NATIONAL TECHNICAL UNIVERSITY OF ATHENS
SCHOOL OF RURAL AND SURVEYING ENGINEERING
DEPARTMENT OF TOPOGRAPHY
LABORATORY OF GENERAL GEODESY

Fast and Convenient Geoid Undulation N Determination in an Urban Area

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FIG Working Week 2011, Bridging the Gap between Cultures
Marrakech, Morocco, 18-22 May 2011

Geoid Undulation N

- The accurate determination of the geoid surface consists one of the **primary targets** in the discipline of geodesy.
- Therefore, **various methods** have been developed, with adequate accuracy of the order of few centimeters.
- **Main advantage** : the opportunity to convert the geometric(h) heights – of no natural significance– to orthometric(H) heights, using the equation $h_i = N_{iD} + H_i$.

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Geoid Undulation N Determination Methods

Methods

- Astrogeodetic leveling.
- Astrogravimetric leveling.
- Satellite altimetry.
- Orthometric (H) & geometric(h) heights.
- Combination of the above.

Geoid maps and **models** in both global and regional scale.

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Global Geoid Models

Earth Geopotential Model 1996 (EGM96)

- Analysis of 55km
- Uncertainty of ±37cm

Earth Geopotential Model 2008 (EGM08)

- Analysis of 9km
- Uncertainty of ±22cm

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Fast and Convenient Direct Determination of N

The great uncertainty of Global Geoid Models → creates the need of local ones → by less labour and time-consuming methods.

THE METHOD : Knowledge of orthometric (H) and geometric (h) heights of uniformed distributed points in the area and determine the undulation N by using the known relationship $N_D = h_i - H_i$.

!!!!! **The examined area should be of limited scale and relatively smooth ground** !!!!!

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Method's Procedure

- Determination of the orthometric (H) heights of the benchmarks.
  - Digital double spirit leveling method (1-2mm).
  - Accurate forward-backward Trigonometric heighting (same order accuracy).
- Establishment of an auxiliary point close (10-15m) from each benchmark for the geometric (h) height determination
- Determination of the geometric (h) heights of the auxiliary points → GPS measurements with the use of a two-frequency (L1&L2) receiver.
- Connection of the auxiliary point with the benchmark using Digital double spirit leveling assuming that $\Delta N=0 \Rightarrow \Delta H = \Delta h$

What makes the Method Fast and Convenient?

- The GPS antenna's placement on **a pole** and not on a tripod, in order to minimize the time required.
- The moving receiver's duration of stay in each point does not exceed **20 minutes.** The determination accuracy cannot be improved even if the receiver stays in that point for more hours.
- The use of a station from **a permanent station network.**

All the abovementioned **minimize the time** required in the field and make the method more **convenient** as less equipment is required.

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The Surface fitting

The equation of the fitting surface can be used in order to determine the val… …tion N_{fi} in other **unl**… …rest.

- The … …mall :
- The …

All the unknown parameters a_i must be tested in order to assess whether they are statistically important for a certain confidence level, using the equation :

$$\sigma_{a_i} \cdot z_{95\%} \leq a_i$$

h_i – … $(\varphi_i-\varphi_o)\cdot(\lambda_i-\lambda_o)$

- The equation … **surface :**

$$h_i - H_i = N_{(\varphi i,\lambda i)} = a_0 + a_1\cdot(\varphi_i-\varphi_o) + a_2\cdot(\lambda_i-\lambda_o) + a_3\cdot(\varphi_i-\varphi_o)^2 + a_4\cdot(\lambda_i-\lambda_o)^2 + a_5\cdot(\varphi_i-\varphi_o)\cdot(\lambda_i-\lambda_o)$$

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The method's accuracy

- The uncertainty of the geoid undulation N in the **known points** of the area is given by the relationship :

$$\sigma_{N_{D_i}} = \pm\sqrt{\sigma_{h_i}^2 + \sigma_{H_i}^2}$$

and it depends upon the determination uncertainty of both geometric and orthometric heights.

