Proper Choice of Data Used for the Estimation of Datum Transformation Parameters

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Key words: Coordinate systems; transformation; estimation, reliability.

SUMMARY

Advances in technologies and science have progressively allowed defining better terrestrial coordinate systems or their reference frames used in surveying. This circumstance always keeps datum transformation from one system, or one frame, to another up to date. Data used for the estimation of datum transformation parameters are the coordinate components of common points whose coordinates are known in both system of datum transformation. Obtaining a reliable set of the parameters requires a proper choice of the common points. What number of the common points should be used? Or, is their distribution suitable? These questions are too hard to decide by rule of thumb. Sometimes, some components of some common points may need to be eliminated while some may be necessary for a reliable solution. In this study, a rigorous method is suggested and useful remarks are made for the proper data choice in datum transformation.

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1. INTRODUCTION

Advances in technologies and science have progressively allowed defining better terrestrial coordinate systems or their reference frames used in surveying. This circumstance always keeps datum transformation from one system, or one frame, to another up to date. As the terrestrial coordinate systems are orthogonal coordinate systems a complete datum transformation between them are established by the seven parameter similarity transformation (Seeber, 1993). However, data used for the estimation procedure of the transformation parameters are the coordinate components of common points whose coordinates are known in both system of datum transformation, so alternative transformation methods in two or three dimension such as affine and polynomials can also be applied in practice to minimize the residuals of coincidence between the common point coordinates.

It is fact that the common point coordinates can be burdened with distortions due to some random and systematic errors and environmental effects such as crustal movements (Kutoglu, 2004). These distortions can be absorbed during the parameter estimation, and can cause the biased transformation parameters. The more the parameters are used for the transformation the more the distortions are absorbed. Obtaining a reliable set of the parameters requires a proper choice of the common points. What number of the common points should be used? Or, is their distribution suitable? These questions are too hard to decide by rule of thumb. Sometimes, some components of some common points may need to be eliminated while some may be necessary for a reliable solution. In this study, a rigorous method is suggested and useful remarks are made for the proper data choice in datum transformation.

2. METHOD FOR PROPER CHOICE OF COMMON POINTS

In a parameter estimation problem by the least square adjustment, redundancy numbers which are the contribution of each data to the degree of freedom of the adjustment are computed by

$$r_i = \left(\mathbf{I} - \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{P}\right)_{ii} \tag{1}$$

where **A** is the coefficient matrix multiplied by the unknown parameters vector of the adjustment, **P** is the weight matrix for the common point coordinates composed of the precision estimations of the coordinates, **I** is the unit matrix (*i* is the running index associated with the data number and *ii* represents the diagonal elements of the resulting matrix on the right hand side).

Using the redundancy numbers one can estimate possible distortions in each coordinate component

$$\nabla_i = \frac{\nabla \mathbf{v}_i}{r_i} \approx \frac{\mathbf{v}_i}{r_i} \quad (2)$$

and then the absorbed amount of the distortion

$$A_i = \frac{1-r_i}{r_i} v_i \quad (3)$$

where $\mathbf{v}_i$ is the residual of the data *i* while $\nabla \mathbf{v}_i$ is the distortion impact on the residual (Baarda, 1967 and 1976).

As seen in the equations increasing redundancy number decreases the possible distortion and the amount of absorption. Accordingly, a proper choice of common points to obtain reliable transformation parameters can be accomplished by regarding the redundancy numbers (Kutoglu, 2006).

3. CASE STUDY

For conducting a sample application, 20 control points have been determined in Istanbul Metropolitan Area, Turkey (Fig. 1). These points have coordinates both in ED50 datum and ITRF94. Let us think these points are coordinated only in ED50. Anyone who wants to produce datum parameters between ED50 and ITRF can use all these points after obtaining their coordinates in ITRF by an observation campaign carried out on the field. On the other hand, someone else can decide to choose the proper points applying the above-mentioned method. Such a proper set of the points proposed by the method are illustrated in Fig 2. During this designation, the transformation method assumed is the three dimensional similarity transformation and the redundancy numbers are taken care of being homogenous and not below 0.50.

