Determination of Minimum Horizontal Curve Radius Used in the Design of Transportation Structures, Depending on the Limit Value of Comfort Criterion Lateral Jerk

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Key words: Jerk (Rate of change of acceleration), Horizontal Curve, Alignment Geometry, Vehicle Dynamics, Standards

SUMMARY

Because of depending on social, cultural and economic developments increasing transportation requirements, as a result of technological developments, vehicles becoming faster and more comfortable and parallel to all developments further increase in the value of time, the significance of road projects have been increasing day by day. Constructing of fast, safe, comfortable and economical transportation systems is an important expectation among the people. To meet the expectations, during the design of transportation structures such as highway, railway, high speed railway, geometric standarts related to the project should be investigated very detailed. The correct selection and implementation of standards is of great importance. Horizontal and vertical curves used in transportation systems are the critical sections on the alignment. An important issue to be addressed in order to increase road safety and comfort during the design of horizontal and vertical curves is determination of the minimum curve radius affected by many factors. While designing of horizontal alignment, smaller than the calculated boundary value of minimum curve radius cannot be used. Therefore, one of the most important boundary values is the minimum curve radius which should be considered in terms of design. Minimum curve radius which is used as design criterion on the horizontal alignment can be derived on the basis of sight lengths, the limit values of superelevation, the limit values of lateral acceleration and the limit values of Jerk (rate of change of acceleration). Evaluation criterion of the alignment geometry in terms of comfort is Jerk. On the horizontal curve, lateral acceleration formed by centrifugal force adversely affects the road safety and reduces vehicle travel comfort. Jerk is defined as the lateral acceleration changing per unit time occurring in horizontal curves. During the curve design, to take into account of Jerk criterion in terms of road safety and comfort is extremely important. In this study, minimum curve radius used as design criterion is derived to take into account of Jerk criterion that can be effective to design comfortable and safety roads which are suitable for road-vehicle dynamic.

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1. INTRODUCTION

In this study as distinct from other studies, minimum horizontal curve radius is derived using the limit value of lateral Jerk. The equations of minimum curve radius related to lateral Jerk are derived by analyzing the equations of lateral Jerk in the literature. Minimum curve radius are calculated by using the equations derived and calculated minimum curve radius are compared with the other minimum curve radius which were calculated with respect to different criterion.

2. DETERMINATION OF MINIMUM CURVE RADIUS

One of the most important boundary values which should be taken into account while designing horizontal alignment is minimum curve radius. Curves combined the different directions of the alignment is critical section of the road. While designing of the road, minimum curve radius must be selected very well for safe and comfortable driving.

Minimum curve radius can be calculated by various methods. The main difference that separates these methods each other is the use of different parameters during the formulation. Minimum curve radius can be determined to take into account superelevation, lateral acceleration, sight distance and lateral Jerk.

2.1 Minimum Curve Radius based on the limit Value of Superelevation

Minimum curve radius can be derived to take into account the limit value of superelevation. Equation of minimum horizontal curve radius depend on the superelevation value is given by American Association of State Highway and Transportation Officials' *A Policy on Geometric Design of Highways and Streets (AASHTO Green Book, 2001)* as follows:

$$\mathrm{R_{min}} = \frac{\mathrm{V}^2}{127(0{,}01\mathrm{e_{max}} + \mathrm{f_{max}})} \tag{1}$$

For railways; equation of minimum curve radius based on the superelevation was expressed as follows: (Baykal,2009).

$$\mathrm{R_{min}} = \frac{\sqrt{\mathrm{b}^2 - \mathrm{u_{max}^2}}\mathrm{V_o^2}}{127{,}14\ \mathrm{u_{max}}} \tag{2}$$

Where;
R_{min} : Minimum curve radius (m)
V : Velocity (km/s)
e_{max} : Maximum superelevation
f_{max} : Maximum allowable side friction factor
u_{max} : Maximum superelevation (m)
V_o : The average speed (km/h)
b : Width of the railway line (m)

2.2 Minimum Curve Radius based on the Limit Value of Lateral Acceleration

The vehicles entered horizontal curve from the linear section of the road move under the influence of various forces. These forces are the centrifugal force, the force created by the weight of the vehicle, lateral friction force and lateral acceleration force.

