Strategies and Methods for Multi-Epoch Deformation Analysis based on Geodetic Networks

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ABSTRACT

There is a long-lasting discussion within geodesy, which methods and strategies are adequate for a rigorous mathematical-statistical analysis of geodetic networks, when measuring results are available for several epochs. In this paper, first general prerequisites and typical problems are outlined, which define the framework for the here presented new concepts. Then the basic ideas for data-driven and model-driven analyses techniques are described, leading to the conclusion that normally a data-driven technique is restricted to (multiple) congruency tests, whereas all further analysis concepts have to be based on some prior information or physical model describing the behaviour of objects. Different computational methods are discussed in detail, like the consecutive two-epoch analysis, the multiple-epoch-congruency test and the - newly developed - hypothesis constrained multiple-epoch analysis. At the end a practical example for congruency analysis of multiple observed levelling networks is given.

I. Introduction

Since decades substantial research within geodesy, engineering surveys and photogrammetry is oriented towards the study of geometric displacements of objects, whether they are engineering structures or parts of the earth's surface. A prominent methodology to get information on the geometry of objects at specific times t_i is based on the use of geodetic networks that are observed n times to determine the geometry of the objects in n epochs. This methodology is the basis for the discussions in this paper.

It is still under discussion, which methods and strategies are adequate for a rigorous and practical mathematical-statistical analysis of these geodetic networks, when measuring results are available for several epochs. Here, we propose to use the so-called *coordinate approach*. This means that the network geometry is represented by a set of coordinates. In a first step, coordinates for the network points are computed from the observations. The analysis of geometric displacements is based on these estimated coordinates including their covariance matrices.

This coordinate approach can also be applied, if the sensor type changes in consecutive epochs. Sensor types can be, e.g., classical surveying, GNSS, digital images and other modern techniques. In the same way the network configuration can change in consecutive epochs.

In the following sections we will treat several methods of the coordinate approach. First, the basic congruency test for two epochs will be treated. Then we will continue with several methods to perform a congruency analysis of multiple epochs. Finally we will treat the hypothesis constrained multi-epoch analysis, and we will give an example. However, before we start, a list of prerequisites from practice is given that are satisfied by the coordinate approach. And we will describe the coordinate approach in more detail.

A. *Prerequisites from Practice*

An often crucial aspect of monitoring with repeated geodetic networks is the analysis, whether some points of interest are moving relative to other points close by or far away. If several points are connected to an object in order to represent the behavior of the object, the total set of points can be separated into these object points and reference points, relative to which the object point movements are monitored, see Fig.1. As the stability of these reference points cannot be guaranteed from the geological stability of an area or possible influences of construction work, it is necessary to analyze and to check reference points simultaneously with or prior to object points!

If a set of stable reference points is detected, it is possible to derive rigid body displacements and internal deformations of any object under study. In comparison with other engineering disciplines, the main and unique advantage of geodetic monitoring is this possibility to determine rigid body displacements.

Several prerequisites have to be covered by the approach, before one can claim it to be rigorous and applicable in practice:

i) The original configuration of points may be different from epoch to epoch, because points are lost and / or added in the course of time.

ii) For long-lasting monitoring projects the type of geodetic sensors can change: While several decades ago levelling and total stations were the only adequate sensors, later GNSS was included, followed by lasertracker, laserscanning and radar systems. Right now digital images from aircrafts and drones come into play.

iii) The approach has to be applicable independently of the dimensions of the network, i.e. for 1D, 2D or 3D monitoring tasks.

iv) It has to be possible to give analysis results to the client after each epoch. These results have to be reliable to allow the client to evaluate the stability of the monitoring object and to take proper action. It is in general not acceptable not to make an analysis of epochs until the end of the monitoring project, when all epochs have been measured.

