Adaptive parametric identification in dam monitoring by Kalman filtering

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ABSTRACT

The contribution presents an implementation of Kalman filtering for estimation of unknown parameters of a well-known hydrostatic-seasonal-time model which is used in dam safety procedures. The unknown parameters present a system state of the KF, which is iteratively improved by new information entering with new observations – in our case measured relative displacements. The analyses showed that the KF can detect statistically significant changes in dam behaviour by testing measurement innovations after a successful initialization phase and stabilisation of the parameters. Since the KF is implemented on the HST-model the algorithm enables detrending of reversible and irreversible deformations, and analysis of a long-term trend for each measurement epoch. The whole process is strongly influenced by the process noise intensity scalar which defines weighting between the process and the measurement noise. Defining an appropriate value of the process noise intensity scalar is the main challenge of the algorithm, where the convergence of a posteriori error covariance matrix is used as the main criterion in the tuning process for the case study. In the case study, time series of measured displacements and water level in impounding reservoir of an embankment dam for a period of 21 years are used to test the proposed algorithm. Measurements were captured with a fully automatized monitoring system based on tachymetric observations.

I. INTRODUCTION

The safety control of engineering structures can be outlined by three phases: monitoring of the structure behaviour and influential parameters by different sensors, modelling and analysing registered observations, and engineering interpretation of the results. Nowadays, the technical development of measurement systems and communication technologies (wireless communication techniques) enable full automatization of high-rate measurement executions, data transfer and storage. On the other hand, the alarming process, which is one of the important steps in the structural safety procedures, is in many cases still not (fully) automated from the computational aspects and possibilities, and is primarily based on engineering experience and decisions.

The development of sensors, communication techniques and high-performance computers has promoted, encouraged or even required the development and testing of enhanced modelling algorithms and evaluation concepts in order to extract all of the information from the data available. The implementation of mathematical techniques prioritizes the models which enable the assimilation of all of the information available, an estimation of essential parameters of dynamic modelling in real time, the processing of measurements with data gaps and methods to handle the complexity of adopted functional models. As such these models can provide an additional supportive tool in decision-making and can distribute more reliable results for alarming procedures.

Geodetic methods and measurement systems have been proven to be reliable and precise techniques for capturing dynamic processes on the object itself, in its surrounding or interrelated deformations of both, and are nowadays part of complex monitoring systems. Geodetic measurement systems can be fully automatized and can capture spatial-temporal processes, i.e. behaviour of the object, deformation patterns, distribution of influential forces and parameters, very precisely and instantaneously.

Long- and midterm time series of measured displacements and other parameters enable testing of algorithms which could potentially be integrated in an automatization of the alarming process.

In this work, the capability of the Kalman filter for an adaptive parametric estimation of unknown parameters of a proposed statistical model in dam safety procedures, which could be implemented as a near real-time algorithm, will be presented. The efficiency of the filtering for detrending reversible deformations from irreversible, estimating long-term trend, and detecting anomalies in a near real-time will be discussed.

After the introduction, the mathematical formulation of the statistical hydrostatic-seasonal-time (HST) model and the principle of a well-known Kalman filtering will be given in Section II. In Section III, a case study is presented with the formulation of the Kalman filter for the HST-model and description of measurements which are used for model testing. The results are presented in the fourth part. Conclusions are summarised in Section V.

II. Mathematical background

A. *Hydrostatic-Seasonal-Time Model*

For modelling the behaviour of dams and their control different analysis methods, based on deterministic and statistical models, (Swiss Committee on Dams, 2003; Salazar et al., 2015), can be used. Models based on mechanical principles - deterministic models - try to describe the influence of changes in the input variables on the response variables with a mathematical model of physical (mechanical, thermal and chemical) behaviour and are often difficult to construct. On the other hand, the statistical models are based on the experience from previous measurements, where the influences of measurable variables are described by basic mathematical functions, which are rather easier to handle as functions in deterministic models. Statistical models, also called empirical models, gain in importance in various scientific fields due to simple and high-rate generation of observation by different sensors, and high computational capacities, (Horvath et al., 2016).

Widely used statistical model for the data analysis and safety control developed primarily for concrete dams and later implemented also for embankment dams is a hydrostatic-seasonal-time (HST) model, (a review manuscript Salazar et al., 2015), which models three main influences on the dam - hydrostatic pressure and temperature influence as elastic effects and irreversible time effect - with basic mathematical functions. The influence of water level in an impounding reservoir - hydrostatic pressure is modelled by a polynomial up to 4th degree (parameters $a_1 - a_4$), the thermal influence is modelled indirectly with trigonometric functions with annual and semi-annual period (parameters $b_1 - b_4$), and for the modelling of long term irreversible deformations different, strictly monotone functions are proposed, (Swiss Committee on Dams, 2003). In Eq.(1) the sum of a linear term, a positive exponential and a negative exponential of reduced time during the analysed period (parameters $c_1 - c_3$) is used. The model prediction $\hat{y}_i$ corresponding to the measured relative displacement y_i for a time point t_i, $t_i: i = 1,2,\dots,N$, can be written as follows:

$$\begin{aligned}\hat{y}_i = a_0 + a_1 \cdot h_i + a_2 \cdot h_i^2 + a_3 \cdot h_i^3 + a_4 \cdot h_i^4 + \\ + b_1 \cdot \sin(w_a \cdot t_i) + b_2 \cdot \cos(w_a \cdot t_i) + \\ + b_3 \cdot \sin(2 \cdot w_a \cdot t_i) + b_4 \cdot \cos(2 \cdot w_a \cdot t_i) + \\ + c_1 \cdot \tau_i + c_2 \cdot e^{\tau_i} + c_3 \cdot e^{-\tau_i} \ ,\end{aligned} \tag{1}$$

where:
a_0 … initial constant
$h_i = \frac{h(t_i) - h_{min}}{h_{max} - h_{min}}$ … relative water level
$h(t_i)$ … measured water level for a time point t_i, $[m]$
h_{max}, h_{min} … maximal and minimal water level, $[m]$, for the time period of modelling
$w_a = \frac{2 \cdot \pi}{\Delta t_a}$ … angular frequency
Δt_a … time step dependent on data; for the case study see sec.III-B
$\tau_i = \frac{t_i - t_1}{t_N - t_1}$ … reduced time for the analysed period $[t_1, t_N]$.

The unknown parameters can be estimated by a multiple linear regression - MLR, (Montgomery et al., 2012), for a defined period and enable separating irreversible deformations, which are usually a consequence of an ageing of the dam, from the reversible influences.

In this work, a solution of the parameter estimation for each time step when new measurements enter in the process is given and discussed. As a well-known algorithm for real-time estimation the Kalman filtering (KF) is deployed.

B. *Kalman filtering*

In the modelling of kinematic processes, we deal with time series of observations and standard techniques of the noise reduction of time series, including filtering and smoothing. KF is a data processing algorithm that estimates the system state from noisy measurements using a least-squares method in the sequential mode. It gives the optimal system state estimate together with a measure of how certain it is that the system state estimate is the true state also for the cases where no redundant observations are available. It performs an optimal solution for a linear process with uncorrelated, white, zero-mean Gaussian process and measurement disturbances. The algorithm is described in several books (Bar-Shalom et al., 2001; Grewal et al., 2001; Gibbs, 2011), and implemented in many non-engineering and, especially, engineering tasks.

The dynamic process can be modelled with two main equations - dynamic plant equation, Eq.(2), and measurement equation, Eq.(3):

$$\mathbf{x}_i^- = \mathbf{F}_{i-1} \cdot \mathbf{x}_{i-1}^+ + \mathbf{G}_{i-1} \cdot \mathbf{u}_{i-1} + \mathbf{w}_{i-1} \tag{2}$$

$$\mathbf{z}_i^- = \mathbf{H}_i \cdot \mathbf{x}_i^- + \mathbf{v}_i \ . \tag{3}$$

Here, $\mathbf{x}_i \in \mathfrak{R}^{nx1}$ and $\mathbf{z}_i \in \mathfrak{R}^{mx1}$ are a system state vector and a vector of measurements, with n system state components and m measurements in each time step. Vectors $\mathbf{w}_{i-1} \in \mathfrak{R}^{nx1}$ and $\mathbf{v}_i \in \mathfrak{R}^{mx1}$ are process and measurement noise, with zero-mean normal distribution, $\mathbf{w}_{i-1} \sim \mathcal{N}(0, \mathbf{Q}_{i-1})$ and $\mathbf{v}_i \sim \mathcal{N}(0, \mathbf{R}_i)$, respectively. Matrices $\mathbf{Q}_{i-1} \in \mathfrak{R}^{nxn}$ and $\mathbf{R}_i \in \mathfrak{R}^{mxm}$ are process and measurement variance-covariance matrices, respectively. Evolving of the system state $\mathbf{x}_i$ in time is described by the state transition matrix, $\mathbf{F}_{i-1} \in \mathfrak{R}^{nxn}$, and the estimation of the predicted measurement value $\mathbf{z}_i^-$ based on the a priori estimated system state $\mathbf{x}_i^-$ is defined by the matrix $\mathbf{H}_i \in \mathfrak{R}^{mxn}$, which describes the relations between the system state vector and the vector of measurements. Term $\mathbf{G}_{i-1} \cdot \mathbf{u}_{i-1}$ relates to the optional control input.

