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Advantages of Using the Mechanics of Continuum to Geometrical Analyse Deformations Obtained from Geodetic Survey

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Paper contents:

- **What is strain analysis, different way how is possible to compute strain - general principles**
- **Advantages of using of strain analysis**
- **Practical example to document of advantages**

Strain Analysis:

- **based on continuum mechanics**
- **Geometrical analysis, describes the change in shape and dimensions of the monitored object**
  - **the goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains.**
- **Physical interpretation is based on the relationship between the causative factors (loads) and deformations**

Strain Analysis:

- Geodetic methods applications is based on repeated measurement and comparison of results of individual epochs of measurements. The vector of point displacement is expressed as a function of coordinates

$$\mathbf{x}_i^o - \mathbf{x}_i^t = \mathbf{d}_i = (u_1, u_2, u_3)_i^T = \mathbf{u}(\mathbf{x}) = (u_1(\mathbf{x}), u_2(\mathbf{x}), u_3(\mathbf{x}))^T, \quad \mathbf{x} = (x, y, z)^T$$

- Where $\mathbf{x}_i^o$ (resp. $\mathbf{x}_i^t$) is the vector of P_i point coordinates of fundamental (resp. actual in t-time) epoch.
- The strain tensor in P_i is defined as a gradient of the function in this point:

$$\mathbf{E}_i = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}_i = \mathrm{grad}(\mathbf{d}_i) = \begin{pmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial y} & \frac{\partial u_1}{\partial z} \\ \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial y} & \frac{\partial u_2}{\partial z} \\ \frac{\partial u_3}{\partial x} & \frac{\partial u_3}{\partial y} & \frac{\partial u_3}{\partial z} \end{pmatrix}_i$$

Strain Analysis:

e.g. for displacements projected to XY of the local coordinate system (min. 3 points with displacements vectors are needed):

$\Delta = e_{11} + e_{22}$	- total dilatation,	$\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$	- total shear,
$\gamma_1 = e_{11} - e_{22}$	- shear strains,	$\varepsilon_1 = \frac{1}{2}(\Delta + \gamma)$	- axis of maximum strain,
$\gamma_2 = 2e_{12}$	- shear strains,	$\varepsilon_2 = \frac{1}{2}(\Delta - \gamma)$	- axis of minimum strain,

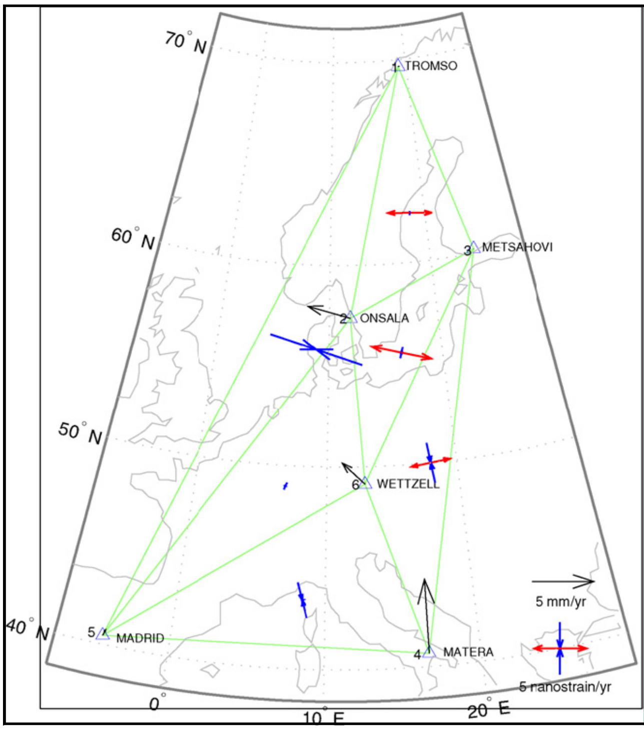
$$\varphi = \frac{1}{2}\operatorname{arctg}\left(\frac{\gamma_2}{\gamma_1}\right)$$ - direction of axis of maximum strain,

$\psi = \varphi + \frac{1}{4}\pi$ for $\omega_{12} > 0$ - direction of shear strain,

$\psi = \varphi - \frac{1}{4}\pi$ for $\omega_{12} < 0$ - direction of shear strain.

