Optimization of GPS Networks with Respect to Accuracy and Reliability Criteria

Mualla YALÇINKAYA and Kamil TEKE, Turkey

Key words: Optimization of GPS networks, second order design - U,m approximation method, accuracy, reliability.

SUMMARY

Optimal design of geodetic GPS networks with respect to accuracy and reliability criteria is an essential part of most geodesy related projects. Whether the datum and point locations of a network was known, the process of determining the optimal baseline configuration and their optimal weights with respect to the selected design criteria can be achieved by optimizing observational plan using Second Order Design (SOD). The scalar design criteria can only satisfy to the limited demands for a network. Thus, criterion matrices, which can be defined as the computed variance-covariance matrix in designing stage that meet many of the accuracy demands, is mostly used. Analytical approximations to the criterion matrices are effective methods of reaching objective functions formulated with criterion matrices.

In this study, observational plan optimization of GPS networks with respect to the accuracy and reliability criteria was aimed to carry out. A criterion matrix was constituted which provides equal sized (homogeneity) and sphere viewed (isotropy) error ellipsoids of points and the radius of the relative error ellipsoids are computed from a function of the distances between points, named chaotic (complete isotropy) Taylor-Karman structured criterion matrix. By an S transformation, criterion matrix's datum adaptation to the network was provided. Then, direct approximation of the inverse criterion matrix (U,m approximation) method was applied to the criterion matrix to compute the optimal weights of observations. In the first step of the solution, optimal baseline configuration was found out by removing the baselines of which weights were negative or near zero (the baselines that have not any contribution to the design criteria) from the observational plan. In the second step of this solution, optimal weights of the remaining baselines in the observational plan were computed. In the last step, redundancy numbers of the baseline components, limit values of the undetected blunders (internal reliability) and the effect of the undetected blunders on the coordinates (external reliability) were calculated, so as to put forth the sensitivity of the network configuration to model errors for consideration, in other words the reliability of the network was determined and new baselines were planned to the baselines of which reliability values were not found out sufficient. As a result of the study, in designing stage to ensure the precision and reliability demands from GPS networks, applicable solutions have been suggested.

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1. INTRODUCTION

Geodetic networks are to satisfy certain accuracy and reliability criteria as to serve the aim of network establishment. In designing stage of a GPS network postulated accuracy and reliability criteria can be formulated and applied by certain optimization procedures. The main optimization purpose of geodetic networks is to establish them with high reliability, good accuracy and low cost. Optimization problem is usually classified into different orders (Grafarend 1974). To fulfill the demands of homogeneity and isotropy conditions in all directions for a network with respect to accuracy, second order design (SOD), which is defined as the problem of finding the observational weight matrix from the configuration of the network, is applicable. The SOD Problem has become rather popular in geodesy after its initial treatment by Grafarend (1974, 1975). Since then many new algorithms have been developed with simulated example and real application, where the criterion matrices with Taylor-Karman-structured are used (Grafarend 1970, 1972). Baarda (1973). Cross (1985), Schaffrin (1985), Wimmer (1981) and Schmitt (1980) have discussed SOD for terrestrial geodetic networks in terms of achieving maximum accuracy and minimum cost. Generally, three basic kinds of approximation methods are used in the SOD procedure. These are the direct approximation of the criterion matrix (direct-HR), iterative approximation of the criterion matrix (iterative-HR) and direct approximation of the inverse criterion matrix (U,m) (Grafarend 1975, Schmitt 1985). Analytical methods used to realize optimization that serves to approach the criterion matrix are: sequential least squares, linear and non-linear programming (Grafarend 1970, Grafarend 1972, Grafarend and Harland 1973, Grafarend 1974, Koch 1982, Schaffrin 1985, Gerasimenko 1997, Seemkooei 2001, Grafarend 2006).

In order to increase the capability of detecting model errors and outliers in a geodetic network, it has to be optimized. Baarda (1968) distinguishes "*internal reliability*" and "*external reliability*". While internal reliability of a control network measures the marginal undetectable error in the measurements, external reliability measures the effect of an undetectable gross error on the network coordinates and on quantities computed from them (Even-Tzur and Papo, 1996).

2. METHODOLOGY

The quantities to be obtained after the measurement for a GPS network are the baselines' vector components and their covariances. The final outcomes of the baseline adjustment procedure are the adjusted baselines, estimated points coordinates and their final covariance matrices. In an optimization procedure in respect to accuracy, criterion matrix defined as artificial covariance matrix of the estimated coordinates is constituted due to the objectives of the network. Then, SOD procedure can be carried out by the U,m approximation method. The initial observational plan can be formed by including all the probable baselines. Final outcomes will be the optimal observational plan (optimal baselines and their weights). A GPS network can be optimized in respect to accuracy and reliability criteria as the following steps:

- A criterion matrix is constituted for the network ensures maximum homogeneity and isotropy conditions.
- Initial design matrix of the first observational plan is formed by all probable baselines taken account.
- U,m approximation method of SOD is applied to the criterion matrix. The baselines, of which weights are found out negative or very close to zero, should be removed from the observational plan, since these baselines do not serve the objective function (criterion matrix). This procedure is continued until all the observation weights are found out to be positive. Finally reduced observational plan is provided.
- Global relative redundancy number (r_0) of the network is, individual redundancy numbers (r_j) of the baselines are, internal reliability criteria (Δ_{0j}) and external reliability criteria (δ_{0j}) are calculated from the reduced observational plan. If some of the individual redundancy numbers are lower than the global relative redundancy and the baselines of which internal and external reliability values are not fit the critical values it is decided that the concerned baselines cannot be checked adequately by the other baselines. As to provide compliance with the boundary conditions new baselines are planned perpendicular to the relevant baselines.
- U,m approximation method of SOD is applied to the criterion matrix by using the new observational plan. The optimal baselines and their optimal weights are produced again. After that, construction and measurement works are begun.