To start the prediction stage of the filter an initial estimate of the system state, $\mathbf{x}_i^-$, and the corresponding a priori error covariance matrix, $\mathbf{P}_i^-$, have to be known. By entering of new measurements, the updating step is performed and an improved a posteriori system state, $\mathbf{x}_i^+$, and the corresponding a posteriori error covariance matrix, $\mathbf{P}_i^+$, are estimated:

$$\mathbf{K}_i = \mathbf{P}_i^- \cdot \mathbf{H}_i^T \cdot \left(\mathbf{H}_i \cdot \mathbf{P}_i^- \cdot \mathbf{H}_i^T + \mathbf{R}_i\right)^{-1} \tag{4}$$

$$\mathbf{x}_i^+ = \mathbf{x}_i^- + \mathbf{K}_i \cdot (\mathbf{z}_i - \mathbf{z}_i^-) \tag{5}$$

$$\mathbf{P}_i^+ = (\mathbf{I} - \mathbf{K}_i \cdot \mathbf{H}_i) \cdot \mathbf{P}_i^- \tag{6}$$

The matrix $\mathbf{K}_i$ is a Kalman gain matrix and weights the measurement residual – innovation, $\mathbf{d}_i = \mathbf{z}_i - \mathbf{z}_i^-$. The algorithm tries to minimise the conditional mean-squared estimation error with respect to the Kalman gain $\mathbf{K}_i$. If the measurement error covariance approaches zero, the gain $\mathbf{K}_i$ weights the residual and information which enters with precise observations more heavily than a priori system state estimate. On the other hand, as the a priori estimate error covariance approaches zero the KF can rely more on the a priori system state estimate and consequently also on the predicted measurement than the actual measurement (Welch et al., 2001).

III. CASE STUDY

A. *Description of Measurements*

The implementation of the Kalman filtering for a parametric estimation in the HST-model is tested on long term coordinate time series of a geodetic point in a permanent geodetic network, which is designed for deformation monitoring of a rock-fill embankment dam and its surrounding. The point is located in the middle of the upstream side on a dam crest. The observations are captured with a high precision total station, which is permanently installed at the point. The monitoring system is fully automatized and deploys displacements every two hours. The permanent geodetic monitoring system represents the main monitoring system of the dam.

The relative coordinates in all three directions of a predefined coordinate system, Fig.(1), are estimated with the geodetic adjustment of observations. Seasonal variations, a strong correlation with the water level in impounding reservoir and an underlying long-term trend of irreversible deformations can be detected in coordinate time series of the analysed point (Gamse *et al.*, 2017a; Gamse, 2017b), where the reversible and irreversible deformations are of the utmost significance in the radial direction.

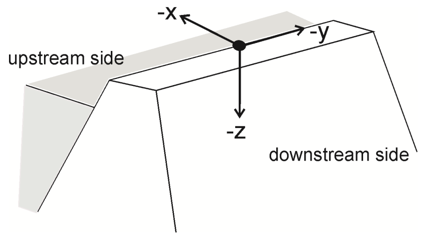

Figure 1. Orientation of the axes of the local coordinate system in the analysed point on the dam crest (Gamse, 2017b)

In the case study monthly median values of radial relative displacements (perpendicular to the dam body, with a negative direction towards the upstream side; x-direction on Fig.(1)) and measured water level, reduced for a constant, for the period of 21 years were available by the dam/data owner and were used for the algorithm testing. Both time series are graphically presented in Fig.(2).

In the evaluation of the proposed model the main intention will be given on the periods with two scenarios:

- for the period where the annual reduced water level in the impounding reservoir is above the reduced minimal water level $H_{min} = 475m$, and
- for the period with a drawdown of the water table, where the annual water level is below the reduced minimal water level H_{min}.

Namely, if the water level reaches or falls below the minimal water level H_{min} within the operating year or even for several adjacent years, this can cause changes in the trend of irreversible deformations, since there is a less back-pressure on the dam induced by the water masses. With the model we try to detect these unknown changes, which could be significant according to the previous behaviour.

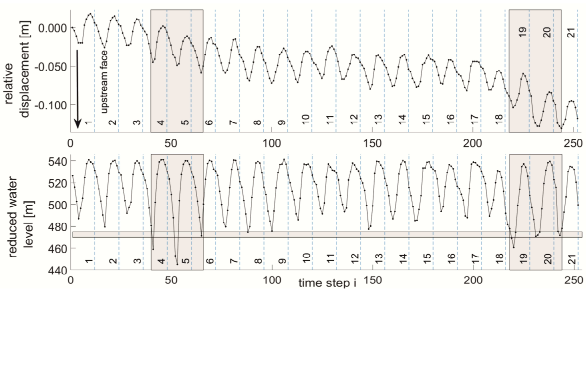

Figure 2. Relative radial displacements, $[m]$, reduced water level, $[m]$, and reduced minimal water level $H_{min} = 475m$

B. *Formulation of the Kalman filter for the HST-model*

The unknown parameters of the HST-model are usually estimated for a given period of measurements where different techniques, for example MLR, can be used. The optimal model can subsequently be used for the prediction of the dam behaviour under similar loads, to separate reversible and irreversible deformations, and to estimate different loads on the dam.