Table 1: Datum parameters obtained from both approaches

Parameter	**Using all the points**	**Using the proper points**
$\mathbf{t_x}$ (m)	87.359±0.017	87.390±0.027
$\mathbf{t_y}$ (m)	91.181±0.017	91.224±0.027
$\mathbf{t_z}$ (meter)	127.494±0.017	127.541±0.027
$\varepsilon_\mathbf{x}$(arcsecond)	-3.808±0.212	-3.658±0.212
$\varepsilon_\mathbf{y}$ (arcsecond)	0.132±0.325	-0.102±0.380
$\varepsilon_\mathbf{z}$ (arcsecond)	1.550±0.190	1.666±0.220
k (ppm)	3.245±0.827	3.170±0.952

The transformation parameters are estimated for both approaches stated above. Table 1 summarizes the obtained results. As seen the table, the parameters obtained from both approach are quite close to each other.

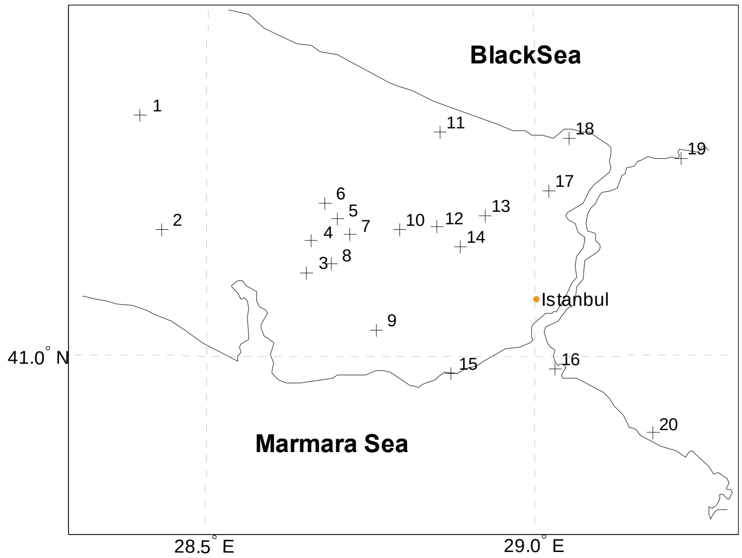

Figure 1. Common points in Istanbul Metropolitan Area

In the first approach, the redundancy numbers change between 0.70-0.95 (Fig. 3). The redundancy numbers over 0.90 belong to the points heaped up in the center area. These values show that there is over control which means the majority of the points in this area do not have an important contribution to the solution. The main disadvantage of that situation is that observing such points for producing common points will not be cost-effective. Secondly, such heaped points may localize a solution.