2.1 Accuracy Optimization of Geodetic GPS Networks

Accuracy optimization of GPS networks can be carried out by U,m approximation method. Effective covariance matrix produced from U,m approximation method is to be overlapped with the criterion matrix. Criterion matrix is to include all the accuracy demands from the network.

2.1.1 Construction of Criterion Matrix

The criterion matrix can be defined as an ideal artificial covariance matrix that ensures all required accuracy conditions from a geodetic network. In the design stage of geodetic networks, criterion matrices can be chosen as global objective functions that represent all the accuracy criteria of the network (Grafarend and Schaffrin 1979).

The general expression for the sub matrix of cross-co-variances between Pi and Pj within the Taylor-Karman (TK) structured criterion matrix is

$$C_{xx} = \begin{bmatrix} \varphi_m(s) & 0 & 0 \\ 0 & \varphi_m(s) & 0 \\ 0 & 0 & \varphi_m(s) \end{bmatrix} + [\varphi_l(s) - \varphi_m(s)] \begin{bmatrix} \frac{\Delta X^2}{s^2} & \frac{\Delta X \Delta Y}{s^2} & \frac{\Delta X \Delta Z}{s^2} \\ \frac{\Delta X \Delta Y}{s^2} & \frac{\Delta Y^2}{s^2} & \frac{\Delta Y \Delta Z}{s^2} \\ \frac{\Delta X \Delta Z}{s^2} & \frac{\Delta Y \Delta Z}{s^2} & \frac{\Delta Z^2}{s^2} \end{bmatrix} \tag{1}$$

where $\Delta X = X_i - X_j$, $\Delta Y = Y_i - Y_j$ and $\Delta Z = Z_i - Z_j$, are the baseline vector lengths between the points i and j, $\varphi_m(s)$ denotes the transversal or cross correlation function and $\varphi_l(s)$ the longitudinal correlation function (Grafarend 1970, Grafarend 1972, Baarda 1973, Grafarend 1974, Schmitt 1980).

When the network is expected to provide the conditions homogeneity and isotropy, TK-structured criterion matrix that has the characteristics; absolute error ellipsoids are to be spherical (isotropy), absolute and relative error ellipsoids are to be equally sized (homogeneity) and relative error ellipsoids are to be defined by correlation functions depended on the distances between network points, can be formed. As a result of the preceding condition only one correlation function can be defined as follows,

$$\begin{aligned} \varphi_m(S) &= \varphi_l(S) = \varphi(S) \\ \varphi(S) &= d^2 - 2c^2 S_{ij(km)} \end{aligned} \tag{2}$$

where d, is the semi axes of point error ellipsoids, c is an arbitrary constant that provides all the $\varphi(S)$ values to be positive.

Experiences with GPS measurements have shown that accuracies of the horizontal components of the baseline vectors ($\Delta X, \Delta Y$) are more or less equal, while the accuracy of the vertical component (ΔZ), is approximately two or more times lower (Even-Tzur and Papo 1996). Thus, the following weights can be adopted: $p_{\Delta X} = p_{\Delta Y} = 1$ and $p_{\Delta Z} = \frac{1}{k^2}$, where $k=2$. These relationships of the baseline weights reflects the cofactor matrix of estimated coordinates (Q_{xx}) in different proportions dependent on the observational plan. Finally, the sub matrix of cross-co-variances between points *i* and *j* within the TK-structured criterion matrix of GPS networks can be designed as follows:

$$C_{xx} = \begin{bmatrix} d^2 & 0 & 0 & d^2 - 2c^2 S_{ij} & 0 & 0 \\ 0 & d^2 & 0 & 0 & d^2 - 2c^2 S_{ij} & 0 \\ 0 & 0 & 4d^2 & 0 & 0 & 4(d^2 - 2c^2 S_{ij}) \\ d^2 - 2c^2 S_{ij} & 0 & 0 & d^2 & 0 & 0 \\ 0 & d^2 - 2c^2 S_{ij} & 0 & 0 & d^2 & 0 \\ 0 & 0 & 4(d^2 - 2c^2 S_{ij}) & 0 & 0 & 4d^2 \end{bmatrix} \tag{3}$$

To compare the criterion matrix with the datum dependent variance-covariance matrix of the estimated coordinates, which is constructed with respect to the observational plan, the criterion matrix can be transformed into the same datum as the variance-covariance matrix of the estimated coordinates as follows:

$$
\begin{aligned}
S &= I - G(G^T G)^{-1} G^T \\
\overline{Q}_{xx} &= S C_{xx} S^T
\end{aligned}
\tag{4}
$$

where I is the identity matrix, G is the orthogonal coefficient matrix and S is the transformation matrix. As to carry out a datum free optimization procedure, the transformed criterion matrix ($\overline{Q}_{xx}$) has to be positive semi-definite like the covariance matrix of the observational plan (Baarda 1973, Schaffrin and Grafarend 1982, Teunissen 1985, Kuang 1992, Even-Tzur and Papo 1996, Marinkovic et al. 2003).