In the proposed attempt we try to combine the previous knowledge of the dam behaviour and a priori information about measurements, and the information which enters in the process at each measurement step. For the basic KF model, with no windowing applied, the estimation follows in one step back – one step forward manner. In our case, we are interested in the estimation of the unknown model parameters, their changes and capability of the stochastic model of the KF to detect statistically significant changes in the dam behaviour described by measuring relative displacements as a consequence of changeable influential forces on the dam.

In order to reduce the complexity of the model as much as possible, we keep only sine and cosine terms with an annual period. Furthermore, for the time component only a constant term and a linear trend, $a_0 + c_1 \cdot t_i$, are used in order to simplify the analysis of changes in the long-term trend. The detailed discussion on the importance of parameters of the hydrostatic and seasonal component is presented in (Gamse et al., 2017a). The simplified HST-model has the following form:

$$\begin{aligned} \hat{y}_i = a_0 + a_1 \cdot h_i + a_2 \cdot h_i^2 + a_3 \cdot h_i^3 + a_4 \cdot h_i^4 + \\ + b_1 \cdot \sin(w_a \cdot t_i) + b_2 \cdot \cos(w_a \cdot t_i) + c_1 \cdot t_i \ , \end{aligned} \tag{7}$$

where:

- $n = 8$ … number of unknown system state components:
  $$\mathbf{x}_i = [a_0 \ a_1 \ a_2 \ a_3 \ a_4 \ b_1 \ b_2 \ c_1]_i^T$$
- $t_i: i = 1,2, \dots, N; N = 252$
- $w_a = (2 \cdot \pi)/(\Delta t_a)$ … an annual pulsation with $\Delta t_a = 12$ months for monthly data
- $m = 1$ … number of measurements in each time step, in our case monthly median value of a relative displacement in radial direction, $z_i = y_i$.

The matrix of the system state equation, Eq.(2), has a form $\mathbf{F}_i = \mathbf{I}_8$.

In order to keep the KF-model as simple as possible and not to involve any further parameters, which should be tuned in the initial phase, the optional control input term is not included in the model in the case study. All unknown influences during two adjacent measurements are modelled as a process noise.

The measurement equation, Eq.(3), has a structure of a MLR model where the unknown coefficients – system state vector $\mathbf{x}_i$, varies over the time. The structure of the matrix $\mathbf{H}_i$ remains the same through the process, whereas its values depend on the time step:

$$\mathbf{H}_i = [1 \ h_i \ h_i^2 \ h_i^3 \ h_i^4 \ \sin(w_a \cdot t_i) \ \cos(w_a \cdot t_i) \ t_i] \ . \tag{8}$$

To start the Kalman filtering, an initial estimate of the system state $\mathbf{x}_0^-$, and the associated a priori error covariance matrix $\mathbf{P}_0^-$ are needed. Their values are very well initialised from the first few measurements as well as from a few repetitions of the KF, and influence strongly the acceleration of the convergence of the estimated system state component standard deviations and learning process. Wrong initial values of standard deviations can even lead to the divergence of standard deviations.

The covariance matrix of observations $\mathbf{R}_i$ is usually known or defined externally using know standard uncertainty of a used measurement system or earlier observations. In our case we make a presumption that the monthly median values of relative displacements are defined with the same precision, which is lower than the precision of each measured displacements. Furthermore, we could assume that the replacement of the high precision total stations during a 21-year period did not influence the precision of monthly median values. Namely, the spread of measured relative displacement values for the time span of one month is influenced mostly by the water level in the impounding reservoir and irreversible deformations (Gamse, 2017b). For the standard deviation of KF measurements, the average value of differences between adjacent monthly median values of relative displacements, $\mathbf{R}_{[1x1]} = 0.006$ m, is taken.

The process noise enters in the system through the process noise matrix, which we define as:

$$\mathbf{Q}_i = \sigma_\omega \cdot \mathbf{I}_8 \ , \tag{9}$$

where σ_ω is a process noise intensity scalar. In order to avoid any further tuning and weighting between three influences, which could be very subjective, we keep the same process noise intensity scalar value for all parameters of interest. In the future research work, tests with different process noise intensity scalars could be performed.

For the defined case study, the matrices $\mathbf{R}_i$ (in our case a scalar), $\mathbf{F}_i$ and $\mathbf{Q}_i$ are constant through the process and independent of time. Further reason that we can omit indexing is evenly spaced data.

