Investigation of Transition Curves With The Lateral Change of Acceleration For Highways Horizontal Geometry

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SUMMARY

Circular horizontal geometry and simple superelevation application are used adequately for safety and comfort in design and application of roads without the need of high speed when combining alignments. Nevertheless they are just not appropriate for high speed roads.

Generally transition curves are used two straight lines with a circle but this conceive not adequate for high speed project and road dynamic. So it has been necessary to investigations new solutions.

New transition curves are defined with lateral change of acceleration. This equation can be used all stipulation related to horizontal geometry and vehicle motion. New curve1 and curve 2 (Tari 1, Tari 2) and classical transition curve (Clothoid and sinuzoidal) are compared with lateral change of acceleration.

Why new generation curves?

- In the recent years with developments in sciences on account of high speeds on motorways and railways have got better the horizontal geometry of roads with respect to road dynamic.
- In design of the combining alignments using an arc of circular curves not adequate for safety and comfort wherefore effects of instantaneous centrifugal force.
- Importance of security
- Comfort of drive
- Deficiency of superelevation which is the applicate our present day
- Abrasion and deformation for highways, railways and vehicles.

Aim of this presentation

- In this study known mathematics functions; curvature, superelevation and lateral change of acceleration functions are examined for classical and new transition curve and then developed new computer programme.
- All transition curves which mentioned in this study are examined with diagrams of lateral change of acceleration. Value of velocity, curve lengt, curve radius and superelevation are entered by all of the programme users in this new software.
- New and classical transition curves according to these diagrams are compared and understanded that suitable by known criterias. To give an example of new curve 1 (Tari 1) and sinuzoidal curve programme codes and illustrated with diagrams

Features of my own programme

- Value of velocity, curve length, curve radius and superelevation are entered by all of the programme users in this new software

- Likewise Start and finish value of coordinate systems are entered by all of the programme users

- New and classical transition curves are compared with regard to lateral change of acceleration by new programme

$$z = \frac{da}{dT}\vec{n}$$

1-Lateral Change of Acceleration (1)

Lateral change of acceleration is very considerable criterion when designed to horizontal geometry for road vehicle dynamic. It is the change of resultant acceleration occurring along the curve normal respect to time. This changes are formed by unbalanced forces, vehicle mass and velocity moving on a curved orbit (Tarı 1997, Baykal 1996)

Why lateral change of acceleration?

- Lateral change of acceleration comprise all of the vehicles motion and horizontal geometry of road.

- z=lateral change of acceleration(m/s3)
- a=resultant acceleration produced by free forces(m/s2)
- T=time(s)
- $\vec{n}$=unit vector along the curve normal

Lateral Change of Acceleration (2)

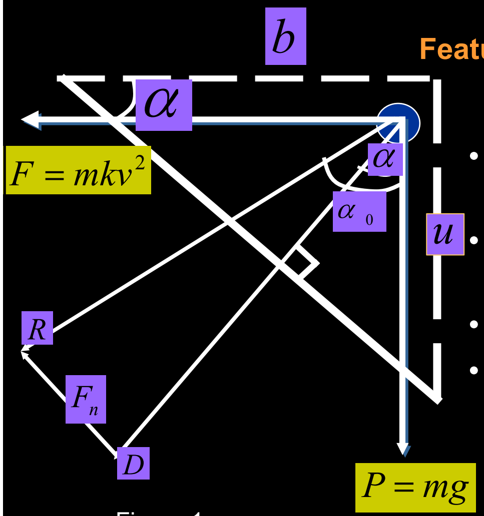


Figure 1

Features of lateral change of acceleration

- Moving on a curved orbit for horizontal geometry of road
- Acting on a vehicle with mass (M) an instantaneous velocity (v)
- Effects of unbalanced forces
- Resultant force of the gravitational and centrifugal forces (R)

Lateral change of acceleration (3)

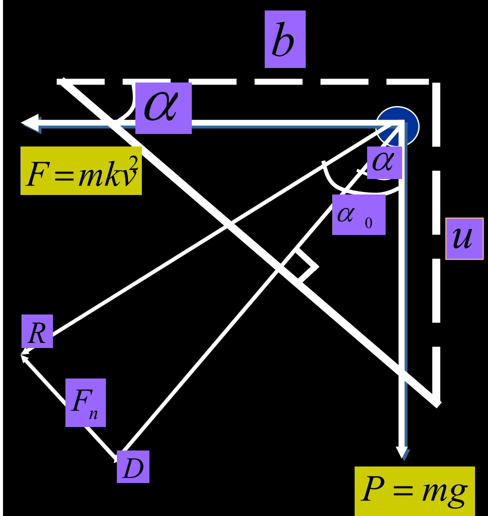


Figure 1

Vehicles moving on a superelevated road are the gravitational force *(P=mg),* the centrifugal force *(F=mkv2)* and the motor force *(Ft=mat).*

According to figure1 resultant force (R) of the gravitational and centrifugal forces can be divided D and Fn components. D component is balanced with the reaction of the road platform.