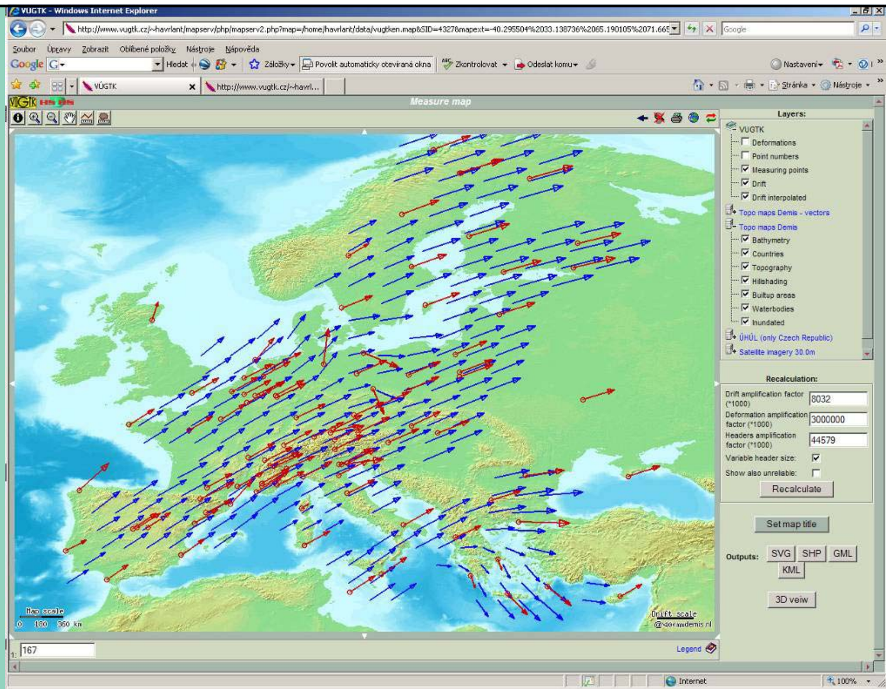
How to determine of Strain tensors from geodetic repeated measurements:

- **First step is determination of displacements of selected points on the object in question (standard task)**
  - **After it determination of displacement field in continuous form by their interpolation (generalized task) can follow**
- **The second step is determination of deformations parameters by geometric analysis of continuum mechanics (strain analysis), usually two ways are used:**
  - **Splitting of the geodetic net in triangles and computation of the strain tensors within each triangle (hypothesis of homogeneous strain field in each triangle self),**
  - **Computation of global strain field from all displacements in the investigated area (hypothesis of homogeneous strain field in the whole area),**

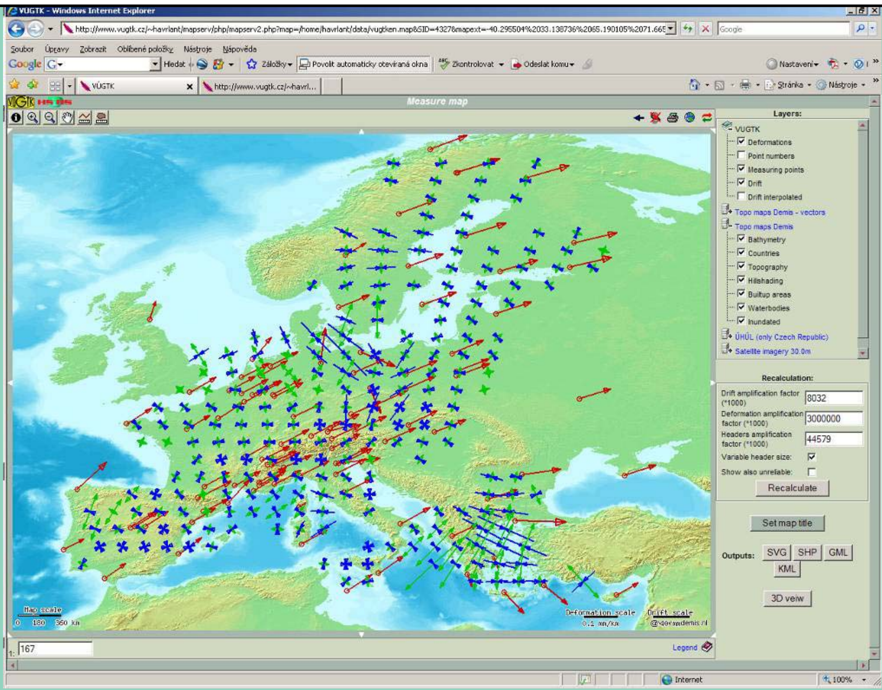


Strain field by using Delaunay triangulation

(Cai and Grafarend, 2007)