The change of velocity with respect to time is called acceleration. The lateral acceleration is created by centrifugal force. Minimum curve radius can be derived to take into account the limit value of the lateral acceleration. It was derived for the horizontal geometry as follows: (Baykal, 2009, p.351)

For highways;

$$R_{min} = \frac{V^2}{12{,}96(\sqrt{1 + e_{max}^2}\, a_Y + e_{max}\, g)} \qquad (3)$$

For railways;

$$R_{min} = \frac{\sqrt{b^2 - u_{max}^2}\, V^2}{12{,}96(b\, a_Y + u_{max}\, g)} \qquad (4)$$

Where; a_Y: Lateral acceleration

According to German RAL guide (Richtlinien für die Anlage von Landstrassen) the maximum lateral acceleration value tolerated by passengers into the car is the value of $1.47 m/s^2$. Some literature values of lateral acceleration are as follows:

For highways; a_y: 2,45 m/s^2 (Schofield; 2001), a_y: 1,47 m/s^2 (Umar; Yayla; 1997).
For railways; a_y: 1,22 m/s^2 (Schofield; 2001), a_y: 0,80 m/s^2 (Esveld; 1989), a_y: 1,0 m/s^2 (Förstberg; 2000), a_y: 0,65 m/s^2 (Megyeri; 1993).

2.3 Minimum Curve Radius based on the Limit Value of Lateral Jerk

The change of the acceleration with respect to time is called Jerk. The concept of Jerk is defined as the third derivative of distance (Schot, 1978). The Jerk is also called the second acceleration (German: Ruck, Beschleunigungsaenderug; French: Saccade). In terms of road safety and comfort, Jerk is an important design criterion. Jerk is a value that used to determine the voyage comfort and is known as comfort criterion while designing of road.

The lateral Jerk is defined as the change of lateral acceleration created by the centrifugal force on the horizontal curve with respect to time. For the safety and comfortable driving, lateral Jerk must be below the predetermined limit values. Lateral Jerk was defined to take into account all factors (Horizontal geometry, vertical geometry, all motion model) as follows (Baybura, 2001):

$$\begin{aligned} \mathrm{Z_y} = \frac{\mathrm{bV}}{\sqrt{\mathrm{u^2+b^2}}} \Bigg\{ & 3k_Y a_T + V^2 \frac{\mathrm{d_{kY}}}{\mathrm{d_l}} \pm \frac{\mathrm{u\,V^2}}{\mathrm{b}\sqrt{1+\mathrm{W^2}}} \frac{\mathrm{d_{kD}}}{\mathrm{d_l}} + \Bigg(\frac{-\,\mathrm{k_Y\,V^2\,u}}{\mathrm{u^2+b^2}} - \frac{g}{b} \\ & + \frac{g\,u^2}{b(u^2+b^2)} \pm \frac{k_D V^2}{\mathrm{b}\sqrt{1+\mathrm{W^2}}} \pm \frac{-k_D V^2 u^2}{\mathrm{b}\sqrt{1+\mathrm{W^2}}\,(u^2+b^2)} \Bigg) \frac{\mathrm{d_u}}{\mathrm{d_l}} \\ & \pm \frac{-u^2\,V^2\,k_D\,W}{\mathrm{b(1+W^2)^{3/2}}} \frac{\mathrm{d_W}}{\mathrm{d_l}} \pm \frac{2\,u\,k_D\,a_T}{\mathrm{b}\sqrt{1+\mathrm{W^2}}} \Bigg\} \end{aligned} \tag{5}$$

Where;

Z_y : Lateral Jerk (m/s^3)
V : Design speed (m/s)
b : Horizontal width of road platform (m), for railways; width of the railway line (m)
u : Superelevation (m)
k_Y : Horizontal curvature (1/m)
k_D : Vertical curvature (1/m)
g : Gravitational acceleration (9,81 m/s^2)
a_T : Resultant tangential acceleration (m/s^2)
W : Longitudinal slope
l : Horizontal length of road