Lateral Change of Acceleration (4)

Fn can be expressed according to figure 1

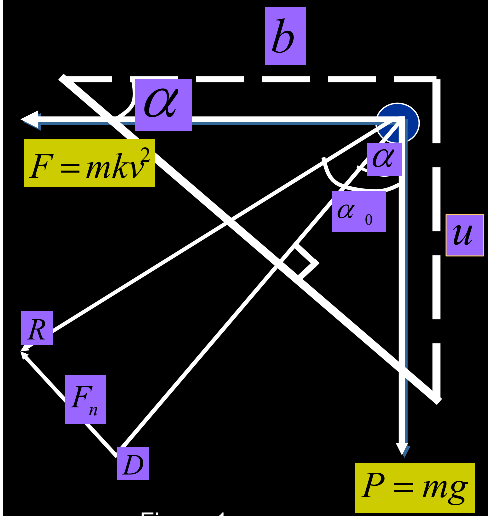


Figure 1

$$F_n = m(kv^2 - g\tan\alpha)\cos\alpha \quad (1.1)$$

can be derived.

$$\tan\alpha = \frac{u}{b} \qquad \cos\alpha = \frac{b}{\sqrt{u^2+b^2}} \quad (1.2)$$

nto (1.1)

$$a_n = \left\{ \left(kv^2 - \frac{g}{b}u \right) \frac{b}{\sqrt{u^2+b^2}} \right\} \vec{n} \quad (1.3)$$

can be derived.

in this equalization $\vec{t}$ is the unit vector along the direction of the tangent of curve. The equation of lateral change of acceleration is obtained as follows;

$$\vec{a} = \frac{dv}{dT}\vec{t} + \frac{b}{\sqrt{u^2+b^2}}\left(kv^2 - g\frac{u}{b}\right)\vec{n}$$

(1.4)

- *u*: superelevation (m)
- *b*: horizontal width of the road platform (m)
- *g*: gravitational acceleration (9.81 m/sec2)
- *k*: curvature of orbiting curve (1/m)
- *an*: acceleration along the direction of the curve normal on an inclined road platform (m/sec2)

$$z = \frac{da}{dT}\vec{n} = \frac{bv}{\sqrt{u^2+b^2}}\left(3ka_t + v^2\frac{dk}{dl} - \frac{kv^2u + gb}{u^2+b^2}\frac{du}{dl}\right) \quad (1.5)$$

This equation of lateral change of acceleration can be used for all conditions related to road horizontal geometry and vehicle motion.(Baykal 1996)

2- Investigate to Lateral Change of Acceleration Function of clothoid curve With Constant Velocity Model (classical curves1)

Superelevation and curvature function of first clothoid curve can be expressed as,

$$\left.\begin{aligned} k_{k,l}(l) &= \frac{l}{L_1 R} \\ u_{k,1}(l) &= \frac{ul}{L_1} \end{aligned}\right\} \quad 0 \le l \le L_1 \qquad (2.1)$$

the function of lateral change of acceleration of the recommended first clothoid curve is derived from (1.5) by taking consideration (2.1) and using the transformation of variable of *t=l/L* and differentiated with respect *l* as follows; (Tarı 1997)

$$z_{k,1}(t) = \frac{L_1^2 v(v^2 - gR\tan\alpha_m)}{L^3 R(t^2 \tan^2\alpha_m + \frac{L_1^2}{L^2})^{3/2}} \qquad 0 \le t < \frac{L_1}{L} \qquad (2.2)$$

where

$$\tan\alpha_m = \frac{u}{b} \qquad (2.3)$$

l: Horizontal length of arc of orbital curve,
L: Total length of arc of orbital curve.

Superelevation and curvature function of the arc of circle can be expressed as,

$$k_{k,2}(t) = \frac{1}{R} \qquad u_{k,2}(t) = u \qquad (2.4)$$

the function of lateral change of acceleration of the recommended arc of circle is derived from (1.5) by taking consideration (2.4) as follows;

$$z_{k,2}(t) = 0 \qquad \frac{L_1}{L} \leq t < \frac{L_1 + L_2}{L} \qquad (2.5)$$

Superelevation and curvature function of second clothoid curve can be expressed as,

$$k_{k,3}(l) = \frac{1}{RL_3}(L - l) \qquad u_{k,3} = \frac{u}{L_3}(L - l) \qquad (2.6)$$