2.1.2 U,m approximation method of Second Order Design

The most probable values of the baseline vector components' weights can be provided by the methods, direct approximation of the criterion matrix (direct-HR), iterative approximation of the criterion matrix (iterative-HR) and direct approximation of the inverse criterion matrix (U,m) (Grafarend 1975, Schmitt 1985). The basic equation for the approximation methods is

$$
A^T P A \doteq \overline{Q}_{xx}^{+} \tag{5}
$$

where A is the design matrix, P is the unknown weight matrix of observations, $\overline{Q}_{xx}$ is the criterion matrix and $\doteq$ shows inconsistent equality. Matrix equation (5) is converted to a set of linear equations

$$
(A^T \odot A^T) p \doteq q \tag{6}
$$

where $\odot$ is the notation of the Khatri-Rao product, q is the vectorised criterion matrix, $p = \mathrm{diag}(P)$ is a vector composed of the diagonal elements of P, $q = \mathrm{vec}(\overline{Q}_{xx}^{+})$, is the vectorised $\overline{Q}_{xx}^{+}$ matrix (Schmitt 1979). By adding inconsistency parameters (d) to the linear equation system given in Eq(6) yields

$$
(A^T \odot A^T) p = q + d \tag{7}
$$

Applying the least squares method to Eq. (7) providing the condition,

$$
\Omega = d^T d = e^T \{[(A^T PA) - \overline{Q}_{xx}^{+}] * [(A^T PA) - \overline{Q}_{xx}^{+}]\} e \Rightarrow \min \tag{8}
$$

unknown weights of baseline vector components can be provided as follows:

$$p = [(A^T \odot A^T)^T (A^T \odot A^T)]^+ (A^T \odot A^T)^T q \tag{9}$$

The baselines, of which weights are found out negative or very close to zero, should be removed from the observational plan, since these baselines do not serve the objective function (criterion matrix). This procedure is continued until all the positive observation weights are found out to be positive. Finally reduced observational plan is provided (Schmitt 1979).

The quality of the approximation of an inconsistent design can be expressed by the difference between the cofactor matrix that is constituted from observational plan and the given criterion matrix as follows:

$$\begin{gathered} D = (A^T PA)^+ - \overline{Q}_{xx} \\ r = \text{vec}(D) \end{gathered} \tag{10}$$

where $r^T r$ is an global measure for the approximation quality of optimization (global test value). Another global measure for the approximation quality of optimization is the largest eigenvalue of matrix B which is calculated as:

$$B = (A^T PA)^+ \overline{Q}_{xx}^+ \tag{11}$$

The largest eigenvalue of matrix B has to approach the value 1 after removing the baselines that have negative weights and on completion of optimization must be close to 1 (Grafarend 1975, Schmitt 1979).

2.2 Reliability Optimization of Geodetic GPS Networks

In order to increase the capability of detecting model errors and outliers in a geodetic network, it has to be optimized. Baarda (1968) distinguishes “internal reliability” and “external reliability”. While internal reliability of a control network measures the marginal undetectable error in the measurements, external reliability measures the effect of an undetectable gross error on the network coordinates and on quantities computed from them. The aim of geodetic networks adjustment is to calculate coordinates so external reliability is of more practical value than internal reliability. There is a strong correlation between internal and external reliability; high internal reliability leads to high external reliability (Even-Tzur and Papo, 1996).

The reliability of a network is considered high when the network can identify even small gross errors. Gross errors in the measurements affect the adjustment parameters; therefore, the reliability of the network is useful as a design criterion (Biacs et al. 1990). In Table 1, certain scalar reliability objective functions, which are to ensure the limit of the critical values, are given.

Table 1. Reliability objective functions

Reliability objective functions		Critical Values
Individual redundancy	$Z = r_j = (Q_{vv})_j P_j$	$Z = r_j > 0.4$
Internal reliability	$Z = \lvert \Delta_{0j} \rvert = m_0 \sqrt{\frac{w_0}{P_j r_j}}$	$Z = \Delta_{0j} < 6 \quad m_j$
External reliability	$Z = \delta_{0j}^2 = \frac{1 - r_j}{r_j} w_0$	$Z = \delta_{0j} < 6$

In the equations above, Q_{vv} is the cofactor matrix of the residuals, P is the weight matrix of the observations, m_0 is the standard deviation of unit weight and w_0 is the lower bound for non-centrality parameter in dependency of the significance level (α_0) and the required minimum power of the test ($1-\beta_0$).