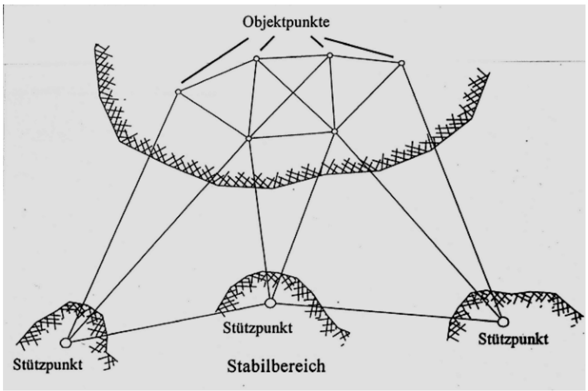

Figure 1: Principle design with reference and object points to derive absolute displacements (Pelzer, 1985)

B. *Coordinate Approach*

To find a way to have an analysis method, satisfying all prerequisites, we propose the so-called *coordinate approach,* as it is the most flexible method and allows for a good interpretation of the displacements and geometry of an object. The coordinate approach requires that, as a first step, for each individual epoch a set of coordinates X_i and covariance matrices $\sum_{xixi}$ is determined by a least squares adjustment. This set has to include both object points and reference points.

As starting point for this paper we have for a series of epochs t_1, t_2, … t_k as starting information:

Epoch t_1:	$\hat{X}_1,\ \hat{\sigma}_{01}^2,\ Q_{x1x1},\ f_1$
Epoch t_2:	$\hat{X}_2,\ \hat{\sigma}_{02}^2,\ Q_{x2x2},\ f_2$
$\vdots$	$\vdots\ \ \vdots\ \ \vdots\ \ \vdots$
Epoch t_k :	$\hat{X}_k,\ \hat{\sigma}_{0k}^2,\ Q_{xkxk},\ f_k$

Here the covariance matrices $\sum_{xixi}$ are split up into cofactor matrices Q_{xixi} and variance factors σ_{0i}^2:

$$\sum\nolimits_{xixi} = \sigma_{0i}^2\, Q_{xixi}$$

This split up allows for testing the *basic hypothesis* of deformation studies: All quantities $\hat{\sigma}_{0i}^2$, derived with f_i degrees of freedom, have to be estimates of the same theoretical variance factor σ_0^2. Additionally, it allows to use theoretical as well as empirical estimates for σ_0^2 within the analysis.

As we want to include the set of reference points into the analysis, coordinate estimates for all points have to be considered, leading, in general, to a singular adjustment model and by this to singular cofactor matrices Q_{xixi}.

II. CONGRUENCY TEST FOR TWO EPOCHS

As a first step one can restrict the statistical analysis to the classical *congruency problem* (term introduced by Niemeier, 1981), i.e. to the question, whether or not statistically significant deviations exist between the geometry of networks in two epochs t_1 and t_2.

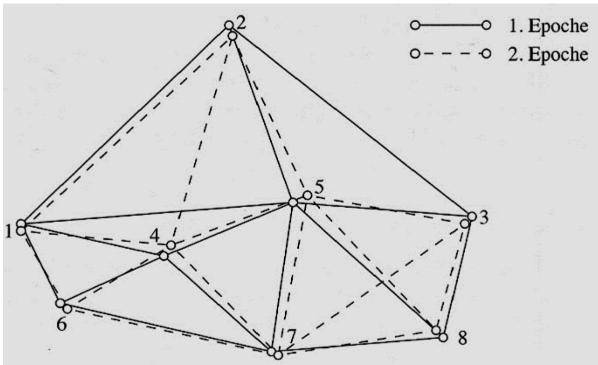

Figure 2: Problems solved by congruency analysis (Niemeier 2008)

With reference to Fig. 2 a congruency analysis answers the question, whether or not the deviations between the locations of all the points are caused by real displacements or are just the effect of uncertainties of the observations, i.e. lay within the unavoidable uncertainty level of the networks under consideration.

Rigorous and approximate analysis approaches for this classical deformation analysis problem can be found in ,e.g, Pelzer (1971, 1985), Niemeier (1979, 1981, 2008), Chrzanowski et al. (1981), Heunecke et al. (2013). Lösler et al. (2017).