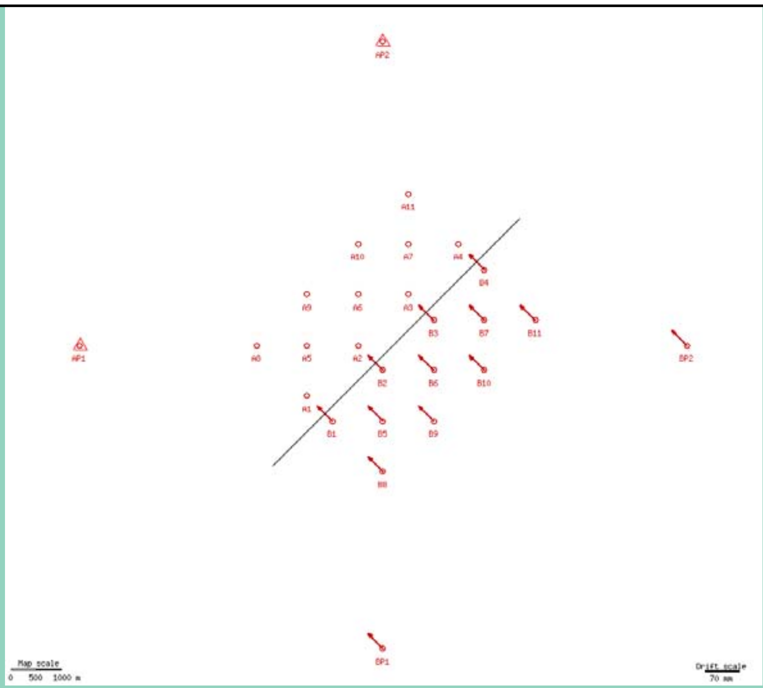
Displacement field



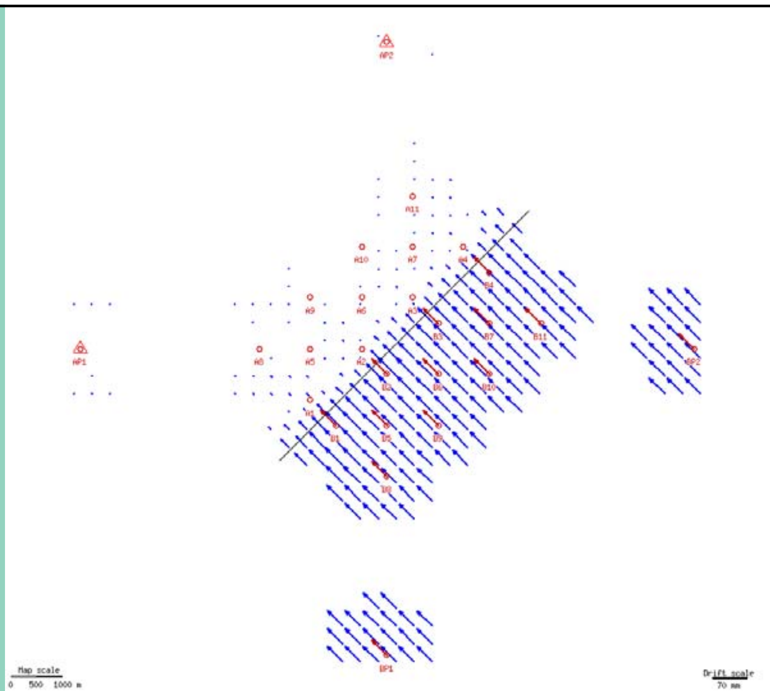
Strain field

Determination of displacements (first step):

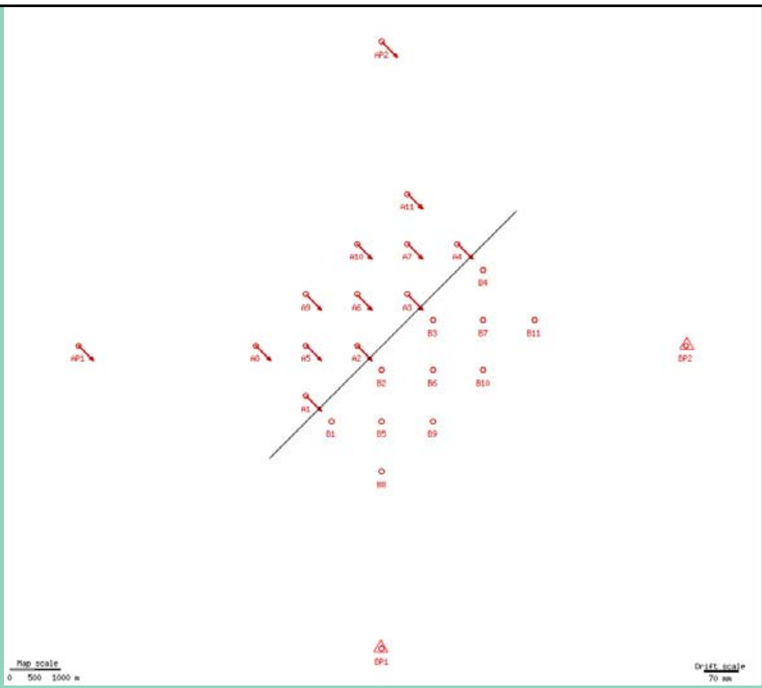
- The first condition is to computing the net adjusting as free network (to avoid of network distortion).
- The resulting character of displacements is given by the conditions for geodetic network placing in the coordinate frame (datum defect by free network adjustment).
- Usually selected points that are expected to be in the stable part of location are chosen like fiducial points (or we can make statistical tests about point stability).
- But: we are never quite sure that our hypothesis about points stability are correct.
- Situation can be shown on models.