So as to reach global relative redundancy number of GPS network at the value of 0.4, first observation plan is constituted with adequate baselines. When optimizing GPS networks in respect to reliability redundancy numbers of the baselines, internal and external reliability of the network are to be taken into consideration as objective functions. New baselines are planned perpendicular to the baselines of which redundancy numbers are under the value of 0.4. Also new baselines are planned perpendicular to the baselines of which internal and external reliability values are higher than the critical values (Dare and Saleh 2000).

A geodetic network can be optimized depending on the reliability objective functions selected from Table 1 as the following steps:

- The global relative redundancy number (r_0) of the network is and individual redundancy numbers of the baselines are (r_j) calculated from the first observation plan.
- If some of the individual redundancy numbers are lower than the global relative redundancy it is decided that the concerned baselines cannot be checked adequately by the other baselines. For that reason, new baselines are planned perpendicular to the relevant baselines.
- Internal reliability criteria (Δ_{0j}) and external reliability criteria (δ_{0j}) are calculated for the network.
- The baselines of which internal reliability values are above the critical values ($6m_j$) are decided that they cannot be checked adequately by the other baselines. For that reason, new baselines are planned perpendicular to the relevant baselines.
- When the cost of the network is taken into consideration, the baselines of which redundancy numbers are $r_j >> r_0$ would be removed from the network.

- The improved session plan is reviewed lastly if it ensures the selected objective function. Then the design is applied to the field (Baarda 1977, Gazdzicki 1976).

3. APPLICATION

To examine the success of U,m approximation method, SOD was carried out for a GPS network, in the province Ordu in Eastern Black Sea Region of Turkey (Fig. 1). In order to decide the baseline vector components to be used for forming of the criterion matrix, calculation of the points' coordinates in WGS84 was necessary. The baseline vectors were calculated before the optimization of the network by subtracting points' coordinates from each other. Universal Transverse Mercator (UTM) Projection coordinates were read from the map of the study area. Then these projection coordinates were transformed to WGS84 coordinates by seven parameter similarity transformation. The baseline vector components $(\Delta X, \Delta Y, \Delta Z)_{WGS84}$ were derived from these coordinates by subtracting them between each other. The criterion matrix was formed based on Eq. (3) using proposed correlation function in Eq. (2).

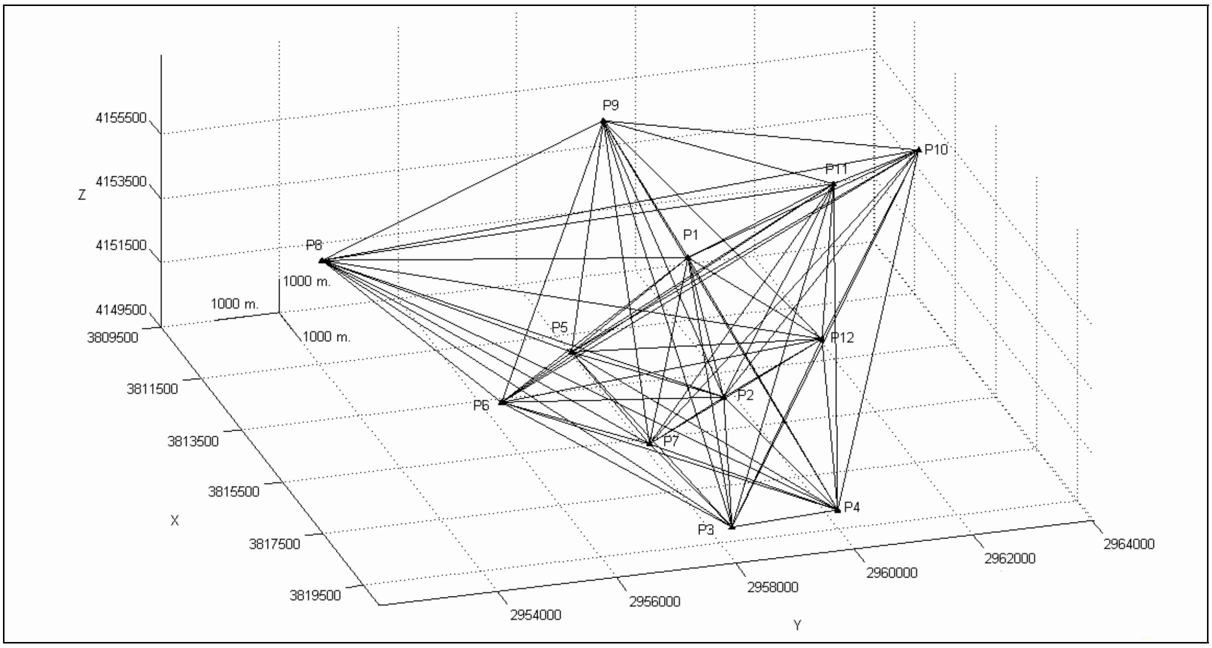

Figure 1. Test network constituted by all probable baselines

Datum transformation was applied to the TK-structured criterion matrix to fix the datum of the criterion matrix with the network. The eigenvalues and semi-axes of point error ellipsoids, computed from the datum dependent criterion matrix, are summarized in Table 2.

