x < x_m - r \\ 1, & \text{for} \quad x_m - r \le x \le x_m + r \\ R\left(\dfrac{x - x_m - r}{c_r}\right), & \text{for} \quad x > x_m + r \end{cases} \quad [5]$$

with x_m denoting the midpoint, r its radius, and c_l, c_r the spread parameters of the monotonously decreasing reference functions (convex fuzzy intervals). The $\alpha-\text{cut}$ with $\alpha \in [0,1]$ of a fuzzy interval $\tilde{A}$ is defined by:

$$\tilde{A}_\alpha := \{x \in X \,|\, m_{\tilde{A}}(x) \ge \alpha\}. \quad [6]$$

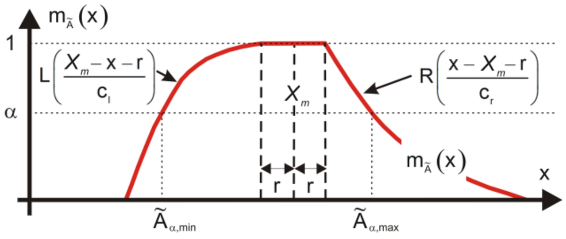


Fig. 1: Fuzzy interval and its $\alpha-$cut

A more general definition and further details in the field of Geodetic science can be found in Neumann (2009).

The LR-fuzzy model (see Eqs. 4 and 5), is based on the idea of a fuzzy functions given in a linear form according to Eq. 2. Instead of using crisp, i.e., a single-valued independent variable vector, on which statistical inference may be done in the case of classical linear regression, fuzzy parameters in the form of, e.g., triangular fuzzy numbers (for some or for all independent variable-vector) with the membership function $A_j(a_j)$ which represents parameter a_j, which have a center (called modal) and spread around the center of the fuzzy number. The membership fuzzy function can be interpreted as follows: the modal value describes the most possible value of the independent variable, while the spread reflects the fuzziness of this variable. Using fuzzy parameters A_j (e. g. in the form of triangular fuzzy numbers) and then applying the extension principle after Zadeh (1965) auf the estimation function, the fuzzy vector of the regression parameter vector can be determined by:

$$\tilde{\boldsymbol{\beta}} = \sup_{\boldsymbol{\beta}=\boldsymbol{f}(x_1,\ldots x_m)} \min\left(m_{\tilde{X}_1},\ldots,m_{\tilde{X}_m}\right), \forall\, \boldsymbol{\beta} \in \mathbb{R}^m \qquad [7]$$

where $m_{\tilde{x}_1}$ is the membership function over the set $\mathbb{R}$ of real numbers with a membership degree between 0 and 1. Due to lack of space, the analytical derivation of the whole procedur will be omitted, and only the computaion procedure will be shortly explained. Fig. 2 describes the Fuzzy-Bayes regression analysis, which comprises two basic Steps:

Step I. The input data is merged in the data matrix called design matrix (**X**). In the example, described below, it includes information about area, living space, age, standard land value and equipping standard. Considering the data as non-fuzzy, its mean x_m comprises the results by fitting with the Bayesian regression analysis with non-fuzzy prior information about the observations. The result of phase I will be partially used as input data sets for the Step II.

Step II. The input values (elements of the designmatrix **X**) with an epistemic uncertainty are fuzzified in this step. For different α-cuts from 0.1 to 1 increasing the vagueness, a large number of simulation (here 250,000) will be computed for different independent realizations $\boldsymbol{X}_{sim,k}$ of the design matrix. The maximal $\tilde{\boldsymbol{\beta}}_{\alpha,\max,\mathrm{MC}}$ and minimal $\tilde{\boldsymbol{\beta}}_{\alpha,\min,\mathrm{MC}}$ of every α-cuts are computed by:

$$\tilde{\boldsymbol{\beta}}_{\alpha,\min,MC} = \min\left[\left(\boldsymbol{X}_{sim,k}^{T}\boldsymbol{X}_{sim,k} + \boldsymbol{V}^{-1}\right)^{-1}\left(\boldsymbol{X}_{sim,k}^{T}\boldsymbol{y} + \boldsymbol{V}^{-1}\underline{\boldsymbol{\beta}}\right)\right]$$
$$\tilde{\boldsymbol{\beta}}_{\alpha,\max,MC} = \max\left[\left(\boldsymbol{X}_{sim,k}^{T}\boldsymbol{X}_{sim,k} + \boldsymbol{V}^{-1}\right)^{-1}\left(\boldsymbol{X}_{sim,k}^{T}\boldsymbol{y} + \boldsymbol{V}^{-1}\underline{\boldsymbol{\beta}}\right)\right] \quad \text{for } k = 1...250,000 \qquad [8]$$