For simplicity, here we assume that the number of points is identical in all epochs and both epochs are adjusted within the same S-System (see: …). The *zero-hypothesis* H_o of this congruency test is given by

$$H_0: \qquad E(\hat{X}_1) = E(\hat{X}_2) \quad ,$$

i.e. under H_0 the statistical expectation E of the coordinate estimates of both epochs are equal.

The common *alternative hypothesis* H_A is :

$$H_A: \qquad E(\hat{X}_1) \neq E(\hat{X}_2) \quad ,$$

what means that there are significant differences between the epochs, if H_A holds. Normally nothing more is specified for a congruency test.

As a first step for the testing procedure itself the difference vector *d* is computed

$$d = X_2 - X_1$$

The corresponding covariance information for this difference vector *d is*:

$$\mathrm{Q}_d = \mathrm{Q}_{\mathrm{x2x2}} + \mathrm{Q}_{\mathrm{x1x1}}$$

The basic test statistic for this global congruency test is given by the ratio, see Niemeier (2008):

$$F = \frac{d^t Q_d^+ \, d}{\sigma_0^2 \; h}$$

Where the "+" indicates the pseudo-inverse, *t* indicates the transposed vector, and *h* is the rank of the cofactor matrix Q_d. Other generalized inverses can be used here, but the pseudoinverse makes it clear that really all points are included into the analysis.

If this empirical quantity F exceeds the 95% quantile of the statistical F-distribution with *h* and ∞ degrees of freedom, the coordinate estimates between the epochs *1* and *2* differ statistically significantly.

Especially the use of the theoretical variance factor σ_0^2 is discussed: several authors recommend to use a combined empirical estimate $\hat{\sigma}_0^2$, instead, to account for the empirical situation more adequately.

The next step of a complete congruency analysis is the *localization* of significant movements for individual points or groups of points (Niemeier 2008). This is, however, outside the scope of this paper.

III. Considerations for Deformation Analysis of Multiply Observed Networks

For the statistical analysis of the results of multiply observed geodetic networks, some conceptual issues have to be considered. It is hardly impossible to analyze multi-epoch networks without at least some information about the deformations to be expected. This means that the approach for further analysis depends on our knowledge (real or assumed) of the behavior of the monitoring object itself.

A. *Data Driven Analysis*

Normally, in our profession we restrict ourselves to the data that are directly available to us, i.e. our own observations resp. results. With proper knowledge and processing of the sensor data we come up with results for each epoch, for which we have sufficient confidence in their precision and reliability.

Any advanced analysis based on just these data has to be restricted to simple models, e.g. no deformations present (see section V), or just linear deformations (kinematic models, see Heunecke et al. (2013).

Of course, during the design phase we use knowledge of the possible behavior of the structure to be monitored to select adequate object and reference points and we determine critical values for displacements in order to select the required precision. However, as long as we do not include a geological or physical model on possible displacements, we remain with a data driven analysis.

B. *Model Driven Analysis*

An extension of this concept is the inclusion of prior information into the analysis approach itself. For most monitored objects some physical model exists, which is shown in the following.

For engineering structures such a model may consist of knowledge about the behavior during the consolidation phase of a foundation, which is, e.g., given by a consolidation function. Alternatively, a mechanical model for the behavior of a structure may have been constructed in the design phase, e.g. a model of the bending of a dam due to the actual water level.

For monitoring of sections of the earth's surface this prior information may consist of knowledge about the existence of active tectonic fissures, the boundary of a landslide effected area or current underground mining activities.

In general these behavior models do not have the same level of confidence as our geodetic results. They are based on well-founded assumptions or derived from theoretical considerations, but they are not severely tested for the actual project.

As a minimum, such a model has to contain information on the:

- geometrical extent of the critical area
- expected movements (direction and size)
- temporal progression of displacements.