$$L_1 + L_2 \leq l \leq L$$

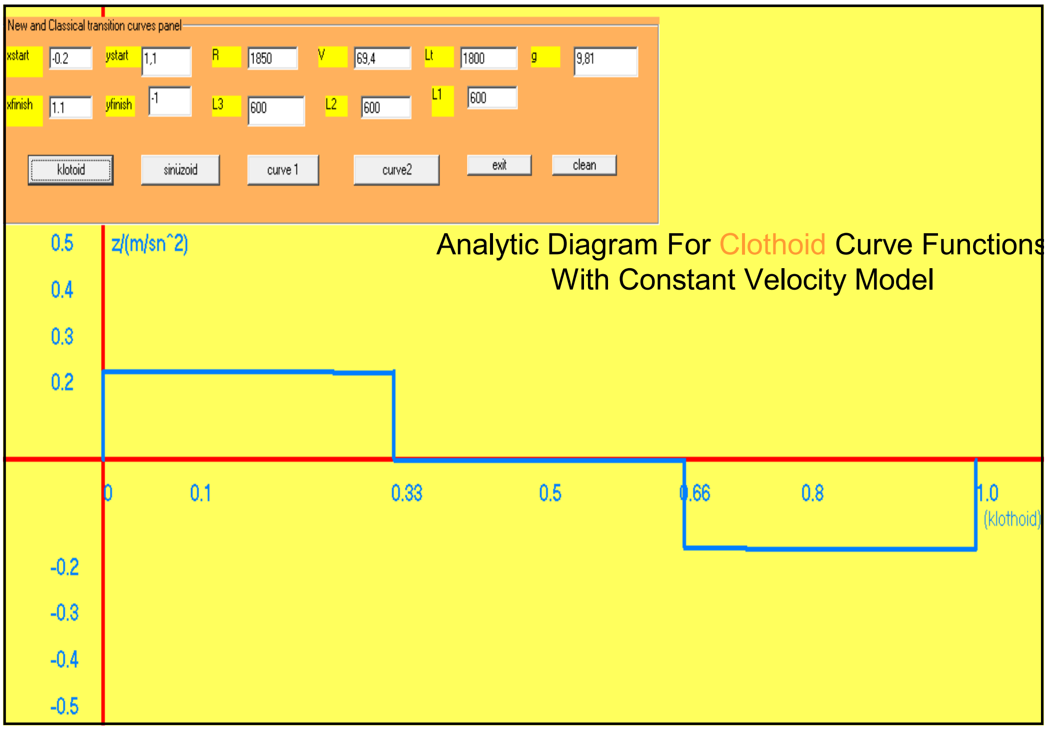
the function of lateral change of acceleration of the recommended second clothoid curve is derived from (1.5) by taking consideration (2.6) as follows;

$$z_{k,3}(t) = -\frac{L_3^2 v(v^2 - gR\tan\alpha_m)}{L^3 R((1-t)^2 \tan^2\alpha_m + \frac{L_3^2}{L^2})^{3/2}} \qquad \frac{L_1 + L_2}{L} \leq t \leq 1 \qquad (2.7)$$

can be derived. (Tarı 1997)

Equalization of (2.2),(2.5) and (2.7) are considerated then examinated with analytical diagram by new programme. In this programme;

Horizontal length of transition curve and arc of circle	*L1=L2=L3=600m*
Superelevation	*u=0.15m*
Total length of arc of orbital curve.	*L=1800m*
Radius	*R=1850m*
Velocity	*V= 230 km/h*



3- Investigate to Lateral Change of Acceleration Function of sinuzoidal curve With Constant Velocity Model

The function of lateral change of acceleration of the recommended first sinuzoidal curve, arc of circle and second circular curve are derived from (1.5) by taking consideration superelevation and curvature function and using the transformation of variable of *t=l/L* and differentiated with respect *l* as follows; (Tarı 1997)

$$z_{s,1}(t) = \frac{N_{s,1} v (v^2 - gR \tan \alpha_m)}{R(1 + Q_{s,1}^2 \tan^2 \alpha_m)^{3/2}} \qquad 0 \le t < \frac{L_1}{L}$$

First sinuzoidal curve
(3.1)

$$z_{s,2}(t) = 0 \qquad \frac{L_1}{L} \le t < \frac{L_1 + L_2}{L}$$

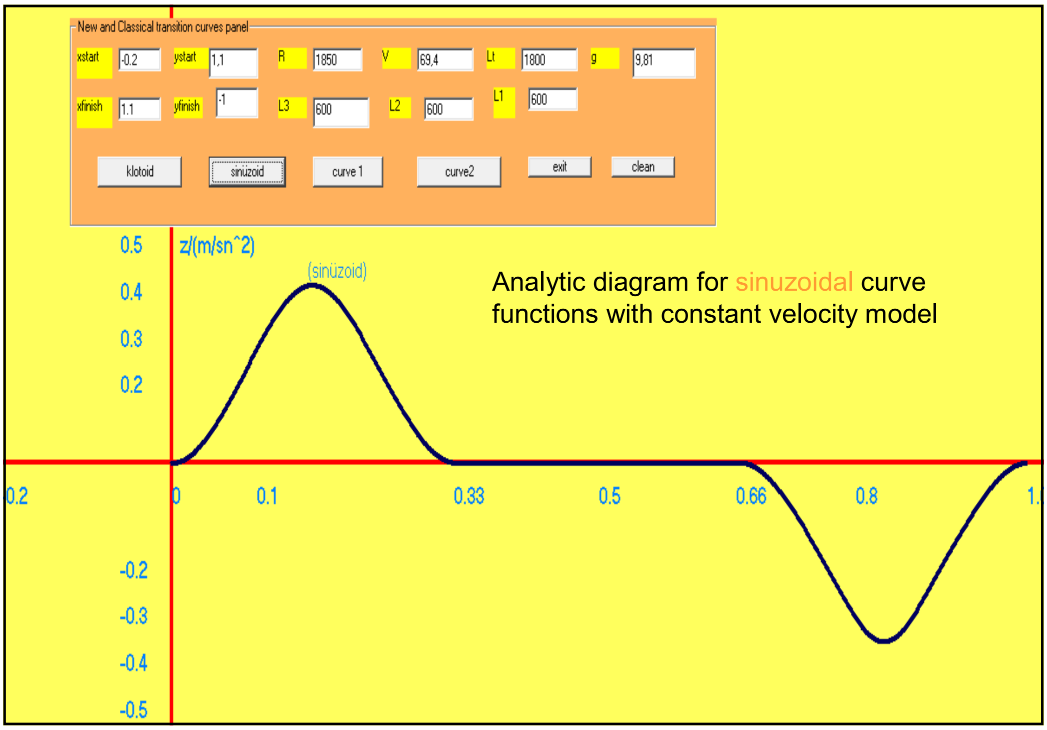
Arc of circle
(3.2)

$$z_{s,3}(t) = \frac{N_{s,2} v (v^2 - gR \tan \alpha_m)}{R(1 + Q_{s,2}^2 \tan^2 \alpha_m)^{3/2}} \qquad \frac{L_1 + L_2}{L} \le t \le 1$$

second sinuzoidal curve
(3.3)