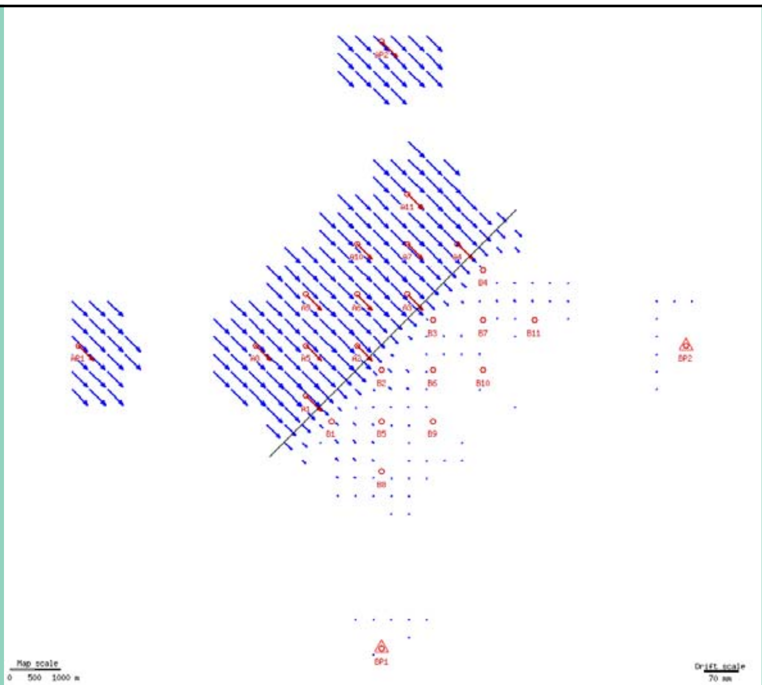
Displacements by fiducial points AP1, AP2



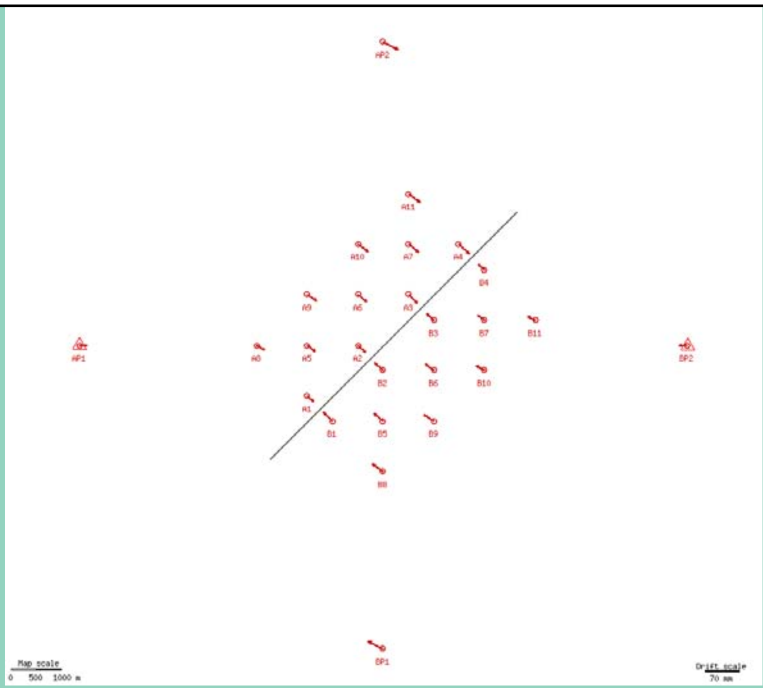
Displacements field by fiducial points AP1, AP2



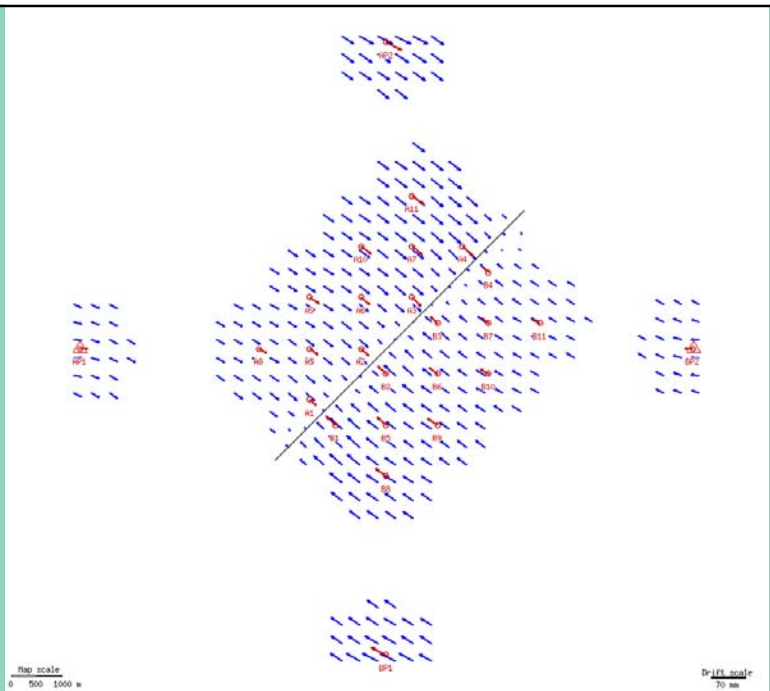
Displacements by fiducial points BP1, BP2