The equation (5) can be edited for horizontal geometry. The equation of Lateral Jerk for horizontal geometry (k_D: 0) is as follows:

$$\mathrm{Z_y} = \frac{\mathrm{bV}}{\sqrt{\mathrm{u^2+b^2}}} \left\{ 3k_Y a_T + V^2 \frac{\mathrm{d_{kY}}}{\mathrm{d_l}} + \left(\frac{-\,\mathrm{k_Y\,V^2\,u}}{\mathrm{u^2+b^2}} - \frac{g}{b} + \frac{g\,u^2}{b(u^2+b^2)} \right) \frac{\mathrm{d_u}}{\mathrm{d_l}} \right\} \tag{6}$$

The value of k_Y in the equation (6) represents the curvature of horizontal geometry at any point. k_Y is equal to the value of $(1/r_y)$. (Baykal, 2009)

$$k_Y = \frac{1}{r_Y} \qquad (7)$$

In the equation (7), the value of r_Y is radius of horizontal geometry curvature. On the horizontal curves, radius of horizontal geometry curvature (r_y) is equal to horizontal curve radius.

In the equation (6), minimum curve radius based on the lateral Jerk criterion was obtained using $1/r_Y$ instead of k_Y. The derivatives of the superelevation and radius of curvature with respect to distance are zero.

$$R_{min} = \frac{3\, V_{max}\, a_T\, b}{\sqrt{u^2 + b^2}\, Z_y} \qquad (8)$$

In the equation (8), the expression of $\sqrt{u^2 + b^2}$ (sloping width of road platform) can be equal to the value of horizontal width of road platform (b).

For highways; equation of minimum curve radius based on the lateral Jerk is as follows:

$$R_{min} = \frac{3V_{max}a_T}{Z_y} \qquad (9)$$

For railways; equation of minimum curve radius based on the lateral Jerk is as follows:

$$R_{min} = \frac{3V_{max}\sqrt{b^2 - u_{max}^2}\, a_T}{b\, Z_y} \qquad (10)$$

Where;

u_{max} : Maximum superelevation (m), b : Width of the railway line (m)

According to American Association of State Highway and Transportation Officials (AASHTO; 2001) standards values of Lateral Jerk ranging from 0,3 to 0,9 m/s^3 have been used for highways. Some literature values of lateral Jerk are as follows:
For highways; Z_y = 0,6 m/s^3 (for residential areas), Z_y = 0,3 m/s^3 (for rural highways) (Schofield; 2001), Z_y = 0,6 m/s^3 (Umar; Yayla; 1997), Z_y = 0,6 m/s^3 (Uren; Price; 1985), Z_y = 0,5 m/s^3 (Manns; 1985).
For Railways; Z_y = 0,5 m/s^3 (Megyeri; 1993), Z_y = 0,2 m/s^3 (Esveld; 1989), Z_y = 0,4 m/s^3 (Förstberg; 2000), Z_y = 0,5 m/s^3 (Evren; 2002).

3. NUMERICAL APPLICATION

Based on the different criterion (Lateral Jerk, lateral acceleration, superelevation), table 1 gives the minimum curve radius from 20 to 130 km/h for highways and table 2 gives the minimum curve radius from 20 to 250 km/h for railways.

Minimum curve radius based on lateral Jerk value from 0.3 to 0.9 m/s^3, minimum curve radius based on superelevation value and minimum curve radius based on lateral acceleration value are given in table 1 for highways.

In table 2 minimum curve radius based on lateral Jerk value from 0.3 to 0.6 m/s^3 and minimum curve radius based on the other criterion are given for railways.