IV. Evaluation of the results

The main goal of modelling and analysing of measured data by any mathematical functional model is to better understand the behaviour of observed structure. In the contribution we test the KF algorithm which could potentially be integrated in monitoring systems of dam safety procedures, but also for other structures. The main goal of the proposed algorithm is to estimate observations entering in the process with each measurement epoch and to estimate the current dam integrity regarding to the previous behaviour. The main question is if the proposed KF algorithm is capable to detect significant anomalies in the dam behaviour, which are captured by measured relative displacements, in an iterative manner.

The convergence properties of the minimisation of the conditional mean-squared estimation error are strongly dependent on the relative magnitude of the process and measurement noise, which is controlled by the process noise intensity scalar σ_ω. Its value strongly defines the filtering process and plays a very important role in the learning process. The main criteria in the tuning of the process noise intensity scalar is the convergence of the trace of the a posteriori error covariance matrix P_i^+. This can be achieved very quickly by assigning a lower value for σ_ω, which means giving a high confidence in the model. This is an extreme situation (under-estimation of the process noise) since we do not allow enough information to enter in the system with new measurements. Similar effects can be observed if the magnitude of $\mathbf{R}_i$ is increased. On the opposite assigning a high value for σ_ω (over-estimation of the process noise) means giving a high confidence in the measurements. In this case we underestimate the information based on the previous knowledge; the model strongly follows measurements and does not remain robust to the outliers in observations, in our case to significant changes in measured relative displacements. This is analogous decreasing the magnitude of $\mathbf{R}_i$.

In practice, we cannot measure the performance of the model with respect to the error measures directly. How can we then recognise if the filter assumptions are met and that the filter is performing correctly in practice?

For the case study, as the first criteria – the convergence of the trace of the a posteriori error covariance matrix P_i^+ is taken. The condition is met for the value $\sigma_\omega = 0.000005$, Fig.(3). But since the process noise could be under-estimated this is not a sufficient criterion for the model consistency or even for an automatization of the evaluation process.

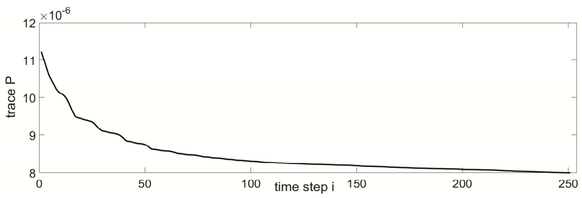

Figure 3. Trace of the a posteriori error covariance matrix P_i^+

A very informative measure of the filter performance is the measurement residual – innovation, $\mathbf{d}_i$, defined as the difference between the observation (measurement) and its prediction made using the information available at time based on the previous knowledge of the process. It is a measure of the new information provided by entering of new measurements in the estimation process, and their values are an important measure of how well the estimator is performing. We can verify if the filter is consistent by applying the innovation magnitude bound test (Reid, 2001) to check that the innovations are consistent with their covariance by verifying that the magnitude is bounded by σ- or $2 \cdot \sigma$-bound, Fig.(4). For a chosen σ_ω value, 9.9% and 1.6% measured displacements lay outside σ- and $2 \cdot \sigma$-bound respectively. The σ- and $2 \cdot \sigma$-confidence bounds correspond to the range of residual values with a 68%- and 95%-probability of being statistically insignificant for the system respectively.

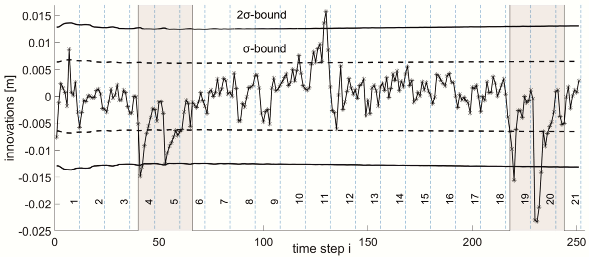

Figure 4: Innovations with σ- or $2 \cdot \sigma$ bound

This simple test is already quite sufficient to check filter consistency. In practice, by performing KF additional test – normalised innovations squared χ^2-test – is applied to check filter consistency and to analyse significant observations. The measure can be used to check the filter consistency by testing if the innovations are consistent with their covariances using the test statistics – normalised innovations squared (applied for example in Lippitsch, 20006):

$$\Omega_{d,k}^2 = \mathbf{d}_i^T \cdot \mathbf{D}_i^{-1} \cdot \mathbf{d}_i, \tag{10}$$

where $\mathbf{D}_i$ is the covariance matrix,

$$\mathbf{D}_i = \mathbf{R}_i + \mathbf{H}_i \cdot \mathbf{P}_i^{-1} \cdot \mathbf{H}_i^T. \tag{11}$$

Under the assumption that the innovation is normally distributed, the normalised innovation $\Omega_{d,k}^2$ follows χ^2-distribution: $P\{\Omega_{d,k}^2 < \chi_{m,1-\alpha}^2 | H_0\} = 1 - \alpha$, with m degrees of freedom or dimension of the normalised innovation vector and α significance level value. For the case study with the number of degrees of freedom $m = 1$ and significance level values of $\alpha = 0.05$ and $\alpha = 0.32$, the confidence regions of the upper one-sided test for the corresponding 68%- and 95%-probability relationship are $\chi_{1,1-0.05}^2 = 3.84$ and $\chi_{1,1-0.32}^2 = 1.08$, Fig.(5). We assume that the correct measurements and non-significant measured relative displacements will be detected within these regions. To perform even more robust test, the normalised innovations squared can be compared to their moving average for each measurement epoch. In Fig.(5) the normalised innovations squared are plotted with the χ^2-significance levels and the moving average.

