

Towards a Full-scale, Numerical Adjustment

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ABSTRACT:

We propose a numerical solution to systems of redundant systems of observation equations. The basic characteristics of this solution are: it is based on a topological, grid-search approach in the R^n space; is suitable for complicated functions deriving from various geodetic observations of deformation monitoring and analysis; it can incorporate weights of observations and compute variance-covariance matrices, i.e. overcome the basic limitations of common numerical solutions. The success of this solution is depicted in the case of the adjustment of a leveling network.

1. INTRODUCTION

Redundant systems of equations are traditionally used on the basis of algebraic techniques (least squares) which require linearization of certain functions. In various fields of geodesy these functions are simple (mostly equations defining distances or azimuths between two points) and the overall solution of the system totally effective.

However, in the broader field of Geodesy, there exist cases of redundant systems of equations based on more complicated functions; for instance in the case of elastic dislocation analysis in a seismic deformation (Okada, 1985) or of magma source identification in an active volcano (Feng and Newman, 2009). In such cases the traditional least squares approach is not possible, and either numerical solutions or grid-search solutions minimizing a certain parameter or maximizing the frequency of a certain parameter are adopted (Stiros et al., 2010).

The basic shortcomings of these solutions are: (1) they cannot accept weighted observations; (2) they cannot provide a full variance covariance matrix; (3) they usually can provide solutions for equations with up to 3 variables; (4) in the cases the unknowns are more than 2, some of the estimates are highly correlated.

In an effort to solve this problem we propose a topological, grid-based solution which permits to include weights for observations and to obtain variance-covariance matrices.

2. METHODOLOGY

We assume a system of n equations f with m unknown variables $\mathbf{x} = (x_1, x_2, \dots, x_i)$, defined by the equations

$$f_j = f_j(x_1, x_2, \dots, x_i) = l_j \quad (1)$$

where, $j = 1, 2, \dots, n$, $i = 1, 2, \dots, m$ and l_j : n measurements each with uncertainty σ_j .

It is also assumed that the system of equations (1) has a solution

$$\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_m) \quad (2)$$

in R^m which is subject to the conditions

$$x_{i,min} < \tilde{x}_i < x_{i,max} \quad i=1,2, \dots, m \quad (3)$$

The geometric/topological significance of this hypothesis is that there exists a common section in the solution of all equations, and this common section represents the solution of the system of equations.

Equation (3) defines a rectangular or a hyper-rectangular in R^m . This hyper-rectangular can be defined by a hyper-grid G , with spacing s_i constant for any two successive points in axis i .

Each measurement l_j in eq. (1) is characterized by an uncertainty σ_j . Introducing a parameter k_j , we define a parameter

$$\delta_j = k_j \sigma_j \quad (4)$$

From eq. (1) we can derive the following inequality

$$|f_j - l_j| < \delta_j \quad (5)$$

A cloud A_j of points of the hyper-grid G will satisfy eq. (5) because of the hypothesis of equation (1). This cloud will obviously depend on the selection of k_j .

Similarly, for each of the n equations (5) a cloud of grid points will be defined, and the common section of all these clouds of grid-points

$$B = A_1 \cap A_2 \dots \cap A_n \subseteq G \quad (6)$$

will represent a cloud of points approximating (containing) the solution of the system of equations (1). The dimensions and the very existence of the cloud of points $B \neq \emptyset$ will depend of course on the selection of k_j , which in fact represents an inverse weight of observations. Finally, from the m -dimensional cloud of grid points their mean value will represent an estimate of the solution $\tilde{\mathbf{x}}$ of the system of defined by eq. (2) and the associated variance-covariance matrix Q can be computed. The overall solution can be obtained using a simple algorithm. This approach can be regarded as a generalization of the determination of the coordinates of a GPS receiver. Measurement of distances from a receiver (the coordinates of which are unknown) to certain satellite permits to locate the receiver on a sphere, the centre of which is the known position

of the satellite and its radius is the measured distance S_i . If the uncertainty σ_i of this distance is taken into account, the location of the satellite will be in the space between homocentric spheres with radii $S_i + \sigma_i$, $S_i - \sigma_i$. Similar measurements from two other satellites permit to locate the receiver at the common section of the three different concentric spaces.

3. CASE STUDY

The proposed topological approach for numerical integration was applied in the case study of a leveling network, but it can be used to solve any other system of non-linear equations. We examine a four-point leveling network with coordinates $x = \{x_0, x_1, x_2, x_3\} = \{0.0, 11.110, 14.560, 13.310\}$ (m). We produced a number of measurements with a standard error of $\sigma_j = \pm 4$ mm between point O (assumed of known coordinate) and the other points, as shown in Table 1. Based on these measurements we shall try to estimate the three coordinates $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ and their variance-covariance matrix. This is a simple linear problem in the R^3 space.