In the tables;

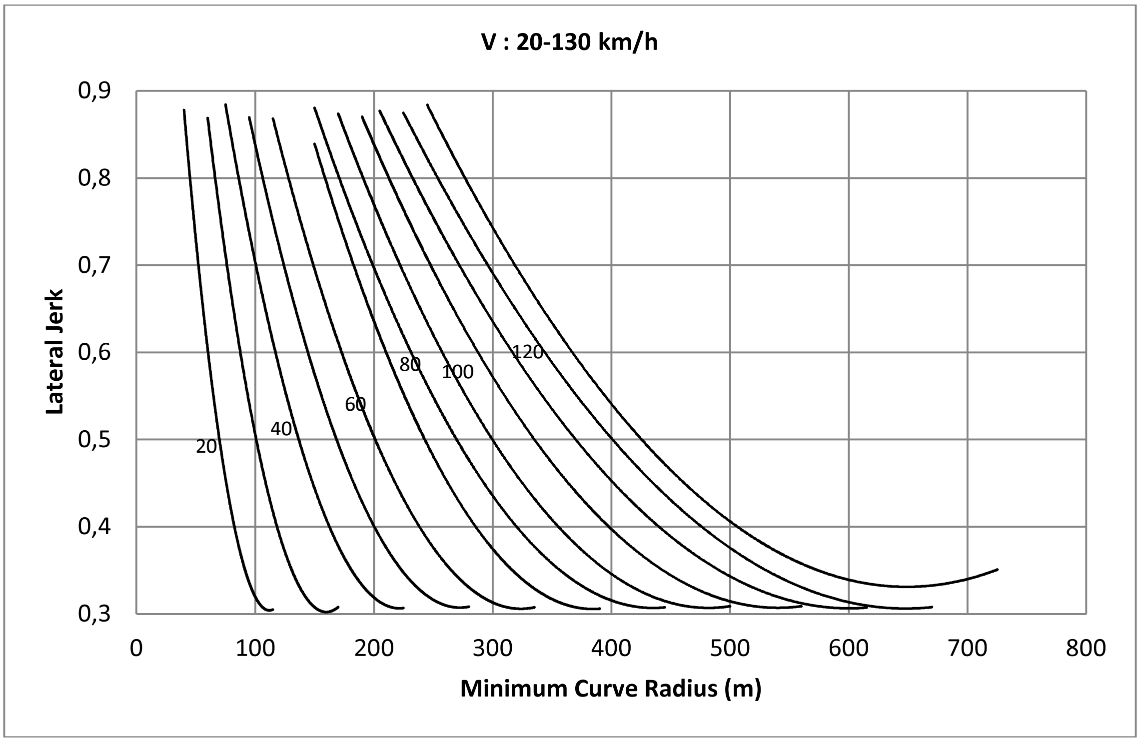
R_{min} : Minimum curve radius
Z_y: Lateral Jerk, Z_y: 0.3-0.9 m/s^3 (AASHTO, 2001)
a_y: Lateral acceleration, a_y: 1.47 m/s^2 for highways (German RAL Guide)
a_y: 0.65 m/s^2 for railways (Megyeri; 1993).
e: Superelevation, e: %4 (AASHTO,2001)
u_{max}: Maximum superelevation, u_{max}: 0.15 m (Baykal, 2009)

V (Km/h)	R_{min} (m)								
	Z_y (m/s^3)							a_y (m/s^2)	e (%)
	Z_y: 0.3	Z_y: 0.4	Z_y: 0.5	Z_y: 0.6	Z_y: 0.7	Z_y: 0.8	Z_y: 0.9	a_y: 1.47	e: 4
20	115	85	70	60	50	45	40	15	15
30	170	125	100	85	75	65	60	35	35
40	225	170	135	115	100	85	75	55	60
50	280	210	170	140	120	105	95	85	100
60	335	250	200	170	145	125	115	125	150
70	390	300	235	195	170	150	170	170	215
80	445	335	270	225	195	170	150	220	280
90	500	375	300	250	215	190	170	280	375
100	560	420	335	280	240	210	190	345	495
110	615	460	370	310	265	230	205	415	635
120	670	500	400	335	290	250	225	495	875
130	725	725	435	365	310	275	245	580	1110