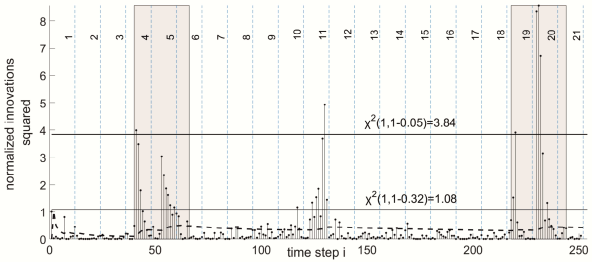

Figure 5: Normalised innovations squared, χ^2-significance level and moving average

Both tests denote three periods of significant innovations and normalized innovations squared. Two of them are observed in years 4-5 and 19-20 with a drawdown of the annual water level below the reduced minimal water level $\mathrm{H_{min}}$, Fig.(2). Another period is in the years 10-11, where the annual water level was kept higher than the average minimal water level.

In Fig.(6) the histogram of innovations and a normal density function with a mean value $\mu = 0.0005\mathrm{m}$ and standard deviation $\sigma = 0.0048\mathrm{m}$ are plotted.

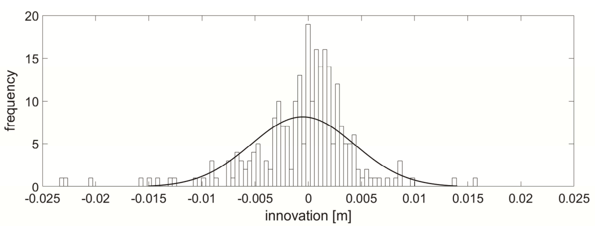

Figure 6: Histogram of innovations with normal density function

The autocorrelation function of innovations in Fig.(7) exposes again some values which lay outside the confidence bounds.

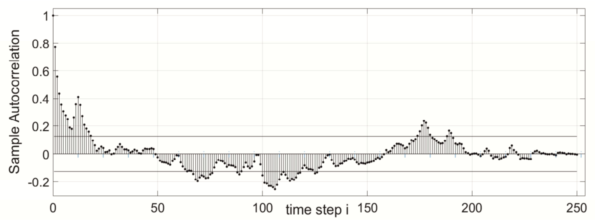

Figure 7: Autocorrelation of innovations

After the weighting between the process and measurement noise is adjusted by process noise intensity scalar value and different tests described above (a) convergence of the trace of matrix $\mathrm{P_i^+}$; b) innovation magnitude bound test; c) normalised innovations squared χ^2-test) are satisfied we analyse system state components – unknown parameters of the simplified HST-model. Based on the KF testing, we can conclude that the σ_ω value significantly influences the behaviour of these parameters. With a small σ_ω value we allow larger changes in parameters since we give a high confidence in the model and the parameters are adjusted to the measurements. By choosing a σ_ω value for which the criteria and conditions of the a-c tests are fulfilled, the parameter values stabilise at some values and expose changes only for the periods for which significant innovations and normalized innovations squared were also detected. Furthermore, the convergence and the initial stabilisation rate depend strongly on the initial values of parameters.

For the case study where we have only monthly median values, which means only 12 observations per year, the initialisation phase has an important role in the whole process of KF. Namely, inadequate initial values of the parameters and their standard deviations can significantly influence the filtering results over several years or even for the whole analysed period.

In Figs.(8-11) the parameter values with σ-bounds are plotted. The parameter c_1 describes the inclination of the linear trend and its changes. The convergence rate of standard deviations for estimated parameters is plotted in Fig.(12), where several iterations were performed to define good initial values since they have an effect on long term KF performance.