Table 2. Eigenvalues and semi-axes of point error ellipsoids computed from the criterion matrix

Points	Eigenvalues			Semi-axes of point error ellipsoids (cm)		
	λ_X	λ_Y	λ_Z	A_H (cm)	B_H (cm)	C_H (cm)
P1	0.51546	0.72397	2.21696	0.72	0.85	1.49
P2	0.44624	0.59452	1.92035	0.67	0.77	1.39
P3	0.58547	0.69749	2.77530	0.77	0.84	1.67
P4	0.62747	0.71823	3.03716	0.79	0.85	1.74
P5	0.51368	0.65644	2.19780	0.72	0.81	1.48
P6	0.61412	0.84281	2.66811	0.78	0.92	1.63
P7	0.50210	0.60324	2.12299	0.71	0.78	1.46
P8	0.79201	1.02238	3.87750	0.89	1.01	1.97
P9	0.74523	0.84930	3.39773	0.86	0.92	1.84
P10	0.74694	1.07778	2.94357	0.86	1.04	1.72
P11	0.62912	0.94723	2.52773	0.79	0.97	1.59
P12	0.54604	0.78572	2.31094	0.74	0.89	1.52

In order to apply the classical U,m approximation method to the criterion matrix, firstly the design matrix (A) was formed by taking all probable baselines into account. Afterwards, the weights of the GPS baseline vector components were computed from Eq. (9) using least-squares. Finally, the baselines that their weights found to be negative or close to zero (p_i <0.1) were eliminated from the observational plan, since the baselines of those weights that are negative or close to zero do not have any connection with the desired precision criteria for the network. Following the fourth iteration, U,m approximation method did not produce any negative or close zero weights so the optimization procedure was finalized with the remaining baselines and their optimal weights. Since the quality of the approximation of an inconsistent design, can be expressed by the difference between the cofactor matrix that is constituted from the observational plan and the given criterion matrix, for each iteration the global test value given in Eq. (10) and the maximum eigenvalue of matrix B given in Eq. (11) were examined (Table 3).

Table 3. Approximation quality values of U,m approximation method for each iteration

	First iteration	Second iteration	Third iteration	Fourth iteration
The number of baselines input for the optimization	66*3 = 198	36*3 = 108	29*3=87	27*3=81
Global test value ($r^T r$)	2.523	12.856	18.602	18.779
Maximum lamda value of matrix B ($\lambda_{\max}$)	3.253	5.508	5.437	5.670
The number of baselines decided to eliminate from observational plan	30*3 = 90	7*3=21	2*3=6	0*0=0

The effective and postulated point error ellipsoids of fourth observational plan derived from U,m approximation method are plotted in Figure 2. It can be seen that sizes and orientations of final effective point error ellipsoids are compatible with the postulated ones.

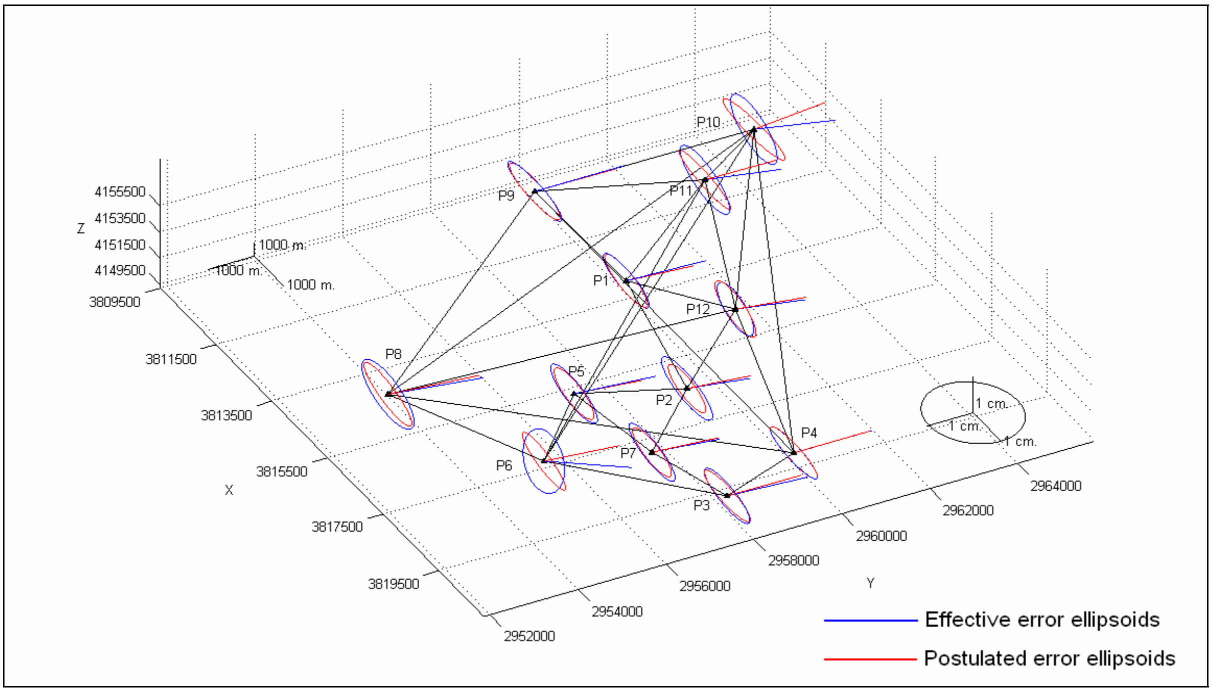

Figure 2. Effective and postulated point error ellipsoids of fourth observational plan derived from U,m approximation method

The effective and postulated relative error ellipsoids of fourth observational plan derived from U,m approximation method are plotted in Figure 3. It can be seen that sizes and orientations of final effective relative error ellipsoids are almost compatible with the postulated ones.