Displacements field by fiducial points BP1, BP2



Displacements by fiducial points AP1, BP2



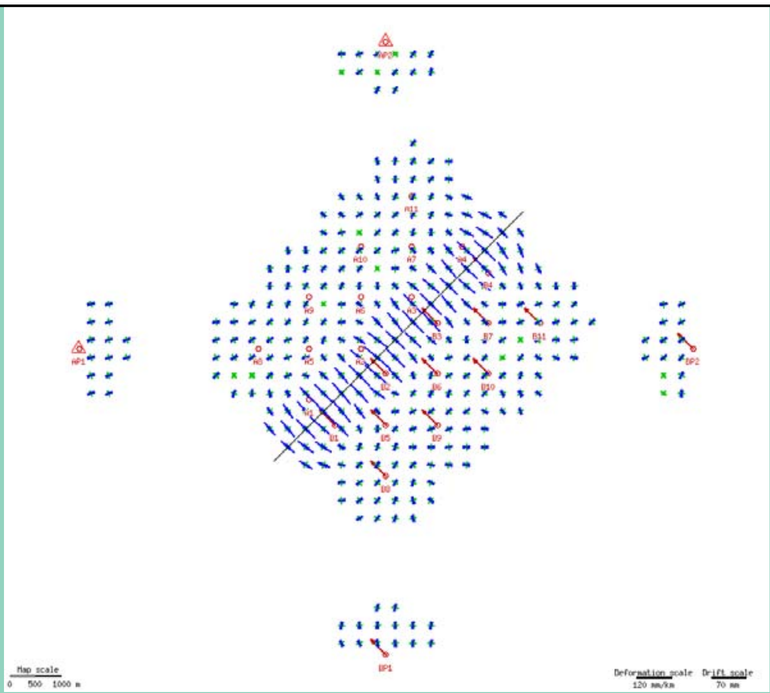
Displacements field by fiducial points BP1, BP2

Conclusion to the determination of displacements (first step):

- We can obtain different displacements from the same data (measurements) in dependence on the geodetic network placing in the coordinate frame (datum defect solution).
- This is one big disadvantage by using only displacements to deformation analyses.

Benefits resulting from Strain Analysis (second step):

- All deformation parameters (strain tensors) are on used coordinate frame independent and insensitive to translation and rotation



Strain field is the same by all possibilities of fiducial points

Practical benefits:

- => It is not necessary to deal with conditions of placing the geodetic network in the coordinate frame (fiducial points declaration by free net adjustment = datum defect solution).
- => To the practical using this means elimination of the errors obtained due to false (erroneous) hypothesis about stability of fiducial points that we consider as stable during processing of the repeated measurement.

Practical benefits:

- => practical example can be GPS antenna exchange (change of phase centre of a new antenna against the old one) of a permanent station at fiducial point of the GPS net (if this point is not included into calculation of the field of displacements and deformation).

Practical benefits:

- => it is not necessary to transform displacements given e.g. in coordinates frame ITRF into ETRF, or to reduce displacements in ITRF by movements of tectonic plate according to some of geodynamic models as APKIM2000 or NNR-NUVEL

Practical benefits:

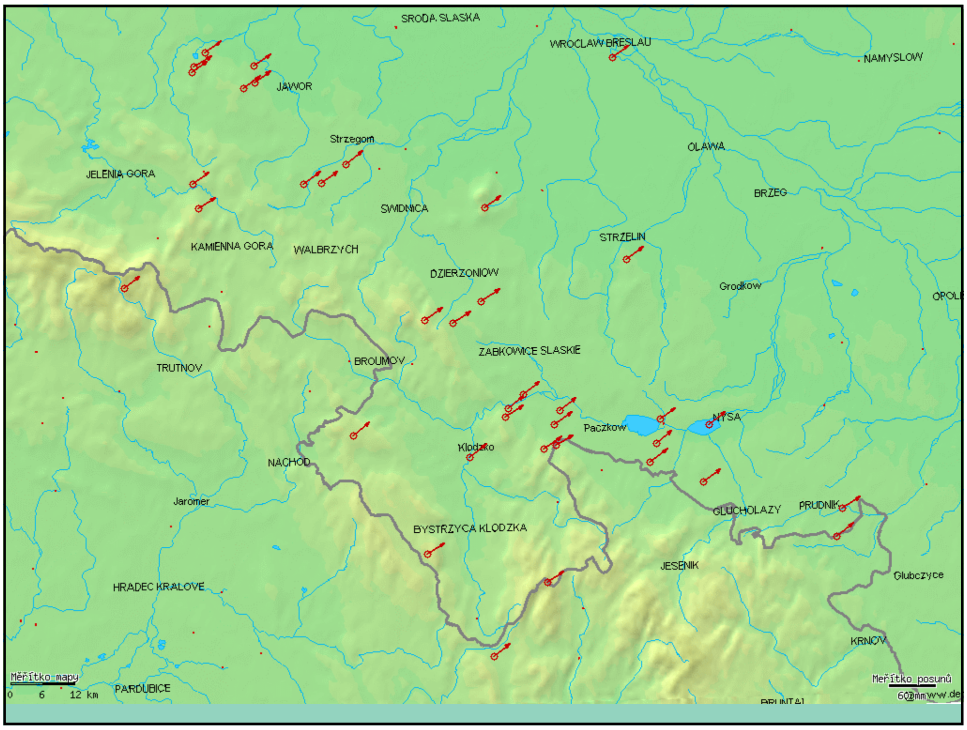
- => it is possible to deduce the real geodynamic activities based on determined deformation parameters (location of faults,…).
- Above all, the real situation is disclosed.