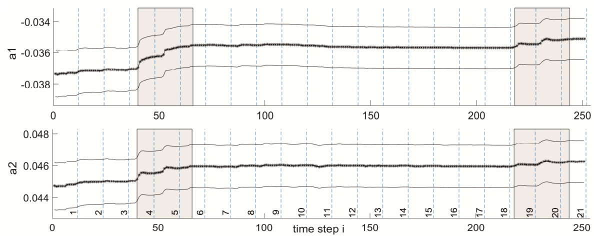

Figure 8: Parameters a_1 , a_2 with σ-bound

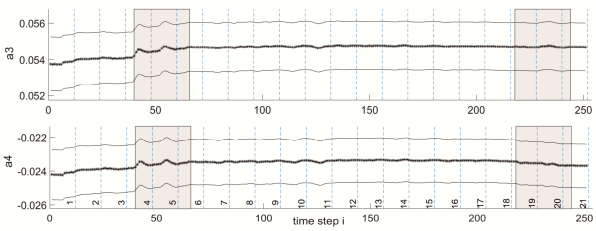

Figure 9: Parameters a_3 , a_4 with σ-bound

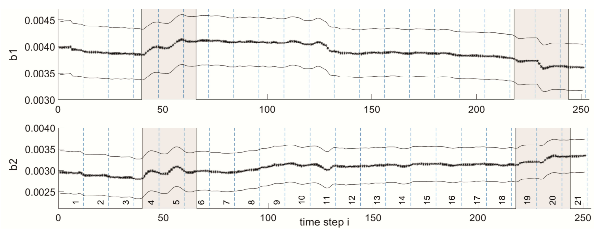

Figure 10: Parameters b_1 , b_2 with σ-bound

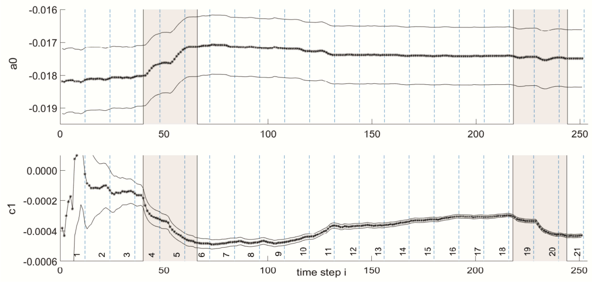

Figure 11: Parameters a_0 , c_1 with σ-bound

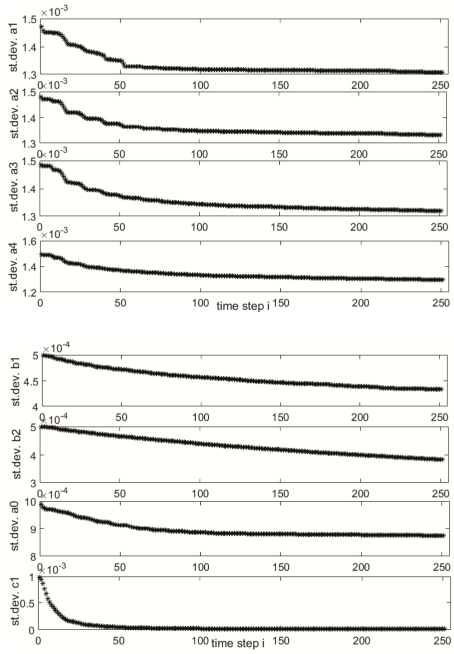

Figure 12: Convergence of standard deviations for estimated parameters

The HST-model enables detrending the reversible and irreversible deformations for each measurement step. Since for the case study where the main purpose of the dam is electricity production, the water level follows an annual period, which is also used for modelling of the temperature influence. In such cases it is difficult to separate reversible deformations due to the water level and the temperature in a manner that they would have practical interpretation. In future work even more simplified HST-model with the excluded parameters b_1, b_2 (thermal effect) and a_3, a_4 (hydrostatic pressure) can be analysed. Namely, the contribution of the temperature effect on the embankment dam analysed is very low and not all parameters of the hydrostatic term are statistically significant, (Gamse, 2017b). Long-term irreversible deformations modelled with a constant and a linear term, and reversible deformation as a sum of hydrostatic and thermal effect are plotted in Fig.(13).

In Fig.(14) measured displacements (red line with dots) and predicted displacements (blue line with stars) with the σ-bound (green lines) of innovations are presented graphically. The KF performance in comparison with the optimal HST-model estimated by the MLR (black dashed line), (Gamse, 2017b), is also presented graphically. Direct interpretation of these two models is not demonstrative since the main principle of the KF is a one-step back/one-step forward estimation, whereas by adopting the MLR we estimate a model for a whole chosen period.

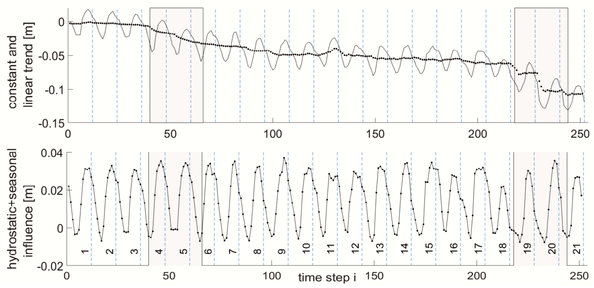

Figure 13: Long-term trend (above) and reversible deformations (below)

V. Conclusions

In (Gamse, 2017b) an attempt of Kalman filtering as a third-order discrete Wiener process acceleration model which describes the dam behaviour with position, velocity and acceleration as system state components was analysed. It was concluded that the model can detect significant anomalies in an iterative manner, but it does not enable to detrend irreversible deformations from reversible.