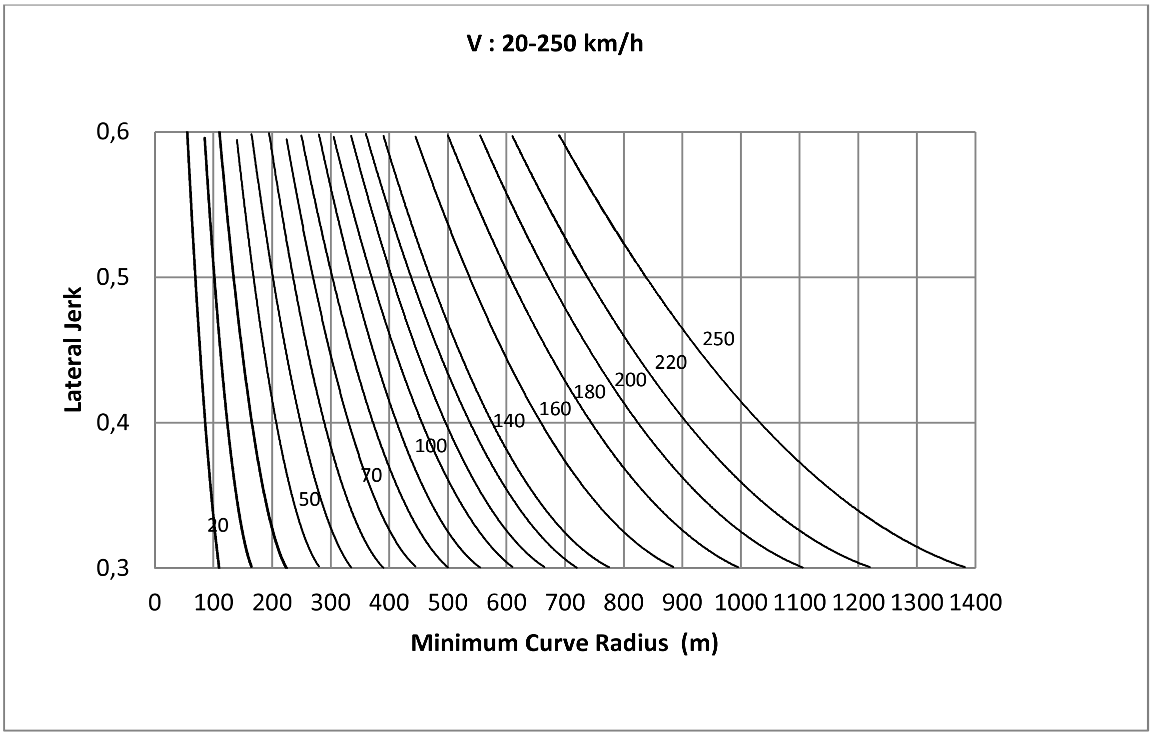
Table 1: Minimum horizontal curve radius for highways

V (Km/h)	Rmin (m)					
	Z_y (m/s^3)				a_y (m/s^2)	u_{max} (m)
	Z_y: 0.3	Z_y: 0.4	Z_y: 0.5	Z_y: 0.6	a_y: 0.65	u_{max} : 0.15
20	110	85	70	55	20	35
30	165	125	100	85	45	70
40	225	165	135	110	75	125
50	280	210	165	140	120	195
60	335	250	200	165	170	285
70	390	290	235	195	230	385
80	445	335	265	225	305	500
90	500	375	300	250	385	635
100	555	415	335	280	470	785
110	610	460	365	305	570	950
120	665	500	400	335	680	1130
130	720	540	435	360	795	1325
140	775	580	465	390	925	1535
160	885	665	530	445	1205	2005
180	995	750	600	500	1525	2535
200	1105	830	665	555	1885	3135
220	1220	915	730	610	2280	3790
250	1382	1040	830	690	2945	4890

Table 2: Minimum horizontal curve radius for railways



Graphic 1: Minimum horizontal curve radius depends on the lateral Jerk for highways



Graphic 2: Minimum horizontal curve radius depends on the lateral Jerk for railways.

For Highways:

1. Z: 0.3 m/s^3 , V: 0-100 km/h $\rightarrow$ $R_J > R_e > R_a$
   V: 100-130 km/h $\rightarrow R_e > R_J > R_a$

R_J : Minimum curve radius based on Lateral Jerk value (m)
R_e: Minimum curve radius based on superelevation value (m)
R_a: Minimum curve radius based on Lateral acceleration value (m)

For safety and comfort, designing horizontal curve radius to take into account lateral Jerk for design speeds from 0 to100 km/h is more acceptable than the others. For design speeds from 100 to130 km/h, R_e must be used.