Different methodologies exist to treat prior information. In section VI prior information is used to setup alternative hypotheses. The validity of these hypotheses can be checked by the given tests.

IV. Congruency Analysis of Multiple Epochs

A. *Starting Models*

In this section a rigorous congruency analysis is presented for multiply observed geodetic networks, i.e. when the object geometry is determined in epochs t_1, t_2 …t_k and results according to section II are given.

Remark: As these epochs are in general not equidistant in time and the number of epochs is limited, we cannot treat them with concepts of time series analysis: No autocovariance function can be derived, no search for periodicities can be done in the time domain, etc.

The starting model for the congruency analysis of k epochs is given in Niemeier (1981, 2008) as:

$$\begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_k \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_{kk} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_k \end{bmatrix}$$

This equation considers the normal situation within an adjustment that one has to linearize the functional model, leading to the incremental observation vectors l_i (observed observations minus computed ones), the residual vectors v_i, the design matrices A_{ii} and the incremental coordinate estimates $\hat{x}_i$. Here no functional relations between the epochs are included; each epoch in itself can be adjusted and analyzed.

The corresponding stochastic model is given by:

$$\Sigma_{ll} = \sigma_0^2 \, Q_{ll} = \sigma_0^2 \begin{bmatrix} Q_{11} & 0 & 0 & 0 \\ 0 & Q_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Q_{kk} \end{bmatrix}$$

In this stochastic model no correlations are included between the epochs, which means that e.g. no remaining effects from non-modelled atmospheric conditions are considered.

B. *Consecutive Two-Epoch Analysis*

A common approach to handle k epochs is repeated application of two-epoch congruency tests. The following strategies can be followed:

i) *Congruency tests for consecutive epochs:*

1 - 2
2 - 3
…
k-1 - k

ii) *Congruency test of each epoch against epoch 1:*

1 - 2
1 - 3
…
1 - k

An important aspect here is the existence of a sufficiently large group of stable reference points during the complete monitoring project. For two-dimensional networks from our own practice we found that we have to have at least 3, better 4 or more stable reference points over all epochs!

Often as a first step within a complex monitoring task just the reference points are analyzed to find the stable points or areas. Then the displacements of all other points are computed relative to the stable reference points. In the example in section VII results for such an analysis are presented.

C. *Cumulative / Sequential Analysis*

In principle, sequential adjustments (Heunecke et al. 2013) allow to update the estimated coordinate vectors X_i, if a network (or even parts of it) is reobserved, i.e. a new epoch is available. The results of epoch 1 are the starting point, which are updated with additional observations of epoch 2, 3, …, k. A standard sequential adjustment is only valid, if the network geometry is stable or the movement model for the behavior is valid throughout the analysis.

For deformation studies, therefore, a modified sequential approach has to be applied:

i) If the congruency test of epoch 1 and 2 leads to the result: “No significant deformations detectable”, it is justified to combine the observations of both epochs, i.e. to make a new adjustment with the functional model:

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix} [\hat{x}_{1-2}]$$

Normally, this combined new adjustment will be restricted to the stable reference points, as one wants to analyze and detect displacements within the object area. This computational model gives better estimates $\hat{x}_{1-2}$ for the coordinates of the points under discussion.

ii) For the next congruency test the coordinate vector $\hat{x}_3$ of epoch 3 will be tested against this combined coordinate vector $\hat{x}_{1-2}$, following the procedure given in section I section B.

If this congruency test leads to the result: “No significant deformations detectable” for the group of points under discussion, it is justified to combine the observations of all three epochs, i.e. to make a new adjustment with the functional model:

$$\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} [\hat{x}_{1-3}]$$

iii) The same procedure is applied for subsequent epochs. Following this concept, one comes up with the *cumulative testing procedure*, originally developed by Niemeier (1979, 1981).

In practice, when in a specific epoch one or more of the discussed points have significant deformations, they get a new point number from that epoch on.