In the presented work we estimate unknown parameters of the simplified HST-model in an iterative manner by adopting KF. The advantage of the KF with parametric estimation is the possibility to estimate individual influences, to detrend reversible and irreversible deformations, and to directly observe the changes in the long-term trend of irreversible deformations at each measurement epoch.

The KF model can detect statistically significant innovations – measurement residuals with the innovation magnitude bound test and normalised innovations squared χ^2-test after the stabilisation of the system state components and their standard deviations is achieved. The statistically significant innovations are strongly correlated to changes in the water level, which deviate from minimal and maximal level. These deviations influence not only changes in the amplitude of reversible deformation but cause also changes in the long-term trend.

The performance of the KF depends strongly on the initial values of the system state components, their standard deviations, standard deviation of measurements, and the process noise intensity scalar. The standard deviation of measurements can be well estimated by the technical specifications of a used measurement system and spread of observations. The whole process can be tuned by the process noise intensity scalar which defines weighting between the process and the measurement noise, and consequently weighting between the predicted and the true measurement. Defining an appropriate value of the process noise intensity scalar was the main chal-

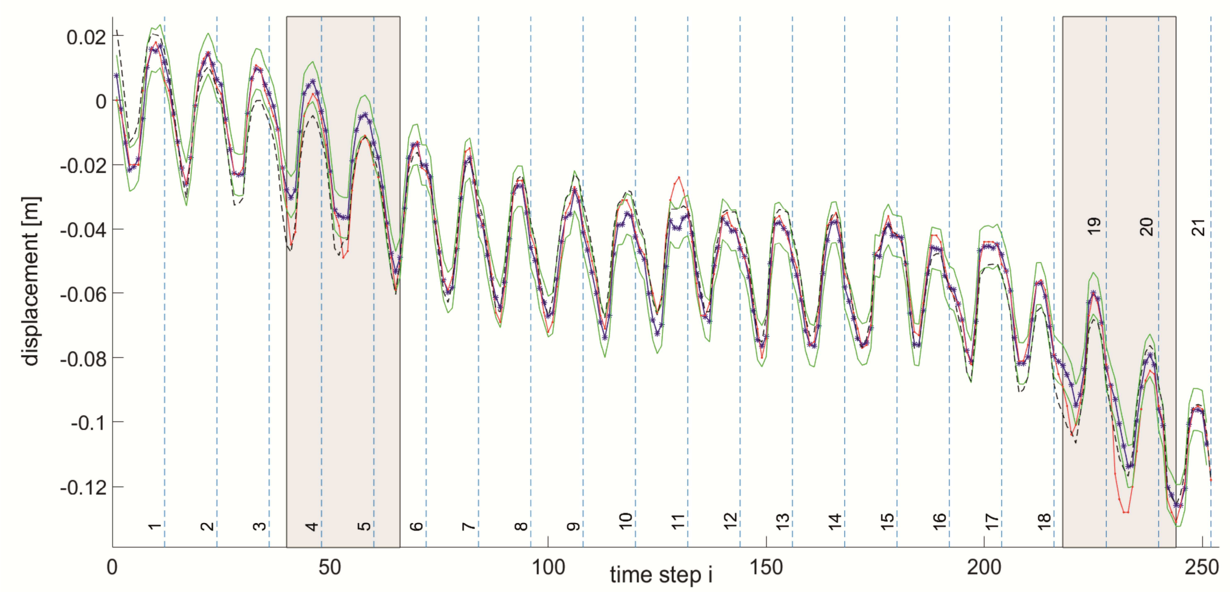

Figure 14: Relative displacements: measured-red line with dots, predicted-blue line with stars, σ-bound of innovations-green lines, optimal HST-model defined by MLR-black dashed line

lenge of the algorithm. As the main criterion the convergence of a posteriori error covariance matrix was taken in the case study.

In the presented work the model was tested on monthly median values. In future work we propose to implement the algorithm on daily measured relative displacements. In the case of detection of some potential correlations, the frequency method such Lomb-Scargle Periodogram can be used for an analysis of underlying periodicities.

Since the knowledge of the system state increases with the filtering process, a constant process noise intensity scalar could slow down the convergence rate, especially if the initial values are not defined accurately enough or for the cases with higher measurement variability. In this case the innovation can be used to validate a measurement prior to it being included in the observation sequence, and the weight between the process and the measurement noise can be appropriately tuned.

Analysis of the significance, meaning, relevance and deployment of the long-term trend and of reversible deformations for the analysed point is not the scope of this work.

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