Equalization of (3.1),(3.2) and (3.3) are considerated then examinated with analytical diagram by new programme. In this programme;

Horizontal length of transition curve and arc of circle	*L1=L2=L3=600m*
Superelevation	*u=0.15m*
Total length of arc of orbital curve.	*L=1800m*
Radius	*R=1850m*
Velocity	*V= 230 km/h*



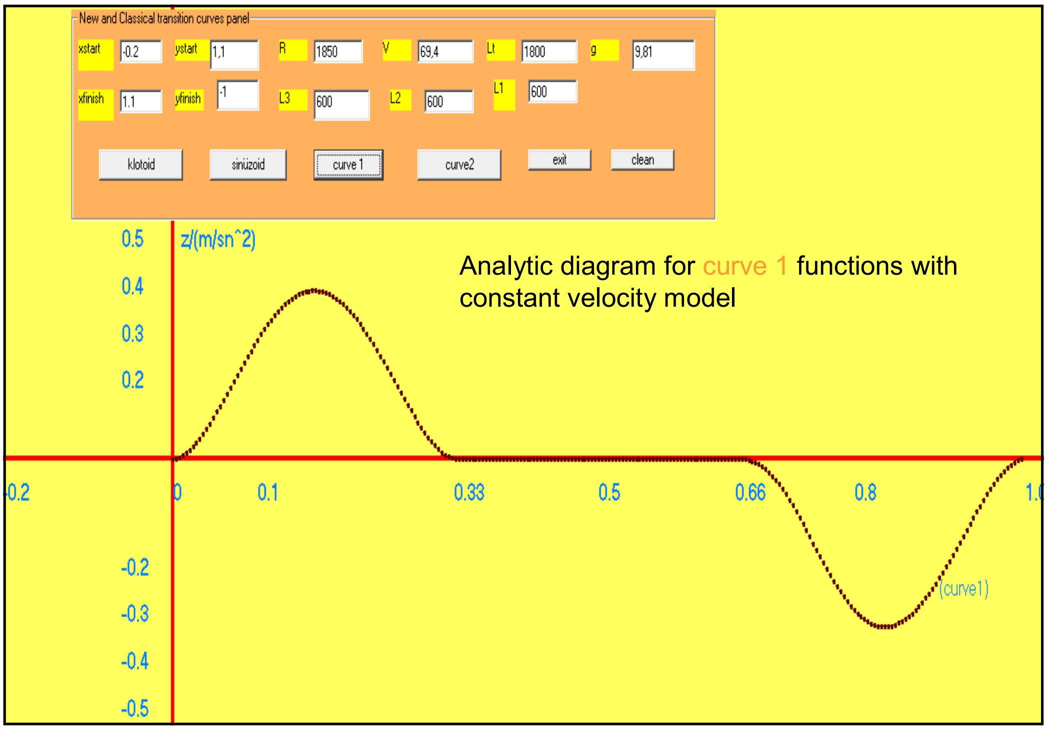
4- Investigate to Lateral Change of Acceleration Function of curve 1 With Constant Velocity Model

The function of lateral change of acceleration of the recommended first curve 1 arc of circle and second curve 1 are derived from (1.5) by taking consideration superelevation and curvature function and using the transformation of variable of *t=l/L* and differentiated with respect *l* as follows; (Tarı 1997)

$$z_{1,1}(t) = \frac{N_{1,1}v(v^2 - gR\tan\alpha_m)}{R(1+Q_{1,1}^2\tan^2\alpha_m)^{3/2}} \qquad 0 \le t \le \frac{L_1}{L} \qquad \text{First curve 1} \quad (4.1)$$

$$z_{1,2}(t) = 0 \qquad \frac{L_1}{L} \le t \le \frac{L_1 + L_2}{L} \qquad \text{Arc of circle} \quad (4.2)$$

$$z_{1,3}(t) = \frac{N_{1,2}v(v^2 - gR\tan\alpha_m)}{R(1+Q_{1,2}^2\tan^2\alpha_m)^{3/2}} \qquad \frac{L_1 + L_2}{L} \le t \le 1 \qquad \text{second curve 1} \quad (4.3)$$