2. Z: 0.5 m/s^3 , V: 0-70 km/h $\rightarrow$ $R_J > R_e > R_a$
   V: 70-90 km/h $\rightarrow R_e > R_J > R_a$
   V: 100-130 km/h $\rightarrow R_e > R_a > R_J$

For design speeds from 0 to 70 km/h, R_Z must be used.
For design speeds from 70 to130 km/h, R_e must be used.

3. Z: 0.7 m/s^3 , V: 0-50 km/h $\rightarrow R_J > R_e > R_a$
V: 50-70 km/h $\rightarrow R_e > R_J > R_a$
V: 80-130 km/h $\rightarrow R_e > R_a > R_J$

For design speeds from 0 to 50 km/h, R_Z must be used.
For design speeds from 50 to130 km/h, R_e must be used.

For Railways:

1. Z: 0.3 m/s^3 , V: 0-70 km/h $\rightarrow R_J > R_u > R_a$
V: 80-110 km/h $\rightarrow R_u > R_J > R_a$
V: 110-250 km/h $\rightarrow R_u > R_a > R_J$

R_J : Minimum curve radius based on Lateral Jerk value (m)
R_u: Minimum curve radius based on superelevation value (m)
R_a: Minimum curve radius based on Lateral acceleration value (m)

2. Z: 0.4 m/s^3 , V: 0-50 km/h $\rightarrow R_J > R_u > R_a$
V: 50-80 km/h $\rightarrow R_u > R_J > R_a$
V: 90-250 km/h $\rightarrow R_u > R_a > R_J$

3. Z: 0.5 m/s^3 , V: 0-40 km/h $\rightarrow R_J > R_u > R_a$
V: 50-70 km/h $\rightarrow R_u > R_J > R_a$
V: 70-250 km/h $\rightarrow R_u > R_a > R_J$

During the design of horizontal curve, the biggest radius calculated with respect to different criteria must be used for comfortable designing.

4. CONCLUSION

Horizontal and vertical curves used in transportation systems are the critical sections on the alignment. An important issue to be addressed in order to increase road safety and comfort during the design of horizontal and vertical curves is determination of the minimum curve radius affected by many factors. In this study as distinct from similar studies, minimum curve radius used as design criterion on the design of horizontal geometry of alignment was calculated to take into consideration lateral Jerk criterion. The equations of minimum curve radius based on lateral Jerk were derived from the equations of lateral Jerk in the literature. Minimum curve radius calculated according to different values of jerk was compared to minimum curve radius calculated according to other methods and the results were interpreted in terms of road safety and comfort. The biggest minimum curve radius calculated with respect to different criterion could be used for the design of comfortable and safety highways and railways.

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BIOGRAPHICAL NOTES

Ahmet Sami KILINÇ was born in 1986 at Ankara/Turkey. He is a Research Assistant at Uşak University, Turkey. He graduated from the Department of Geodesy and Photogrammetry Engineering at Afyon Kocatepe University in 2009. He is MSc student and currently study on his MSc thesis. His MSc thesis is about highway and railway horizontal geometry. His research interests are Engineering Surveying, GPS and its applications, Highways and Railways Alignment Geometry, Deformation Measurements and Analysis.

Tamer BAYBURA was born in 1966 at Sulz am Neckar/Germany. In 1991 he graduated from The University of Selcuk; Department of Engineering and Architecture Faculty as a Geodesy and Photogrammetry Engineer. He finished his M.Sc. in 1994 at Graduate School of Natural and Applied Sciences that is within The University of Selcuk in Konya/Turkey. His PhD completed in 2001 at Graduate School of Natural and Applied Sciences that is within The Technical University of Istanbul. He worked as an research assistant in The University of Selcuk; Department of Engineering and Architecture Faculty between 1992 and 2000. Continuing from 2000, and current to date, he is working as Assistant Professor in Afyon Kocatepe University; Department of Engineering Faculty. Baybura has many studies on the area like highway and railway alignment geometry and deformation measurements and analysis.

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