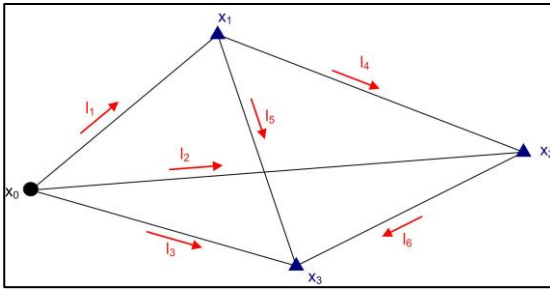


Figure 1. The 4-point leveling network (height of point O assumed known) of our case study. The six observations l_j are shown.

Measurements (m)	
l_1	11.113
l_2	14.562
l_3	13.314
l_4	3.448
l_5	2.204
l_6	-1.252

Table 1. Measurements l_j of the elevation difference between the four points.

Each of the six measurements leads to a linear equation of the type

$$f_1 = x_1 - x_0 = l_1 \quad (7)$$

and adopting for simplicity weights $k_j=0.75$, a flat value of $\delta_j=3$ mm was defined for all observations.

The next step was to define the 3-dimensional grid G necessary for our solution. We computed a preliminary solution and

then selected a 3-D grid around this solution. This grid was selected with a total width of $2\text{cm} (=5\sigma_j)$ in each of the three axes and step $s = 0.5$ mm between each grid point in each axis; hence it was defined by 41 points in each axis and $41^3 = 68,921$ grid points in total.

The following step was to define the 6 inequalities of equations (5), of the type

$$|f_j - l_j| < \delta_j \quad (8)$$

with $\delta_j = 3.0\text{mm}$, corresponding to each of the 6 observations (Table 2), shown below, and define the set (cloud) of grid points A_j satisfying each one of them. This was done using a grid-search algorithm which identifies the points which satisfy each of the inequalities. Then we found the common section of all 6 clouds of grid points. In graph form, the clouds of points A_1 to A_6 and B are shown in Fig. 2. At a final step we estimated the mean value of their coordinates and the elements of the corresponding variance-covariance matrix (Table 3).

The net adjustment was made also on the basis of conventional least square techniques, and the results are also summarized in Table 3. Estimates of coordinates from both techniques are quite similar, but the variances in the topological approach are much larger. These variances can of course be optimized adopting a different “weight” factor k_j .

equation system	cloud
$ x_1 - l_1 < \delta_j$	A_1
$ x_2 - l_2 < \delta_j$	A_2
$ x_3 - l_3 < \delta_j$	A_3
$ (x_2 - x_1) - l_4 < \delta_j$	A_4
$ (x_3 - x_1) - l_5 < \delta_j$	A_5
$ (x_3 - x_2) - l_6 < \delta_j$	A_6

Table 2. The system of inequalities formed and the corresponding cloud of points A_j satisfying each inequality.

elevation	real values	our estimates ($\pm\sigma$)	lsqr estimates ($\pm\sigma$)
x_1	11.110	11.1121 (± 0.00225)	11.1125 (± 0.00110)
x_2	14.560	14.5629 (± 0.00225)	14.5627 (± 0.00110)
x_3	13.310	13.3137 (± 0.00250)	13.3137 (± 0.00110)

Table 3. Hypothetical and adjusted coordinates (in m) of the network.

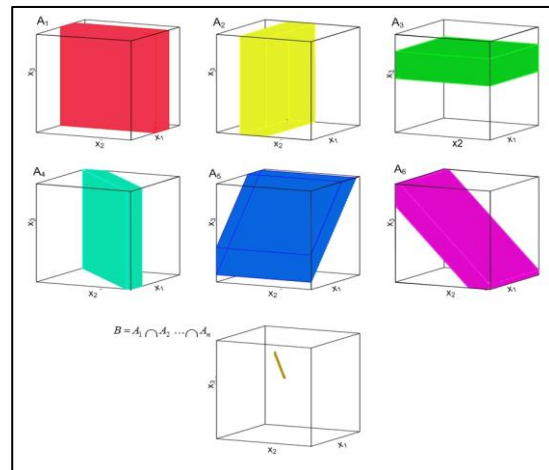


Figure 2. Perspective view of the sets (clouds) of grid points A_j and of their common section B .

4. DISCUSSION AND CONCLUSION

The proposed approach permits a numerical, grid-search-based solution in redundant systems of equations free of the limitations noticed in the Introduction. In particular, weighted observations and full variance-covariance matrices can be computed. In addition, simultaneous solutions for systems of equations with more than 3 unknown variables can be obtained, avoiding highly correlated solutions obtained by other numerical, especially grid-search techniques.

5. REFERENCES

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