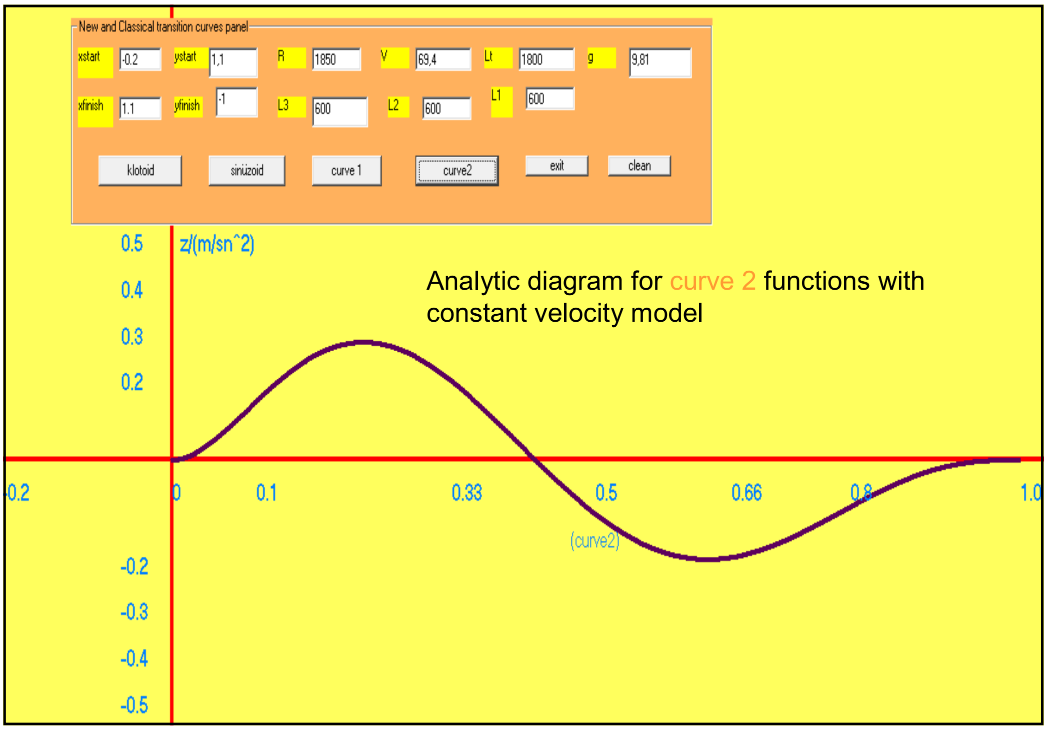
5- Investigate to Lateral Change of Acceleration Function of curve 2 With Constant Velocity Model

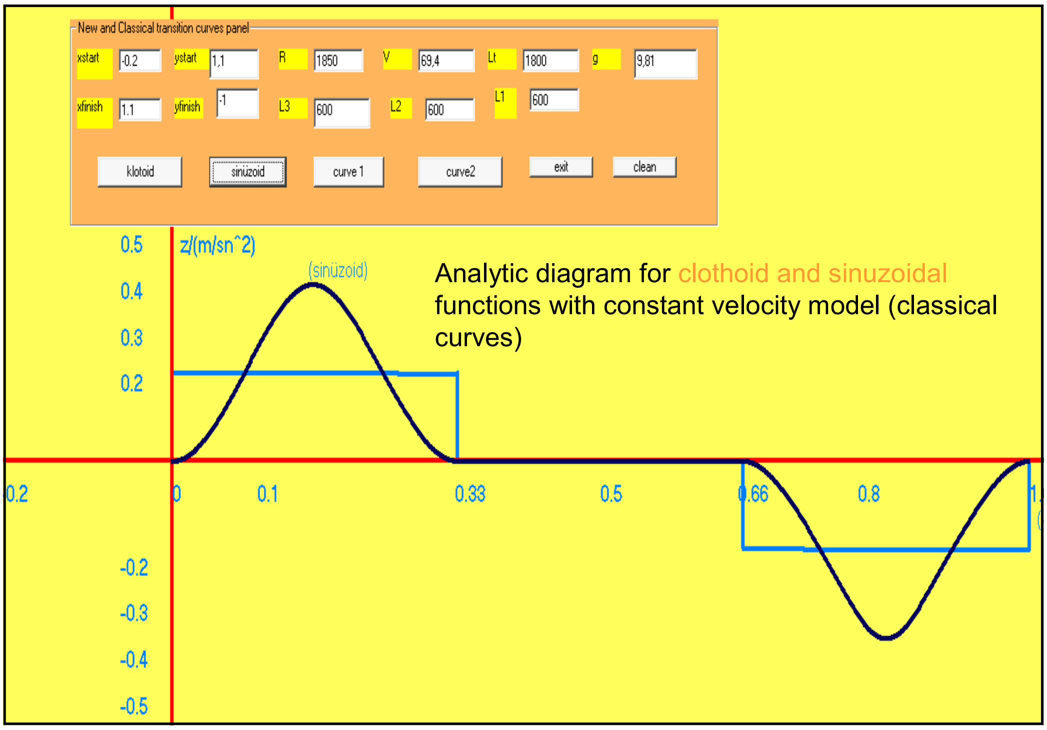
The function of lateral change of acceleration of the recommended curve 2 is derived from (1.5) by taking consideration superelevation and curvature function and using the transformation of variable of *t=l/L* and differentiated with respect *l* as follows; (Tarı 1997)

