Trigonometric Levelling at Extremely Short Lines of Sight

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Abstract

Trigonometric levelling is usually considered as method used for determining height differences when spirit levelling is not applicable and/or when efficiency prevails accuracy. This paper analyses possibilities of trigonometric levelling utilization when lines of site are extremely short. Extremely short line of sight is considered as the line of sight shorter than standard lengths i.e. shorter than 1.8 m. For the purpose of power aggregate's rotor on power plant "Đerdap" upon a Danube dimension determination, precise measurements of directions and zenith distances were provided. Horizontal distances are obtained from adjusted values of points coordinates with estimated accuracy by means of least squares and as such, were used for height difference determination by standard formula for trigonometric levelling. This paper aims to research the obtained accuracy of height differences measured from five positions inside of power aggregate's rotor circle with radius less than 1 m, which means that horizontal distance between stations and points marked on the part of rotor was between 0.5 m and 1.2 m. Slope distances were longer but accordingly the zenith distances were bigger. Measurements were provided into tight space and under artificial light.

Key words: Key words: zenith distances, precise measurements, extremely short line of sight, trigonometric levelling

1 INTRODUCTION

Trigonometric levelling as method for height differences determination is not considered as a high precise method because of different limitations especially air refraction. It is, however, very efficient in height differences determination because of easiness for distance and zenith distance measurements by total stations. Trigonometric levelling is also very useful in situations when spirit levelling is not applicable at all as in some tight areas and in cases of high structures. Consequently the choice of method for height differences determination is limited by accuracy, efficiency and/or applicability.

Generally speaking, accuracy of trigonometric levelling is limited by accuracy of measured distances and zenith angles, lengths and values of zenith angles between points whose height difference is to be determined and the mathematical model which is used for height differences determination. Smaller zenith angles and shorter distances shall provide higher accuracy of height differences because of less influence of measurements errors in model.

In this paper the possibility of reaching maximal accuracy of height differences determined by trigonometric levelling method was researched in condition when lines of sights were extremely short. Extremely short lines of sights mean that the used instrument has ability to measure when lines of sights are shorter than standard visibility i.e. shorter than 1.8 m.

Trigonometric levelling is widely researched in literature (Ceylan, A., Inal, C., Sanlioglu, 2005; Grgić, I., Lučić, M., & Kršulović, D. 2010; Ceylan, A., and O. Baykal. 2008; GUO, Z. H., & ZHENG, J. F. 2004.; Becker, J. M., Lithen, T., & Nordquist, A. 1988; Soycan, M. 2006) which means that it is applicable and useful method for height differences or height of points itself determination. Geodetic refraction (air refraction) is main limit for utilization of trigonometric levelling because of its negative influence on height differences determination. Very large interval of refraction coefficient variation between -5 and 15 depending on height above ground and micrometeorological conditions (Brunner, F. K., and A. Kukuvec. 2011) and practical impossibility to determine it with needed accuracy is the main limit for accuracy of trigonometric leveling. Refractive index of air was widely researched (Owens, James C. 1967; Ciddor, P. E. (1996).; Shelby, R. A., Smith, D. R., & Schultz, S. 2001) and it is possible to state that no practical and efficient solution is possible to found for this influence in order to improve accuracy of trigonometric levelling. The only possibilities are based on double-sided measurements, shortening lines of sight and measuring near zenith angles of 90^o. Errors of measured values (distances and zenith angles) also limit the accuracy of trigonometric levelling. This paper aims to investigate possibilities of trigonometric leveling to reach maximal accuracy with extremely short lines of sight. Because those measurement were performed with aim to solve another problem it was not possible to control values of zenith angles which were extremely deviated from 90^o, reaching even 136^o.

2 METHODOLOGY

Models for height differences determination by trigonometric levelling method are (as well known from literature) , depending of available measurements, as follows:

$$h_S = f_S(S, Z, \varepsilon) \tag{1}$$

$$h_D = f_D(D, Z, \varepsilon) \tag{2}$$

where:

- h - height difference
- f – function mapping measured values into the height difference;
- S – slope distance;
- D – horizontal distance;
- Z – zenith angle and
- ε – air refraction angle.

Both models (1) and (2) requires measured zenith angle and both models are limited by influence of air refraction ε which is very difficult for determination and significantly limits the accuracy of height differences.

Concrete formulae for models (1) and (2) read as follows when slope or horizontal distances are known (measured or determined), respectively:

$$h_S = S * \cos(Z + \varepsilon) \quad (3)$$

$$h_D = D * \operatorname{ctg}(Z + \varepsilon) \quad (4)$$

Structures of formulae (1) and (2) imply that air refraction angle is unavoidable and difficulties of its determination significantly limit the accuracy of height difference between two points nevertheless of used model.