The result is a combined Fuzzy-Bayes-Regression model in full accordance to Zadehs' extension principle for the fuzzy part (Zadeh, 1965). All parameters can be fuzzified by the process and can have Bayesian parts, as well (according to Neumann 2009, Eq. 3.67). By means of the application of the extension principle, the estimation function can be given as fuzzified parameter vector $\tilde{\boldsymbol{\beta}}$ beneath, the model provide Fuzzy-Bayes Confidence Intervals. A prediction can be carried out. The predicted output value has also fuzzy and Bayesian parts.

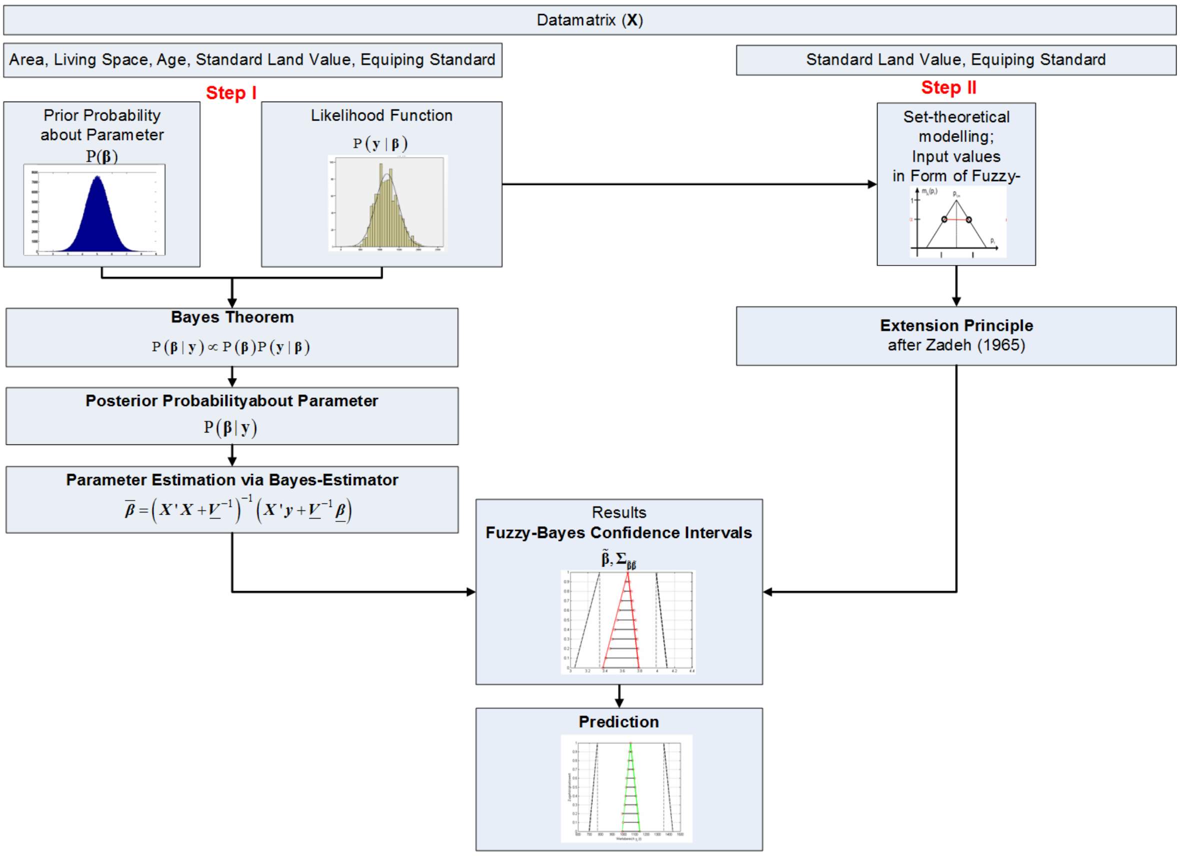


Fig. 2: Flowchart of the analysis process in context of the example presented in this paper.