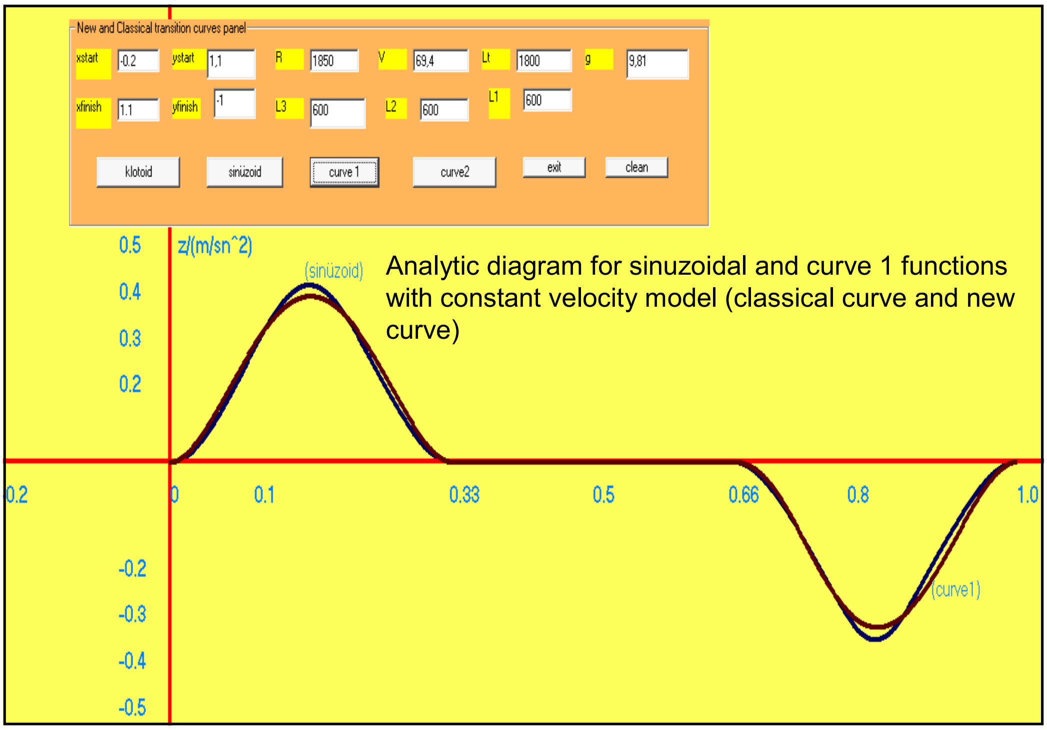
$$z_2(t) = \frac{N_2 v(v^2 - gR\tan\alpha_m)}{LR(1+Q_2^2\tan^2\alpha_m)^{3/2}} \qquad 0 \le t \le 1 \qquad \text{Curve 2} \quad (5.1)$$

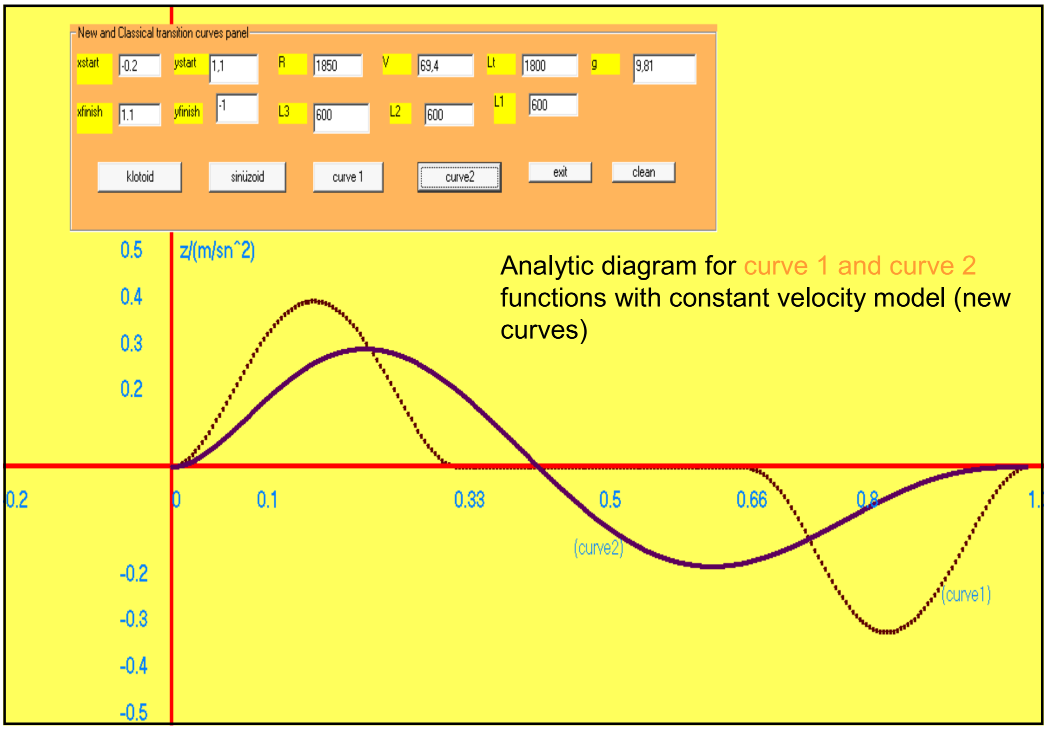
$$Q_2 = \frac{823543}{6912}(t^7 - 4t^6 + 6t^5 - 4t^4 + t^3)$$