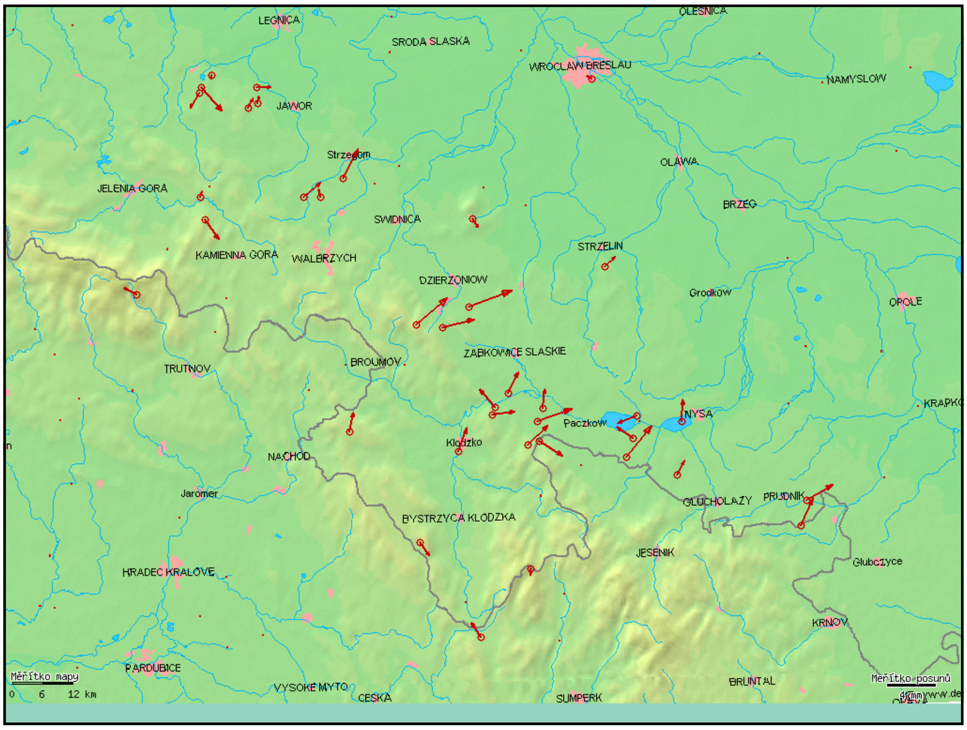
Practical benefits:

- Strain analysis can be used as technological and scientific base for communication between specialists of different professions because such information is used to further studies, physical interpretations and determining causative factors.

Practical example:

- Deformation network GEOSUD in the area of Polish Sudetes and the Fore-Sudetic Block discussed in (Cacon et al. 2005).
- displacements in ITRF are approximately of the same size, 24 to 27 mm /year, and approximately of the same direction.
- “residual” displacements after reduction to local system with model APKIM2000 are of different character with size from 0,3 to 3,7 mm / year.