- The uncertainty of the geoid undulation N_{f_i} in the **unknown point** of the area is given by the equation :

$$\sigma_{N_{f_i}} = \pm\sqrt{\sigma a_o^{\,2} + (\varphi_i - \varphi_o)^2 \cdot \sigma a_1^{\,2} + (\lambda_i - \lambda_o)^2 \cdot \sigma a_2^{\,2} + 2 \cdot (a_1^{\,2} + a_2^{\,2}) \cdot \sigma\varphi^2}$$

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Application

Test area :

2km * 4km

Athens center,

Greece

16 benchmarks

Application

Measurements :

- **Spirit levelling** and accurate **forward-backward trigonometric heighting** method for the orthometric heights ➡ accuracy from ±1mm to ±4mm .
- **GPS measurements** with the use of one **permanent station** of the Hellenic Positioning System (HEPOS) for the geometric heights ➡ accuracy ±6mm to ±23mm .

Various tests were performed in the examined area to find the equation that best suits. The following plane equation was finally chosen :

$h_i - H_i = N_D(\varphi_i,\lambda_i) = 38.5227+252.0681\cdot(\varphi_i-\varphi_o)+140.6205\cdot(\lambda_i-\lambda_o)$

with standard deviation $\acute{o}_0 = \pm 11\text{mm}$

By using the above equation the geoid undulation N of **other unknown** points can be calculated with an uncertainty of ±1cm.

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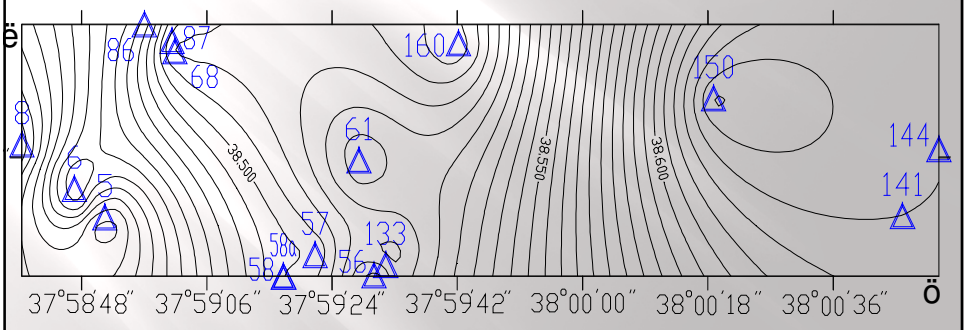
Map presentation

- A local geoid map was also created using the interpolation method Kriging.



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Global Model enrichment

- New values of the geoid undulation N were determined by the EGM08 in the known 16 points of the tested area.
- These values were adjusted with those which were directly determined by using orthometric and geometric heights.
- The following solution was reached after the enrichment :

$$\delta N_i = = -0.5898+190.6845\cdot(\varphi_i-\varphi_o)-113.5694\cdot(\lambda_i-\lambda_o)$$

With standard deviation $\hat{\sigma} = \pm 2.2 cm$.

This procedure is used for the global model's enrichment with terrestrial data.

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Concluding remarks(1)

- The determination of **local geoid models** is indispensable especially in urban areas where more infrastructure works are carried out.
- The geoid undulation N can be determined **directly** in an **urban area** by an uncertainty of about **±2cm**.
- **Twenty minutes** of observations are enough for the h determination with an uncertainty of ±2cm, which makes the method very fast.
- The use of **permanent stations** leads to fast determination of the geometric height in an urban area by using a double frequency receiver mount on a pole.

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Concluding remarks(2)

- The procedure of the fitting of a **plane surface** in a concrete area proved to be adequate and can be used for the majority of infrastructure works
- The **enrichment** of the EGM08 by using the available terrestrial data really improves the global model's result. The differences between the EGM08 after the enrichment and the terrestrial data are of the order of about 1.5 cm, before the enrichment the differences were about 6cm.

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THANK YOU FOR YOUR ATTENTION

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