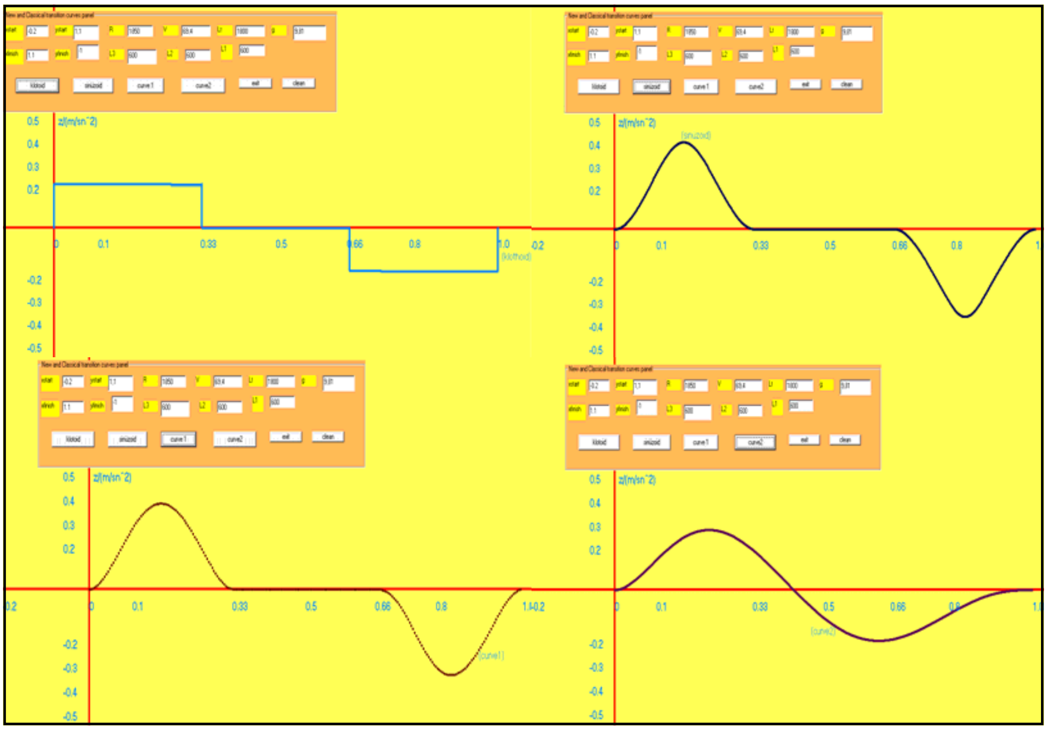
$$N_2 = \frac{823543}{6912}(7t^6 - 24t^5 + 30t^4 - 16t^3 + 3t^2)$$




New and Classical transition curves panel
xstart -0.2 ystart 1,1 R 1850 V 69,4 Lt 1800 g 9,81
xfinish 1.1 yfinish -1 L3 600 L2 600 L1 600
klotoid siniüzoid curve 1 curve2 exit clean
z/(m/sn^2)
(sinüzoid)
Analytic diagram for clothoid and sinuzoidal functions with constant velocity model (classical curves)
0.5
0.4
0.3
0.2
-0.2
-0.3
-0.4
-0.5
0.2
0
0.1
0.33
0.5
0.66
0.8


New and Classical transition curves panel
xstart -0.2 ystart 1,1 R 1850 V 69,4 Lt 1800 g 9,81
xfinish 1.1 yfinish -1 L3 600 L2 600 L1 600
klotoid siniüzoid curve 1 curve2 exit clean
z/(m/sn^2)
(sinüzoid)
Analytic diagram for sinuzoidal and curve 1 functions with constant velocity model (classical curve and new curve)
0.5
0.4
0.3
0.2
-0.2
-0.3
-0.4
-0.5
0.2
0
0.1
0.33
0.5
0.66
0.8
1.0
(curve1)

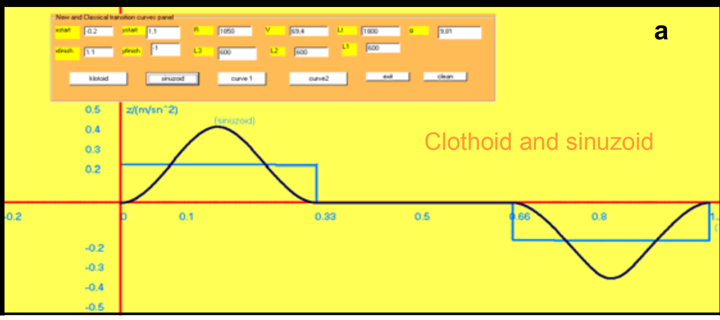

New and Classical transition curves panel
xstart
-0.2
ystart
1,1
R
1850
V
69,4
Lt
1800
g
9,81
xfinish
1.1
yfinish
-1
L3
600
L2
600
L1
600
klotoid
siniüzoid
curve 1
curve2
exit
clean
z/(m/sn^2)
Analytic diagram for curve 1 and curve 2 functions with constant velocity model (new curves)
(curve2)
(curve1)



CONCLUSION (1)

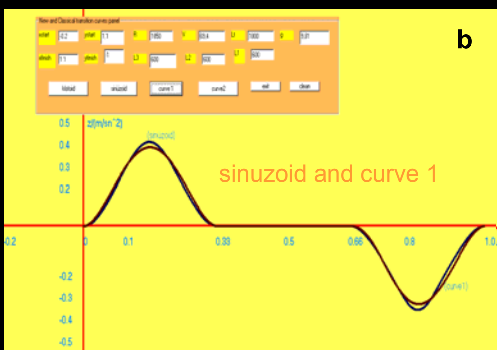
This study and new programme examined that new and classical transition curves according to these diagrams are understanded that suitable by known criterias for horizontal geometry of higways and railways.