LEGNICA
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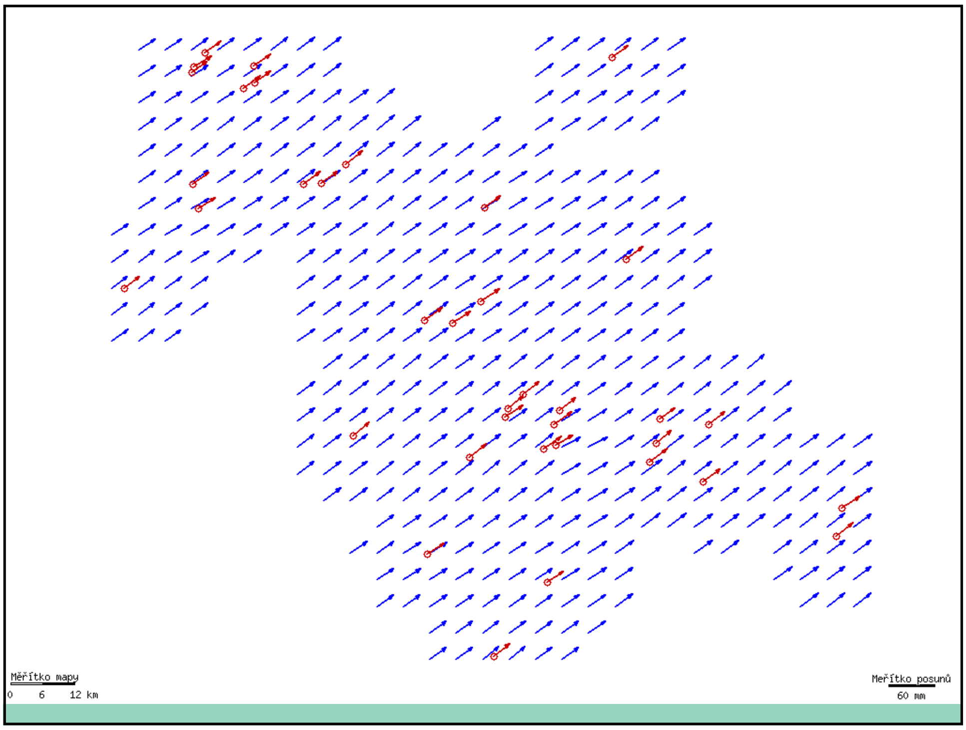
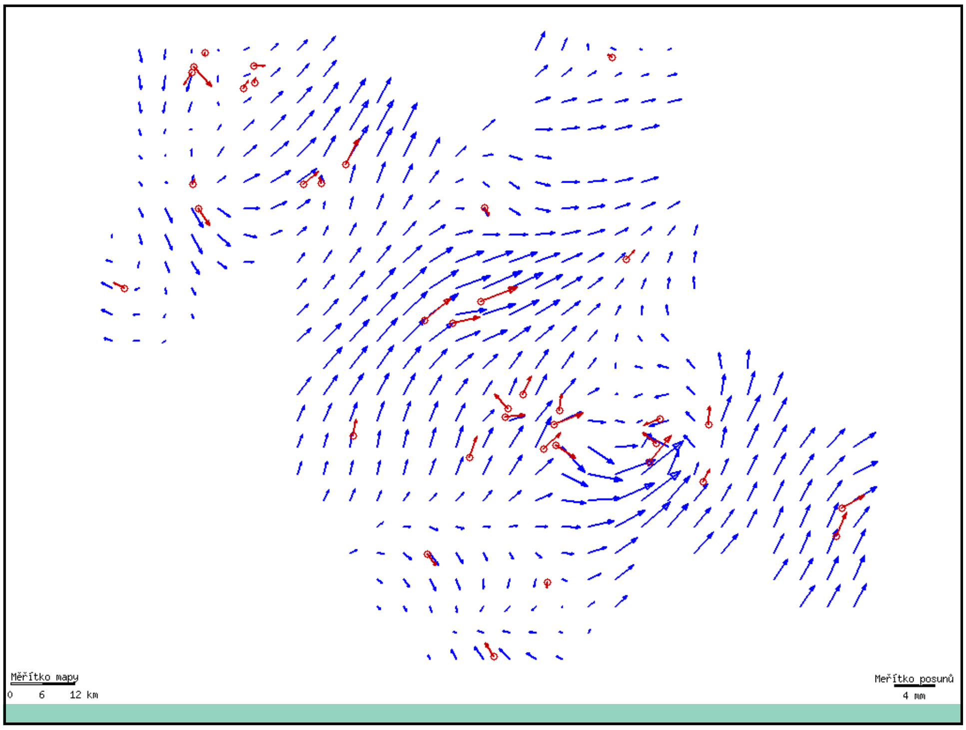
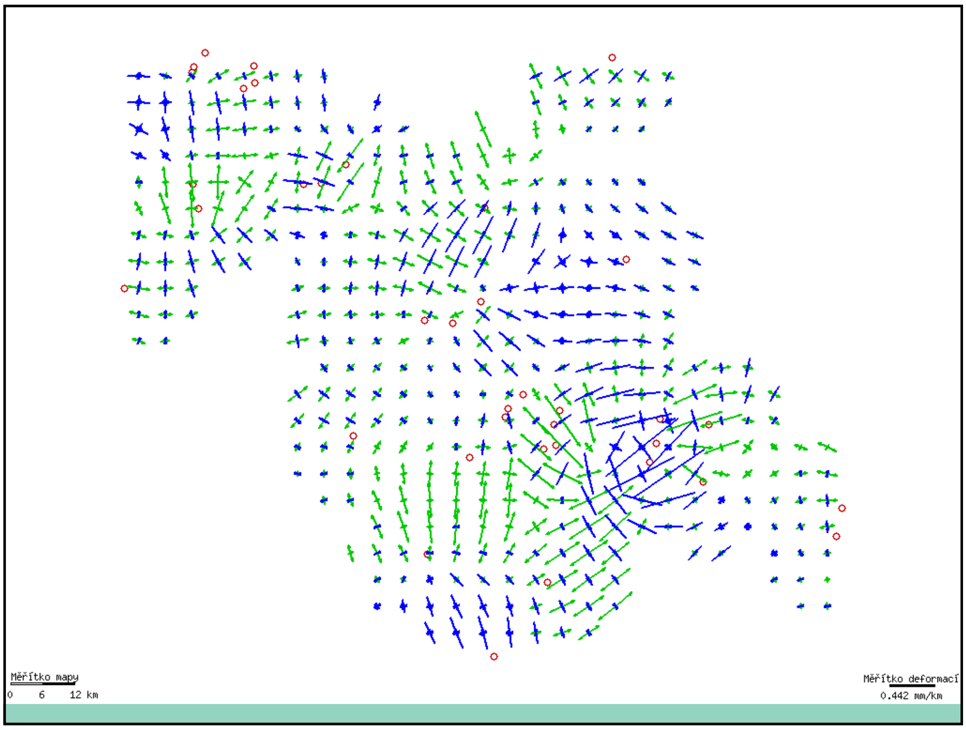
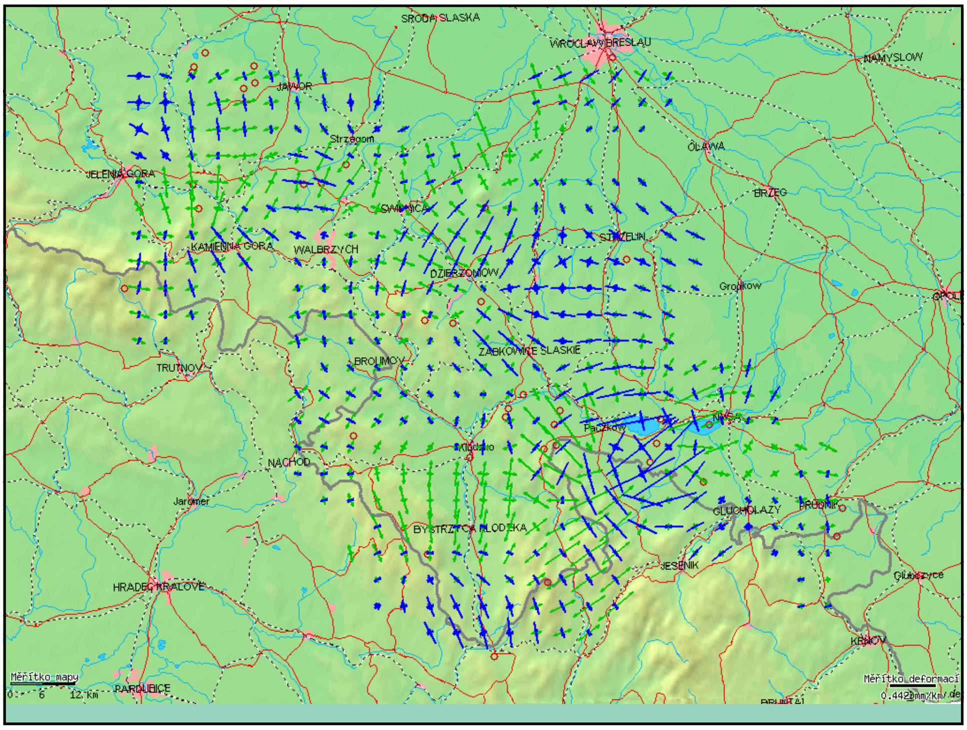
SWIDNICA
STRZELIN
KAMIENNA GORA
WALBRZYCH
DZIERZONIOW
Grodkow
OPOLE
BROUMOV
ZABKOWICE SLASKIE
TRUTNOV
Paczkow
NYSA
Klodzko
NACHOD
Jaromer
GLUCHOLAZY
PRUDNIK
BYSTRZYCA KLODZKA
JESENIK
Glubczyce
HRADEC KRALOVE
KRNOV
PARDUBICE
VYSOKE MYTO
SUMPERK
BRUNTAL
Měřítko mapy
0 6 12 km
Měřítko posunů


Měřítko mapy
0 6 12 km
Měřítko posunů
60 mm


Měřítko mapy
0 6 12 km
Meřítko posunů
4 mm


Měřítko mapy
0 6 12 km
Měřítko deformací
0.442 mm/km



Conclusion:

- the deformation analysis by application of theory of continuum mechanics (fundamental condition is homogeneity of the researched territory) is more objective dynamic indicator in the researched area than the only calculus and representation of point displacement vectors.

- For practical computation of strain tensors from repeated measurements of geodetic networks can everybody use on-line application : *www.vugtk.cz/~deformace*

- Or we can help you with processing of your data.

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Thank you for your attention

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