D. *Conclusion for this section*

It can be concluded that a multi-epoch analysis within this data-driven analysis is restricted towards multi-epoch congruency tests, i.e. to an analysis which points and/or which areas are statistically stable. This is an important analysis tool, but does not fulfill advanced requirements as they exist today.

V. Hypothesis Constrained Multi-Epoch Analysis

As mentioned above, one possibility to deal with prior information is to use hypotheses that account for the available displacement model. These hypotheses constrain the adjustment model for several epochs (Velsink, 2016, 2018). An analysis with such a model is called here a hypothesis constrained multi-epoch analysis.

A. *Characteristics of the method*

As outlined in section II the monitored points (both object and reference points) constitute a separate geodetic network for each epoch. Hence, if points have been measured in *n* epochs, there are *n* networks, but in this approach every point has a different name in each epoch. All networks are adjusted in a joint least-squares adjustment, in which the networks are linked together by constraints on the parameters. Using a Gauss-Markov model with constraints (e.g. Kourouklis and Paige 1981, Niemeier 2008, Koch 2013), the coordinates of each point appear *n* times in the parameter vector. If "no deformation" is the starting point for the deformation analysis, the constraints merely state that the coordinates of each point are equal in all epochs. It may happen that this model is rejected, when submitted to a statistical test. Then, more specific tests of the constraints (one-, two- or three-dimensional tests for 1D, 2D or 3D respectively) are used to detect which points are rejected most clearly. These points can be *relaxed* by giving them some large standard deviation. A new adjustment and subsequent testing will show, whether the model is now accepted. Then, one or more deformation hypotheses should be formulated concerning some or all of these relaxed points, based on assumptions about physical reality. These deformation hypotheses are incorporated in the adjustment model as *reformulations* of the deformation constraints. The reformulations are based on one or more mathematical functions that describe the movement(s) of one or more points through several epochs.

The observations of the adjustment model are *coordinates* that have resulted from separate adjustments of each epoch (Velsink, 2016). It is assumed that full covariance matrices of the preceding adjustments are available. If they are missing, the analysis can still be done, but with less precise results.

A notable characteristic of the model is the incorporation of *transformations* in the parameter vector. The transformations are similarity or congruence transformations between the epochs (that is, between the coordinates of different epochs in the observation vector). They take care that all coordinates in the parameter vector are estimated relative to the same geodetic datum. This approach has three advantages. First, the coordinates in the observation vector, and their covariance matrices, do not need a previous transformation to a common S-system. Secondly, no S-transformations are necessary to test for movements of datum points. And thirdly, it is guaranteed that testing the model will test the equality of form (in the case of similarity transformations), or form and size (in the case of congruence transformations), of the different networks (epochs). Differences in geodetic datum will have no influence on the testing results. Moreover, the testing results are invariant for changes in the geodetic datum of the coordinates of any epoch.

Here, it is worth noting that fixing the geodetic datum of the coordinates of all epochs can be done by fixing some coordinates (in the parameter vector; for example six or seven in 3D) of *just one* epoch. The coordinates of datum points in all other epochs need not to be fixed. This means, that datum points do *not* have to be stable points (nonmoving points)! This means as well that datum points and reference points are two different types of points; they can be different points; it is even possible to use object points as datum points.

B. *Adjustment model*

Several approaches exist to solve a linear Gauss-Markov model with constraints on the parameters. Here an approach is chosen that leads to a singular covariance matrix of the observation vector. Singularity of the covariance matrix does not have to pose problems, because several methods exist to get a rigorous least-squares solution of such an adjustment model (Velsink, 2018). It is not necessary to use large weights (as approximation to infinite weights), which would yield a nonrigorous solution and could cause numerical instabilities.