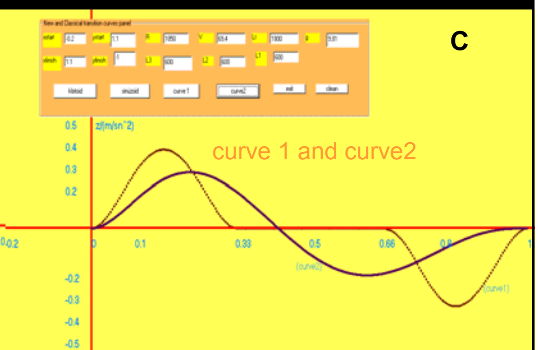
According to criterion 1, diagram of lateral change of acceleration functions of contunity and discontinuties in the form of jump very considerable for travelling comfort. This curves for suitable this criteria except to clothoid curves (Tarı 1997). (figure a)



CONCLUSION (2)

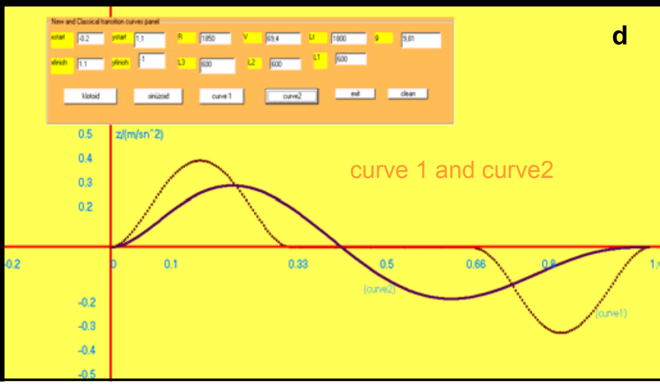
According to criterian 2 amplitudes of lateral change of acceleration is very considerable. In literature 0.3-0.6m/sec3 values are the maximum values of lateral change of acceleration for travel comfort (Tarı 1997). Curve 2 more suitable than Sinüzoidal and curve 1 for criterian 2 (figure b and c).





CONCLUSION (3)

According to criterian 3, break affect and irregular change of the lateral change of acceleration of diagrams at the point where transition curves and arc of circle are combined. Break affect is determined that differences between slope values of two tangents on this diagram at this point (Tarı 1997). Because of this, curve 2 is the best curve for travelling comfort and security for high speed highway and railway projects (figure d). New programme support this criterians which are mentioned above



Using the Curve 1 and Curve 2 in design of high speed railways and highways systems give an oppurtnity for travel comfort and security beside this curves prevent to wear on vehicle wheels and rails in railways

Part of Programme codes for Sinuzoidal Curve

```
Private Sub Command2_Click()     sinuzoidal curve
Dim xstart, xfinish, ystart, yfinish, R, V, Lt, L1, L2, L3, g
xstart = Val(Text1):xfinish = Val(Text2):ystart = Val(Text3):yfinish = Val(Text4):R = Val(Text5)
V = Val(Text6):Lt = Val(Text7):L1 = Val(Text8):L2 = Val(Text9):L3 = Val(Text10):g = Val(Text11)
Scale (xstart, ystart)-(xfinish, yfinish)
Line (0, ystart)-(0, yfinish), QBColor(12)
Line (xstart, 0)-(xfinish, 0), QBColor(12)
Dim x2, y2, Q1, N1, Q2, N2
For x2 = 0 To 1 Step 0.001
Q1 = (Lt * x2 / L1) - (1 / 2 * 3.14 * Sin(Lt * x2 / L1 * 2 * 3.14))
N1 = (1 / L1) - (1 / L1 * Cos(Lt * x2 / L1 * 2 * 3.14))
Q2 = ((Lt - Lt * x2) / L3) - (1 / 2 - 3.14 * Sin((Lt - Lt * x2) / L3 * 2 * 3.14))
N2 = (-1 / L3) + (1 / L3 * Cos((Lt - Lt * x2) / L3 * 2 * 3.14))
Select Case x2
Case 0 To (L1 / Lt): y2 = (N1 * V * (V ^ 2 - g * R * 0.1)) / (R * (1 + Q1 ^ 2 * 0.1 ^ 2) ^ 1.5)
Case (L1 / Lt) To ((L1 + L2) / Lt): y2 = 0
Case ((L1 + L2) / Lt) To (Lt / Lt): y2 = (N2 * V * (V ^ 2 - g * R * 0.1)) / (R * (1 + Q2 ^ 2 * 0.1 ^ 2) ^ 1.5)
Case Else
End Select
PSet (x2, y2), QBColor(1)
Next
```

Thanks for your attention

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