4. APPLICATION IN THE SUBMARKET OF SINGLE AND TWO-FAMILY HOUSES IN OSNABRUECK

The evaluation was performed in the region of the expert committee for land values Osnabrueck for the spatial sub-markets of the city and the district of Osnabrueck (Fig. 3). Osnabrueck is located in the southwest of Lower Saxony in Germany. The analysed region is very heterogeneous: while the city of Osnabrueck is a regional metropolitan centre, the northern district is particularly very rural. The transactions (purchases) are taken from the automated purchase price collection of the expert committee.

About 280 cases of purchase were done in 2007 (blue circles); these are used as prior information. The 135 purchase of the year 2008 are used as likelihood data (red circles). The analysis includes the functional submarket of free-standing single and two-family homes; as an objective regression was used to purchase price per living area. Significant variables of the selected sample were the area of the lot, the standard land value, the age of the building, the area of living space and the equipment.

Both data sets deviate only slightly from each other. The means are nearly similar; the extent of the data does not vary very strong: Within the regression analysis, possible outliers at the edges of the data were eliminated. This justifies combined use as prior information and likelihood data. The approach is processed with 250,000 simulations.

Fig. 3: The used purchases of the spatial submarket in the region of Osnabrueck - in blue: purchases from 2007, in red: 2008.

5. FIRST RESULTS

The advantage of the Bayesian approach lies in the higher accuracy of the target values by using informative prior information. The functional relationship itself is not improved. Other work like in Weitkamp & Alkhatib (2014) prove the advantage of using prior data in areas with only few transactions and as result only few likelihood data. It is expected, that for the Fuzzy-Bayesian model same advantages could be determined. The difference lies only in the additional consideration of fuzzy variables and therefore in the consideration of epistemic uncertainty sources.

In this example, the design matrix is fuzzified for two rows: standard land value and equipment standard. As consequence of e.g. $\boldsymbol{X}_{sim,k}^{T}\boldsymbol{X}_{sim,k}$, as part of Eq. 8 to compute the parameters of interest, the hold equation is a strongly non-linear function with fuzzy input variables.

Looking at the regression coefficients $b_i > |1|$, the coefficients were instable, because of the low number of simulations (250,000). With a number of data of approx. 400, more than 160,000 combinations within the fuzzified equation exist. The search over the hold range of values of each $\alpha - \text{cut}$ exceeds the computing capacity. For this reason, the minima and maxima search according to Eq. 8 is reduced to realizations of $\boldsymbol{X}_{sim,k}$ where each influence variable is only introduced with its maximum and minimum value of its $\alpha - \text{cut}$.

As a result of i.e. the quadratic non-linearity, the left side within the minima search of the triangular function is left side screwed with a curvature to the outside for regression coefficients $b_i > |1|$. The right side (maxima search) is right side screwed with a curvature to the inside (e. g. Standard Land Value in Fig. 4). The phenomena turns around for regression coefficients $b_i < |1|$. These parameters are stable (e. g. Area in Fig. 4). As a fact, the uncertainty area is unsymmetrical. The solution using prior information is due to the higher possible number of combinations more instable than without. This is broadly discussed also in Neumann (2009, p. 124).