Root mean square errors (RMSE) for models (3) and (4) are respectively:

$$m_{h_S} = \sqrt{m_S^2 * \cos^2(Z + \varepsilon) + S^2 * \sin^2(Z + \varepsilon) * m_Z^2 + S^2 * \sin^2(Z + \varepsilon) * m_\varepsilon^2} \quad (5)$$

$$m_{h_D} = \sqrt{m_D^2 * \operatorname{ctg}^2(Z + \varepsilon) + \frac{D^2}{\sin^2(Z + \varepsilon)} * m_Z^2 + \frac{D^2}{\sin^2(Z + \varepsilon)} * m_\varepsilon^2} \quad (6)$$

Analysing structure of formulae (5) and (6) immediately follows:

- The shorter distance reduce RMSE for both models;
- The zenith angle near horizon (Z≈90º) reduces RMSE for both models and consequently
- Shorter distances and zenith angles near horizon reduce the influence of air refraction.

The complexity of trigonometric levelling appears when, in case of impossibility of choice, shortening distances causes increasing of zenith angles. At the fixed distances in case of zenith angle changes away from 90º the RMSE for models (3) and (4) behave in opposite directions:

- if slope distance are measured then RMSE will be reduced and
- if horizontal distance are known then RMSE will be increased.

In the case of very short distances and in absence of air fluctuations the term which describes influences of air refraction angle could be neglected and formulae (3), (4), (5) and (6) could be written as follows, respectively:

$$h_S = S * \cos Z \quad (7)$$

$$h_D = D * \operatorname{ctg} Z \quad (8)$$

$$m_{h_S} = \sqrt{m_S^2 * \cos^2 Z + S^2 * \sin^2 Z * m_Z^2} \quad (9)$$

$$m_{h_D} = \sqrt{m_D^2 * \operatorname{ctg}^2 Z + \frac{D^2}{\sin^2 Z} * m_Z^2} \quad (10)$$

Analysing structure of formulae (9) and (10) the question about the errors of measured distance and zenith angles influences on height difference appears. Assuming that accuracy of measured distances and zenith angles could be reached as it is given in catalogues of most precise total stations on the market we obtain for RMSE of height differences as follows:

$$m_{h_S} = \sqrt{0.00025^2\mathrm{m}^2 \ * \cos^2 135^{\mathrm{o}} + 1.8\ \mathrm{m}^2 * 1 * (\frac{0.5"}{206265"})^2} = 0.00018\ \mathrm{m} \qquad (11)$$

$$m_{h_D} = \sqrt{0.00025^2\mathrm{m}^2 * \mathrm{ctg}^2\, 135^{\mathrm{o}} + \frac{1.8^2\ \mathrm{m}^2}{\sin^2 135^{\mathrm{o}}} * m_Z^2} = 0.00025\ \mathrm{m} \qquad (12)$$

It means practically that RMSE for height differences at extremely short lines of sight approximately equals to the RMSE of distances.

Accordingly to this it follows that hypothesis about equality of obtained and theoretical values of height differences could be formulated as follows:

H_0: Obtained RMSE of height differences at extremely short lines of sight equals the RMSE of distances and
H_0: Obtained RMSE of height differences at extremely short lines of sight not equals the RMSE of distances.

Test statistics is (Perović, 2005):

$$F = \frac{(m_h^O)^2}{(m_h^T)^2} \sim F_{1-\alpha,f_1,f_2}$$

where:

- F – statistics;
- m_h^O – RMSE of height differences obtained by measurement;
- $(m_h^T)^2$ – RMSE of height differences obtained by calculation;
- α – significance level;
- f – degree of freedom and
- $F_{1-\alpha,f,\infty}$ – quantile of F distribution for significance level and degree of freedom f.

Also, in the case of redundant measurements, it is possible to compare the equality of same height differences between two same points measured from different stations.

Bearing in mind that height difference between two points is obtained as:

$$\Delta h = h_j - h_i \qquad (13)$$

immediately follows

$$m_{\Delta h} = \sqrt{m_{h_j}^2 + m_{h_i}^2} \qquad (14)$$

Assuming the worst cases for models (9) and (10) obtained by formulae (11) and (12) and assuming $m_{h_j} = m_{h_i}$ it is obtained $m_{\Delta h} = 0.00025$ m for model (9) and $m_{\Delta h} = 0.00035$ m for model (10).

Hypothesis about equality of two height differences obtained from different position of instrument could be formulated as follows:

H_0: height differences obtained from two different positions are equal and
H_0: height differences obtained from two different positions are not equal.

The test statistics is

$$t = \frac{dh}{m_{dh}} \sim t_{1-\alpha,f}$$

where:

- t– statistics;
- dh – difference of height differences between two points obtained from the different positions;
- $m_{\Delta h}$ – RMSE of height differences between two points;
- α – significance level;
- f – degree of freedom and
- $t_{1-\alpha,f}$ – quantile of F distribution for significance level and degree of freedom f.