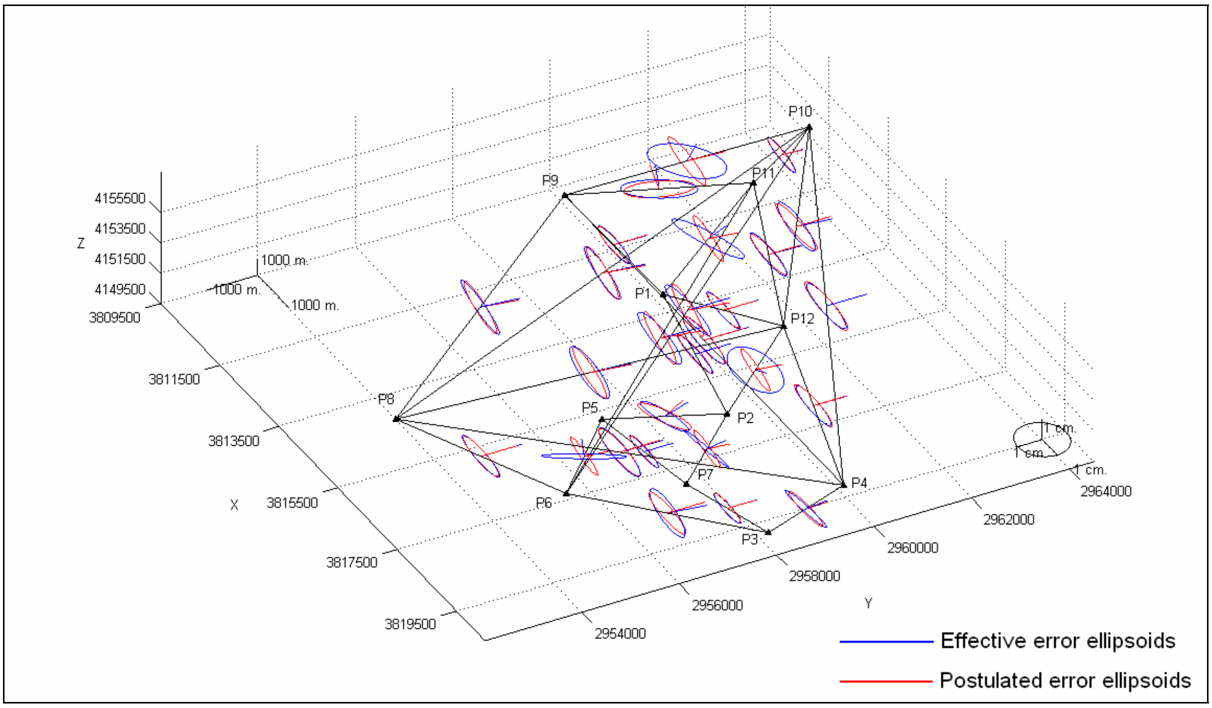

Figure 3. Effective and postulated relative error ellipsoids of fourth observational plan derived from U,m approximation method

In the reliability optimization of the reduced GPS observational plan, it was aimed that global redundancy number is to be greater than the value of 0.4, individual redundancy numbers are to be close to the global relative redundancy, internal reliability criterion values are to be under the "$6m_j$" critical values and external reliability criterion values are to be under the "6" critical value. The optimal weights produced by U,m approximation method of SOD and redundancy numbers of the baseline vector components are shown in Table 4.

Table 4. Weights and redundancy numbers of the designed GPS network baselines

Baselines	Optimal baseline weights			Redundancy numbers (Critical value=0.4)		
	$p_{\Delta X}$	$p_{\Delta Y}$	$p_{\Delta Z}$	$r_{\Delta X}$	$r_{\Delta Y}$	$r_{\Delta Z}$
P1-P2	0.10809	0.98211	0.08193	0.98	**0.17**	0.70
P1-P9	0.46128	0.92786	0.10579	0.45	**0.16**	0.49
P1-P11	0.51092	0.08610	0.22103	0.77	0.94	**0.30**
P1-P12	0.95056	0.43541	0.06339	**0.25**	0.59	0.76
P2-P5	0.94147	0.40174	0.08986	**0.26**	0.54	0.67
P2-P7	0.95893	0.63738	0.37830	0.54	0.67	**0.23**
P2-P12	0.76996	0.37566	0.27538	0.40	0.80	**0.26**
P3-P4	0.98771	0.65727	0.37302	**0.17**	**0.35**	**0.14**
P3-P6	0.15131	0.27462	0.02898	0.72	0.57	0.79
P3-P7	0.93445	0.97161	0.16636	**0.17**	**0.21**	**0.37**
P4-P8	0.23879	0.14269	0.02647	0.61	0.74	0.78
P4-P9	0.20073	0.12902	0.01589	0.69	0.81	0.86