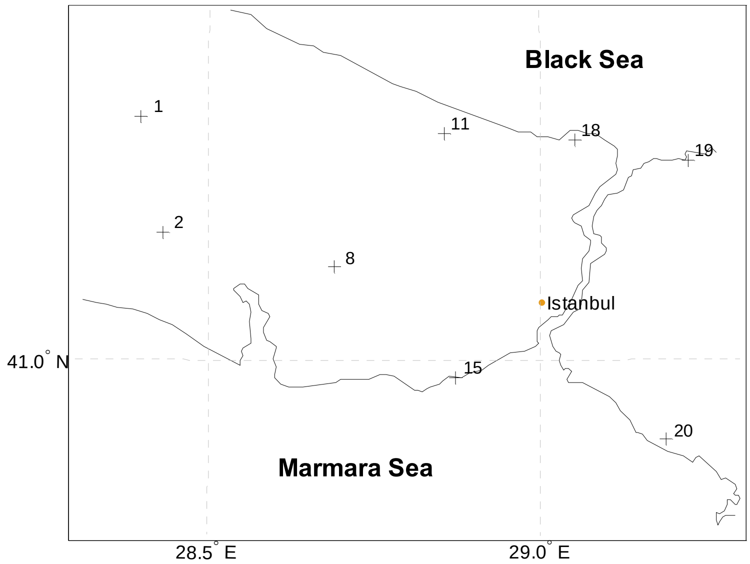

Figure 2. A proper choice of common points

After applying the proposed method, almost all of the points in the centre area have been eliminated, and the number of the common points has been reduced from 20 to 8 whose redundancies range between 0.55-0.84 (Fig. 2 and 4). However, the remaining points have still produced similar parameters to the previous ones. Also, use of the method provides some further information about the data such as possible distortion and absorption (Fig. 5). These further information allow practitioners to make more rigorous analysis for the point selection.