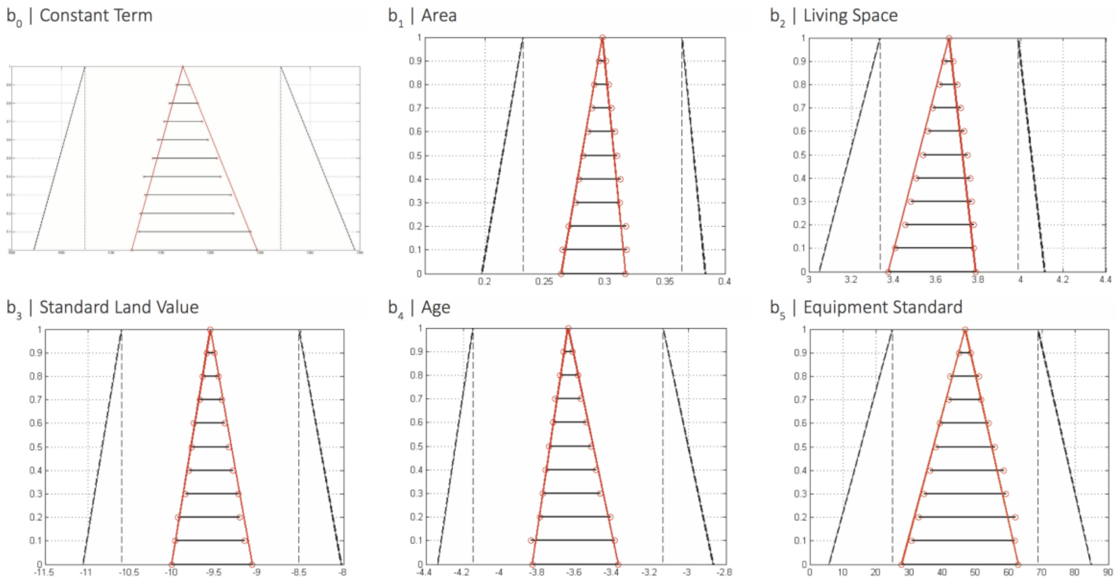
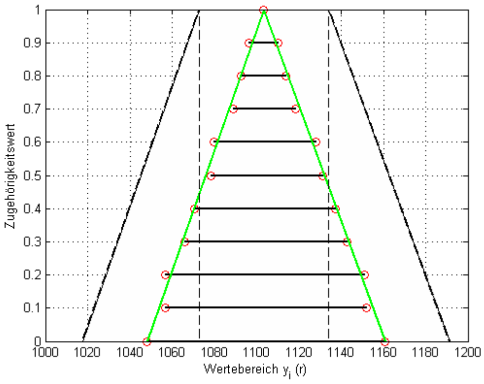


Fig. 4: Regression coefficients of the regression analysis (Fuzzy-Bayes with prior information).



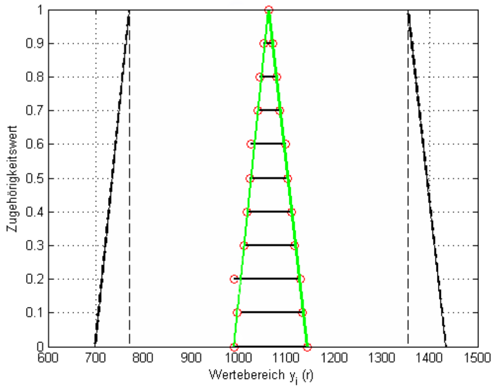
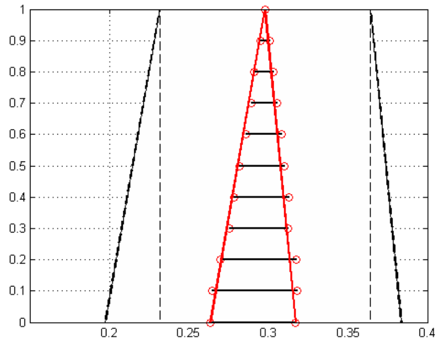


Fig. 5: Predicted Target of Price per Living Space as result of informative prior information.

Fig. 6: Predicted Target of Price per Living Space as result of non-informative prior information.

Fig. 5 shows the predicted price with prior information and Fig. 6 without prior information. The red bullets show the minima and maxima of the alpha-cuts (the green lines connect the alpha-cuts for zero with the alpha-cut for 1). Instead of a Fuzzy-Bayesian confidence interval of nearly 700 EUR/m^2, the Fuzzy-Bayesian confidence interval with using prior information contains only a quarter of this: 170 EUR/m^2 (in alpha-cut zero). The range of values for the epistemic uncertainty is given by the lower and upper bound of each $\alpha-\text{cut}$. This is for the prior information within 110 EUR/m^2 instead of 150 EUR/m^2 for the non-prior $\alpha-\text{cut}$ for $\alpha = 0$. This means an improvement of 25%.

As a result of the fuzzified influence variables, the regression coefficients are also fuzzified and have a Fuzzy-Bayes confidence interval, as well (see Fig. 7 and Fig. 8).