P4-P10	0.18082	0.16830	0.01813	0.71	0.72	0.86
P4-P12	0.17157	0.63687	0.08594	0.77	**0.38**	0.57
P5-P6	0.79245	0.95237	0.39484	0.47	0.48	**0.12**
P5-P7	0.77557	0.99002	0.15151	0.43	**0.19**	0.45
P6-P8	0.63845	0.52998	0.07529	**0.34**	**0.29**	0.48
P6-P10	0.18826	0.03912	0.01777	0.70	0.92	0.86
P6-P11	0.14801	0.02543	0.00385	0.75	0.94	0.97
P8-P9	0.17250	0.14444	0.08944	0.78	0.76	0.41
P8-P10	0.09647	0.13865	0.02995	0.83	0.69	0.77
P8-P12	0.06574	0.03805	0.01416	0.86	0.91	0.89
P9-P10	0.09851	0.10911	0.03682	0.86	0.78	0.78
P9-P11	0.47837	0.16789	0.04175	0.46	0.69	0.80
P10-P11	0.99327	0.97959	0.42781	**0.18**	**0.22**	**0.15**
P10-P12	0.01814	0.01192	0.03375	0.95	0.96	0.80
P11-P12	0.12697	0.72597	0.05952	0.85	**0.20**	0.75

In Table 4, it can be seen that redundancy numbers of the baselines written in bold are under the critical value. The internal and external reliability values of the baselines related to the reduced observation plan are given in Table 5.

Table 5. Internal and external reliability values of the designed GPS network

Baselines	Internal reliabilities and critical values						External reliabilities (Critical value = 6)		
	$\lvert\Delta_{0\Delta Xi}\rvert$	$6m_{\Delta Xi}$	$\lvert\Delta_{0\Delta Yi}\rvert$	$6m_{\Delta Yi}$	$\lvert\Delta_{0\Delta Zi}\rvert$	$6m_{\Delta Zi}$	$\delta_{0\Delta X_i}$	$\delta_{0\Delta Y_i}$	$\delta_{0\Delta Z_i}$
P1-P2	5.51	8.18	**7.81**	**4.80**	11.45	14.40	0.57	**8.91**	2.60
P1-P9	6.78	6.81	**9.64**	**5.73**	13.07	13.68	4.43	**9.26**	4.11
P1-P11	5.19	6.82	5.22	8.68	**13.09**	**10.71**	2.21	0.10	**6.14**
P1-P12	**7.20**	**5.36**	6.22	7.19	10.82	14.13	**7.00**	3.31	2.26
P2-P5	**6.39**	**4.93**	6.59	7.23	10.40	12.77	**6.66**	3.73	2.81
P2-P7	5.02	5.54	5.37	6.57	**12.10**	**8.63**	3.67	2.84	**7.40**
P2-P12	6.48	6.65	5.43	7.27	**12.80**	**9.81**	4.86	2.01	**6.73**
P3-P4	**8.37**	**5.16**	**7.75**	**6.86**	**16.19**	**9.14**	**8.87**	**6.47**	**9.84**
P3-P6	6.34	8.07	6.63	7.48	11.93	15.90	2.49	3.51	2.06
P3-P7	**8.31**	**5.19**	**6.97**	**4.78**	**13.56**	**12.34**	**8.73**	**7.78**	**6.24**
P4-P8	6.72	7.86	6.70	8.63	13.42	17.77	3.21	2.39	2.13
P4-P9	6.30	7.85	6.01	8.09	13.05	18.17	2.69	1.96	1.61
P4-P10	6.14	7.75	6.26	7.95	12.08	16.84	2.57	2.50	1.59
P4-P12	5.52	7.29	**6.38**	**5.93**	12.19	13.77	2.16	5.07	3.49
P5-P6	7.50	7.70	6.70	6.99	**17.45**	**8.99**	4.27	4.13	**10.93**
P5-P7	5.06	5.94	**6.99**	**4.55**	12.05	12.09	4.62	**8.31**	4.44
P6-P8	**7.51**	**6.54**	**9.17**	**7.46**	16.00	16.65	5.60	**6.19**	4.15
P6-P10	5.98	7.52	6.49	9.33	12.58	17.49	2.60	1.21	1.62
P6-P11	5.95	7.71	6.43	9.33	11.79	17.41	2.34	1.03	0.72
P8-P9	6.13	8.10	6.88	8.99	15.17	15.37	2.15	2.25	4.78
P8-P10	5.78	7.91	7.26	9.03	12.47	16.46	1.79	2.70	2.16
P8-P12	4.17	5.58	6.16	8.83	11.94	16.86	1.62	1.22	1.44
P9-P10	5.09	7.09	6.50	8.59	11.10	14.68	1.59	2.15	2.14
P9-P11	6.18	6.29	6.70	8.35	10.48	14.06	4.33	2.68	2.00
P10-P11	**7.72**	**4.92**	**8.34**	**5.81**	**14.45**	**8.45**	**8.54**	**7.62**	**9.45**

P10-P12	5.29	7.94	4.93	7.47	10.58	14.23	0.10	0.10	1.98
P11-P12	5.52	7.62	**9.58**	**6.35**	10.52	13.65	1.70	**8.12**	2.32

When Table 5 is examined, it can be seen that internal and external reliability values of the baselines written in bold are under the critical value. As to provide the compliance all the individual reliability criteria with their critical values certain baselines were added to the reduced observational plan (Figure 4).