The following system of equations for the coordinate model, in its linearized form, is used:

$$\underbrace{\begin{pmatrix} b \\ z_d \\ z_g \end{pmatrix}}_{l} + \underbrace{\begin{pmatrix} v_b \\ v_d \\ v_g \end{pmatrix}}_{v} = \underbrace{\begin{pmatrix} P & -F \\ Z_d & 0 \\ Z_g & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 0 \\ Z_\nabla \\ 0 \end{pmatrix}}_{C} \underbrace{\begin{pmatrix} \hat{x} \\ \hat{f} \\ \hat{\nabla} \end{pmatrix}}_{p}.$$

On the left hand side, *b* contains the incremental stochastic observations, which is split in *n* subvectors:

$$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix},$$

in which b_1, b_2 and b_n contain the incremental coordinates of epoch 1, 2 and n, respectively. Vector z_d contains nonstochastic observations and is used for the constraints. Its values are zeros. Vector z_g contains, likewise, nonstochastic observations, but now to define the geodetic datum. Vector v contains the random residuals for the corresponding observations.

Vector p contains the incremental parameters of this Gauss-Markov model. Vector $\hat{x}$ contains the incremental coordinates of all epochs. It is split, in the same way as vector b, in n subvectors, one for each epoch. Vector $\hat{f}$ contains the incremental transformation parameters. Vector $\hat{\nabla}$ contains q deformation parameters, which describe the movements of a subset of points during several epochs. Consider, for example, some points that are attached to a subsiding building. If the subsidence of these point is linear during three epoch intervals, $\hat{\nabla}$ contains the rate of change of the linear subsidence during this intervals and q = 1.

The design matrix contains several submatrices. If the order of coordinates in $\hat{x}$ is the same as in vector b, submatrix P is a unit matrix. Submatrix F contains the partial derivatives of the observed coordinates relative to the transformation parameters. Appropriate approximate values have to be used. A practical way is to use an affine transformation, for which it is easy to compute approximate values, and to constrain this transformation in such a way that a similarity or congruence transformation results (Velsink, 2015, 2016). As the model already uses constraints, these contraints on the affine transformation parameters are easily implemented.

Submatrices Z_d, Z_g and Z_∇ contain the coefficients for the deformation constraints, the datum constraints, and the deformation parameters, respectively.

The corresponding stochastic model is:

$$\Sigma_{ll} = \sigma_0^2\, Q_{ll} = \sigma_0^2 \begin{bmatrix} Q_b & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The advantage of adding the constraints as additional nonstochastic observations to the model, is the testability of the constraints. The formulas to test for one or more biases in the observations are the same for both stochastic and nonstochastic observations. They are given below.

C. *Hypothesis testing*

If, initially, no deformation is assumed, no parameters $\hat{\nabla}$ exist. Therefore, the null hypothesis can be written as:

$$l + v = Ap_A, \ \text{ with } p_A = \begin{pmatrix} \hat{x} \\ \hat{f} \end{pmatrix}.$$

If this model is tested by an overall model test, and rejected, an alternative model can be formulated:

$$l + v = Ap_A + C\hat{\nabla}.$$

This alternative model is tested against the null hypothesis. The test statistic F_q for the generalized likelihood ratio test is (Velsink, 2018):

$$F_q = \frac{r^t C (C^t Q_r C)^{-1} C^t r}{q\, \sigma_0^2},$$

with q the degrees of freedom, which is the number of elements in vector $\hat{\nabla}$.

Vector r is computed using the null hypothesis, and it is the vector of coefficients of the residual vector v, when it is expressed as a linear combination of the columns of matrix Q_{ll}. Matrix Q_{ll} is singular in the adjustment model used here, but a vector r (called the vector of reciprocal residuals) exists (Velsink, 2018). Hence:

$$v = Q_{ll} r.$$

Matrix Q_r is the cofactor matrix of r. Several methods exist to compute r and Q_r (Velsink, 2018) in such a way that:

$$l + Q_{ll} r = A\, p_A \ \text{ and } \ A^t r = 0,$$

in which $A^t r = 0$ ensures that the result is a least-squares solution of the null hypothesis (Kourouklis and Page, 1981).

Test statistic F_q has a statistical F-distribution with q and ∞ degrees of freedom (if the observations are normally distributed).