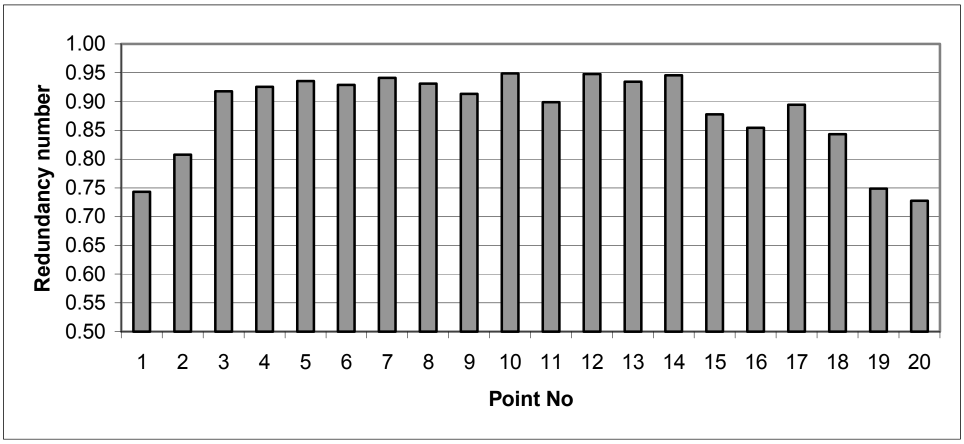

Fig. 3. Redundancy numbers in the case that all the points are used

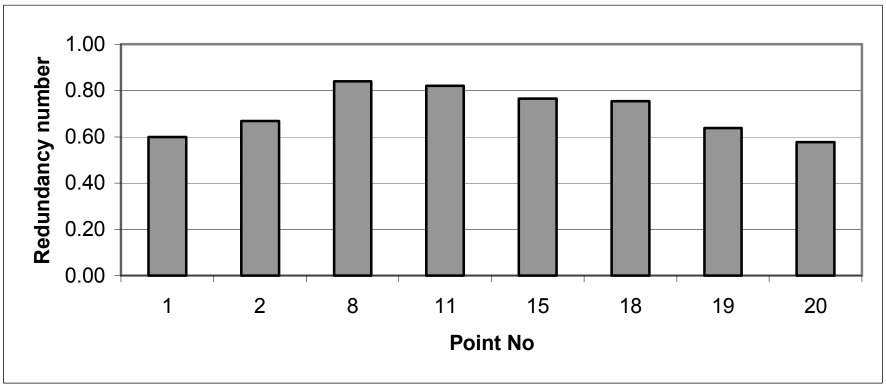

Fig. 4. Redundancy numbers for the selected points

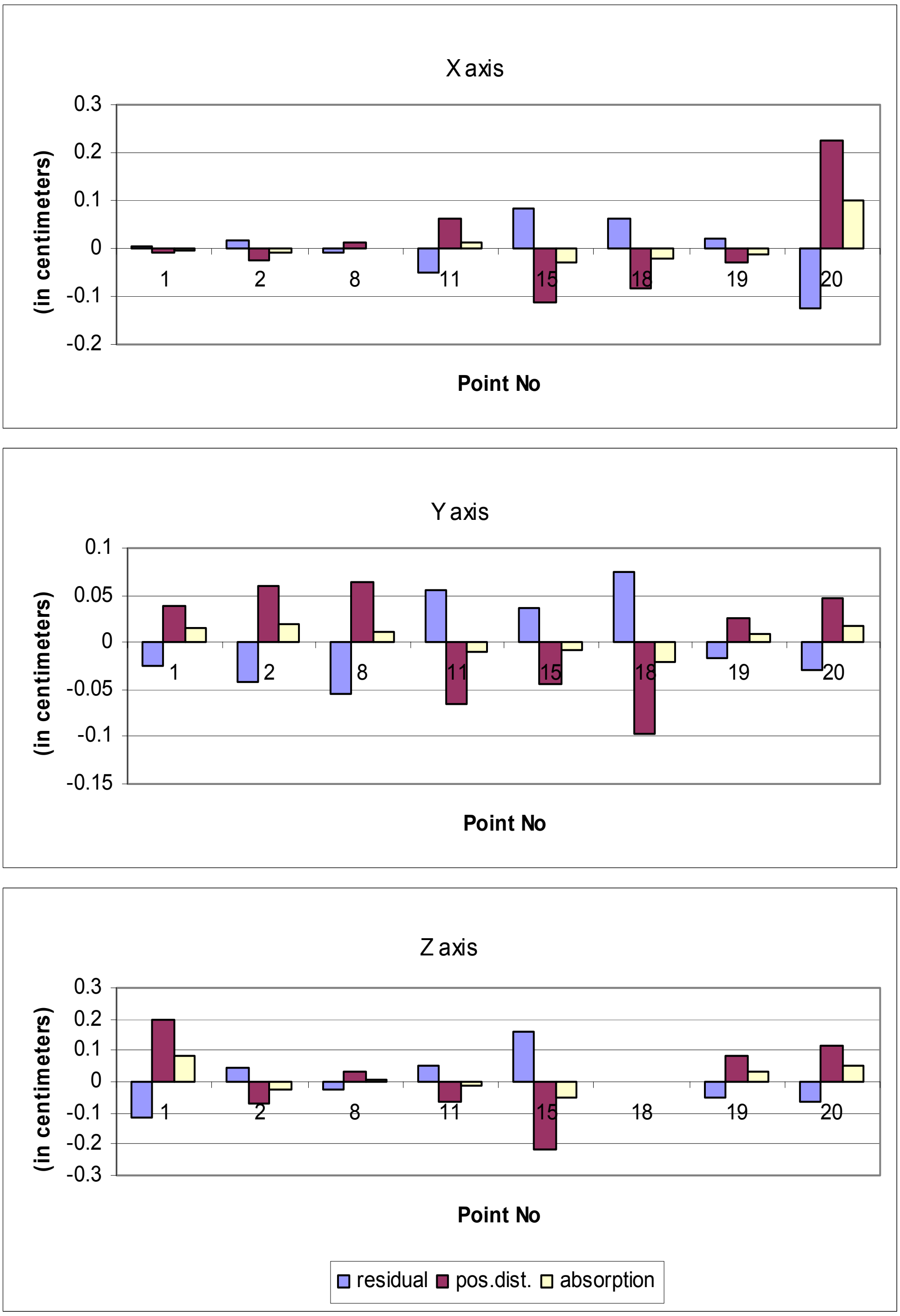

Fig. 5. Residuals, possible distortion and absorption in coordinates of the selected points

4. CONCLUSION

In this study, a useful method is proposed for the common point selection in datum transformation. Applying this method, one can

- avoid unreliable transformation parameters caused by improper number and distribution of the common points,
- avoid producing redundant common points, and thus provides cost-effective solutions,
- provide further information, such as possible distortion and absorption, about the common points, so one can carry out a more rigorous analysis for the point selection. For instance, some components of some common points may need to be eliminated while some may be kept for a reliable solution.

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BIOGRAPHICAL NOTES

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