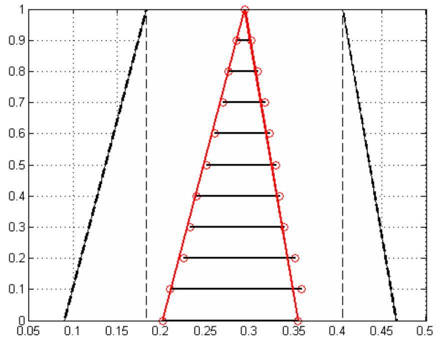


Fig. 7: Regression coefficient of standard land value as result of informative prior information.

Fig. 8: Regression coefficient of standard land value as result of non-informative prior information.

By looking at the range of values for each $\alpha-\text{cut}$, the same result as for the predicted price from Fig. 5 and 6 can be recognized: the range of values for the regression coefficients is more narrow for the use of prior information; without prior information the minima and maxima spread wider.

The difference in the prediction case (Fig. 5 and 6) and the estimation of the coefficients (Fig. 4) lies in the mathematical structure for the propagation of the fuzzy input variables. The estimation from Eq. 8 is strongly non-linear and the prediction case is a linear equation. Therefore, in the prediction case, the alpha-cuts (red bullets) should be theoretically identical with the green line in Fig. 5 and 6. The difference lies only in the limited number of simulation.

6. CONCLUSIONS

In this paper, a Fuzzy-Bayes-Approach is introduced. In the design matrix, the input values of standard land value and equipping standard are extended by means of fuzzy sets. The difference to a standard Bayesian Approach lies only in the treatment of the uncertainties and not in the functional model of the estimation of the target values.
In summary it can be stated that the fuzzification leads to reasonable results. The use of fuzzy variables leads to a more realistic uncertainty budget. The gain of more realistic and reliable uncertainty measures for real estate examples leads to the conclusion of a need for further work with fuzzy variables.
Further works should be done in areas with few transactions. Also, a comparison with a pure Bayesian approach should be carried out. A next step would be a fuzzification, which is based knowledge from several experts instead of a decision made by only one expert.

BIOGRAPHICAL NOTES

Dr.-Ing. **Hamza Alkhatib** received his Dipl.-Ing. in Geodesy and Geoinformatics at the University of Karlsruhe in 2001 and his Ph.D. in Geodesy and Geoinformatics at the University of Bonn in 2007. Since 2007 he has been postdoctoral fellow at the Geodetic Institute at the Leibniz Universität Hannover. His main research interests are: Bayesian statistics, Monte Carlo Simulation, modelling of measurement uncertainty, filtering and prediction in state space models, and gravity field recovery via satellite geodesy.

Prof. Dr.-Ing **Alexandra Weitkamp** received her diploma (Dipl.-Ing.) in “Geodesy” at the University of Hanover in 1999. She passed the highest level state certification as “Graduate Civil Servant for Surveying and Real Estates” in Lower Saxony in 2001. After two-year experience at Bayer AG, she returns to Leibniz Universität Hannover. In 2008 received her Ph.D. in “Geodesy and Geoinformatics” at the University of Bonn. Until 2014, she has been postdoctoral fellow at the Geodetic Institute at the Leibniz Universität Hannover. Since October 2014, she became Chair of Land Management at Technical University of Dresden. Her main research interests are: adaption of innovative evaluation methods for valuation, stakeholders in rural and urban development, and decision-making methods.

Dipl.-Ing. **Sebastian Zaddach** has been a research assistant and is a Ph.D. candidate in real estate valuation at the Geodetic Institute at the Leibniz Universität Hannover, Hanover, Germany. He received his Dipl.-Ing. in Geodesy and Geoinformatics from Leibniz Universität Hannover in 2007. He holds the highest level state certification in Lower Saxony for surveying and real estate. Since April 2014 he holds the position as head of the real estate valuation department for the City and County of Osnabrueck.

Prof. Dr.-Ing **Ingo Neumann** received his Dipl.-Ing. and Ph.D. in Geodesy and Geoinformatics at the Leibniz Universität Hannover in 2005 and 2009, respectively. Since 2012 he is Full Professor in the field of Engineering Geodesy and Geodetic Data Analysis at the Geodetic Institute of the Leibniz Universität Hannover. His research areas are: adjustment theory and error models, multi-sensor systems, quality assessment, geodetic monitoring, terrestrial laser scanning, and automation of measurement processes. He is active in national and international scientific associations and an official delegate of the German and International Organization of Standardization (DIN and ISO).

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