Many alternative hypotheses, each characterized by a different matrix C, can be tested. The degrees of freedom of different alternative hypotheses are, in general, not equal. This means that a larger test statistic does not necessarily mean a better hypothesis. To determine the best alternative hypothesis a decision criterion is needed. Several have been proposed, for example Akaike's Information Criterion and the test ratio (Velsink, 2018). When one of the alternative hypotheses is accepted as the best one to describe the deformation(s), it can be incorporated in the adjustment model. It becomes the new null hypothesis:

$$l + v = (A \quad C) \begin{pmatrix} p_A \\ \hat{\nabla} \end{pmatrix}.$$

D. *Minimal Detectable Deformations*

Linked to the described method of testing for biases, is the possibility to compute *minimal detectable biases*. In the context of constraints that describe deformation patterns, they become "minimal detectable *deformations*". They are computed with the following equation:

$$\sigma^2 \lambda_0 = \nabla_0^t C^t Q_r C \nabla_0,$$

in which λ_0 is the reference noncentrality parameter. It is the equation of an ellipse, ellipsoid or hyper-ellipsoid, depending on the value of q (Velsink, 2017). λ_0 is a value, computed from two chosen probabilities: the chosen significance level of the test and the chosen minimal power to detect a bias of size λ_0. Vector ∇_0 describes the minimal detectable deformations.

The minimal detectable deformations provide an elegant method to describe the capability of a geodetic network to detect deformations. They can be used to describe the quality of a deformation analysis *before* any measurement has been done. Thus, they can be used as well to describe *standards* for deformation analysis in an application domain (for example stability checks of buildings, of construction works, of subsiding areas).

E. *Conclusion for this section*

The conclusion is that the adjustment model of a hypothesis constrained multi-epoch analysis can be used to test intricate deformation hypotheses. A deformation hypothesis can state that several points are moving through several epochs. Moreover, it can state that some points follow a different deformation pattern than other points.

From physical considerations of the monitored object(s), credible hypotheses should be derived. These deformation hypotheses can be tested. If one of them is considered the best one, according to some decision criterion, this hypothesis can be incorporated into the adjustment model, thus providing an amended adjustment model, which contains the deformation. This amended adjustment model should be accepted by an overall model test. If not, the search is for a deformation hypothesis, not yet considered before, that explains the observations better, and is substantiated by physical considerations. This improved hypothesis can be formulated and tested in the same way.

VI. Levelling networks within Rhenish Bight: Multi-epoch congruency tests

As example for a multi-epoch congruency test, the analysis the an extended leveling network with several epochs is outlined here, see Niemeier and Zeimetz (2018).

Within the Rhenish Depression, near Cologne, Germany, since decades there is an intensive lignite extraction. This mining activity is accompanied with continuous, large scale groundwater withdrawal to keep the open pits with a depth of up to 400 m free of water. These activities result in surface subsidence, reaching several meters, with sometimes critical effects on the safety of infrastructure lines and houses.

To determine the current subsidence rates an extensive network of levelling lines has been established and observed periodically. These networks are classified in two groups:

- main levelling network, observed about every five years within an observation time of three months.

- local densifications within villages and cities, observed according to associated mining activities in that area.

The specific task was to analyze the main levelling networks with epochs 1985, 1993, 2001, 2005, 2009 and 2013. Most important was the discovery of reference points, which are statistically proven to be stable during this time span of 28 years.