Difference of height differences is explained more preciously by following expression

$$dh = \Delta h_{ji}^{k} - \Delta h_{ji}^{l}$$

which means that height differences between points j and i are determined from different positions k and l. Consequently RMSE of dh shall be obtained by following formula:

$$m_{dh} = \sqrt{m_{\Delta h_{ji}^{k}}^{2} + m_{\Delta h_{ji}^{l}}^{2}}$$

Described methodology is utilized on the case study based on results obtained in practical measurements.

3 RESULTS AND DISCUSSION

For the purpose of power aggregate's rotor on power plant "Đerdap" upon a Danube dimension determination, precise measurements of directions and zenith distances were provided. Measurements were provided into tight space and under artificial light. The conditions during measurements are illustrated on figure 1.

Horizontal distances are obtained from adjusted values of points coordinates with estimated accuracy by means of least squares and as such, were used for height difference determination by using model (10). Height differences between thirty two (32) points were measured from five positions inside of power aggregate's rotor circle with radius less than 1 m, which means that horizontal distance between stations and points marked on the part of rotor was between 0.5 m and 1.2 m.

Fig.1. Conditions during measurements

Horizontal distances were obtained from coordinates of every stations and points on rotor and their RMSE was obtained by following formula:

$$m_D = m_0\sqrt{g^T Q_x g} \tag{15}$$

where:

- m_D – MRSE of horizontal distance;
- m_0 – MRSE from adjustment;
- g – vector of linear terms for every function of distance and
- Q_x – cofactor matrix from adjustment.

For MRSE of horizontal distances is obtained to belongs to interval $m_D \in (0.012\ \text{mm}, 0.042\ \text{mm})$. According to hypothesis about equality of theoretical and practical RMSE it could be expected very high accuracy of height differences. Empirical standard deviations for height differences between points were obtained by formula:

$$m_{\Delta h_{ji}} = \sqrt{\frac{1}{5}\sum_{k=1}^{5} m^2_{\Delta h^k_{ji}}}$$

From measurements the values of empirical standard deviations were obtained in interval $m_D \in (0.022\ \text{mm}, 0.929\ \text{mm})$. For twenty three of thirty one height differences null hypothesis about equality of empirical and theoretical standard deviations is accepted and in eight cases is rejected, i.e. in twenty three cases the condition $F < F_{0.95,4,131} = 5.6559$ was satisfied. Testing hypothesis about equality of height differences showed that in fifteen cases null hypothesis was accepted and in sixteen cases was rejected, i.e. in fifteen cases the condition $t < t_{0.95,4} = 2.7764$ was satisfied.

Results for height differences obtained by trigonometric levelling at extremely short lines of sight, show that gross errors exist in differences between theoretical and practical values. Also, results of hypotheses testing showed that empirical values significantly differ from theoretical values in significant number of cases. The difference is higher in testing hypothesis about equality of height differences than in testing hypothesis of height differences

RMSE equality. But bearing in mind that results were obtained from practical work whose main aim was not to determine height differences it is possible to accept hypothesis that trigonometric levelling at extremely short lines of sight possess potential for further investigation. Further investigation could be based on the existing data (research the gross errors in measurements, eliminating results with gross errors from further analysis and utilizing advanced statistical methods for analysis) as well as on special experiment design for investigate the possibilities of trigonometric levelling at extremely short lines of sight (perform measurement under better conditions of visibility, with enough time for measurement and with controlled length of lines of sight as well as controlled values of zenith angles).

4 CONCLUSION

Obtained empirical results of height differences determined by trigonometric levelling method significantly differ from theoretical values. The sample of thirty one height differences even could be treated as too small for reliable conclusions, showed out significant number of null hypothesis rejected (26% null hypothesis rejected about equality of theoretical and empirical RMSE and 52% null hypothesis rejected about equality of height differences between same points determined from different positions).

Nevertheless of those results, number of accepted null hypotheses and the fact that results were obtained in real conditions and that main aim of measurement was not to determine the height differences imply possibility of further investigation of height differences determination by utilizing trigonometric levelling method at extremely short lines of sight.

Possibilities for further research are based on further investigations of existed data and on design of experiments with aim to research the real limits of accuracy of height differences determination by trigonometric levelling at extremely short lines of sight.

Further investigation of existing data shall encompass gross errors analysis of zenith angles (this analysis was not performed here and only statistical hypothesis about height differences were tested). Design of experiments shall encompass the minimum level of flux of light to provide visibility for precise measurements as well as the way for points marking. Design of experiment shall also provide measurement with different zenith angles and different lengths of lines of sight.

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