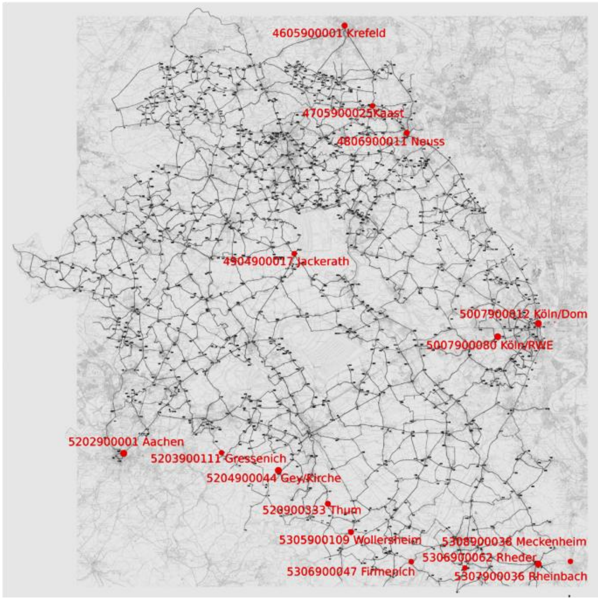

Fig. 4: Configuration of the main levelling networks covering the lignite mining area within the Renish bight. In **red** the potential reference points are marked.

In Fig. 4 the potential reference points are marked in red. This selection was made in cooperation with geologists and considers the boundaries of the groundwater withdrawal.

For these potential reference points consecutive two-epoch congruency tests were performed according to section IV part B, especially with the combinations 1993-1985, 2001-1993, 2005-2001, 2009-2005 and 2013-2009. Then - as independent analysis, covering the total time span - the first and the last epoch have been analyzed: 2013 – 1985.

The summarized results are given in Fig. 5, indicating the stability of the southern edge of the area. Further reference points in this are just above significance level, i.e. show limited movements, only. This result is in accordance with the geological stability assumption of the Eifel Edge. Besides, the area around Cologne can be assumed to be stable, as well.

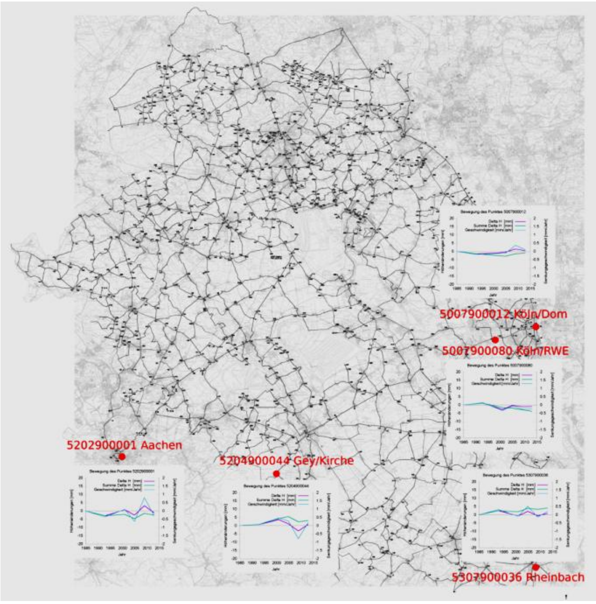

Fig. 5: Stable reference points after multi-epoch congruency tests for the main levelling lines within Rhenish bight.

As indicated within Fig. 5, for some stable points local effects were found, e.g. for the Aachen station, which are not yet significant but give hint for further studies.

Summarizing, this concept of multiple two-epoch congruency tests has proven to be applicable for the definition of stable reference points.

VII. Conclusions

The coordinate approach for the analysis of geometric displacements has been proposed and main advantages have been outlined. Starting from the congruency test for two epochs, several methods for multi-epoch analyses have been treated. The essential differences between data driven and model driven approaches have been investigated.

Various modifications of the classical congruency tests have been presented as example for a data-driven analysis. It could be shown that these concepts can be adapted to the analysis of several epochs and are a valuable, flexible analysis tool.

Then the hypothesis constrained multi-epoch analysis has been presented as an example of a model driven approach. Its advantages have been shown regarding the definition of the geodetic datum, the elimination of the need to have stable reference points, the possibility to yield quantified minimal detectable deformations, and, most notably, the inclusion of prior information or physical models into the geodetic adjustment model.

A numerical example, finally, has shown an application field of a data driven multi-epoch analysis for large scale levelling networks.

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