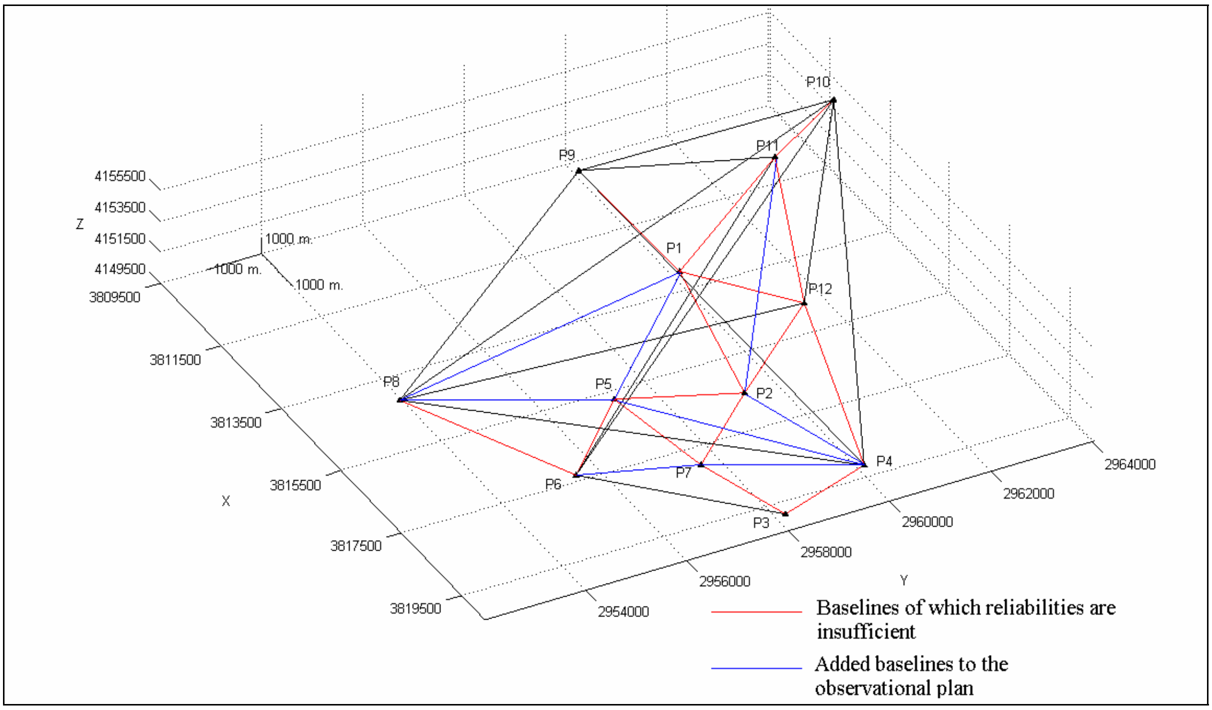

Figure 4. The baselines of which reliabilities are insufficient and the added baselines to the observational plan

Finally, U,m approximation method was applied to the criterion matrix again with the design matrix of the new observational plan. The last outcomes were the optimal baselines and their optimal weights. The last observational plan was ensured not only the optimal weights of the baselines provided by U,m approximation method of SOD but also the reliability conditions for the network.

4. CONCLUSION

Whether the datum and point locations of a network was known, the process of determining the optimal baseline configuration and their optimal weights with respect to the selected design criteria can be achieved by optimizing observational plan using Second Order Design (SOD). Accuracy optimization of GPS networks can be carried out by direct approximation of the inverse criterion matrix (U,m approximation method of SOD). Effective covariance matrix produced from U,m approximation method is to be overlapped with the criterion matrix.

In this study, SOD of a GPS network was carried out with respect to the accuracy criteria. A criterion matrix was constituted which provides equal sized (homogeneity) and sphere viewed (isotropy) error ellipsoids of points and the radius of the relative error ellipsoids were computed from a function of the distances between points, named chaotic (complete isotropy) Taylor-Karman structured criterion matrix. Then, U,m approximation method was applied to the criterion matrix to compute the optimal weights of observations. Optimal baseline configuration was found out by removing the baselines of which weights were negative or near zero from the observational plan. In the second step of this solution, optimal weights of the remaining baselines in the observational plan were computed. In the last step, redundancy numbers of the baseline components, limit values of the undetected blunders (internal reliability) and the effect of the undetected blunders on the coordinates (external reliability) were calculated, so as to put forth the sensitivity of the network configuration to model errors for consideration, in other words the reliability of the network was determined and new baselines were planned to the baselines of which reliability values were not found out sufficient. So that capability of the network geometry in detecting model errors and outliers was increased. Finally, U,m approximation method was applied to the criterion matrix again with the design matrix of the new observational plan. The last outcomes were the optimal baselines and their optimal weights. The last observational plan was ensured not only the optimal weights of the baselines provided by U,m approximation method of SOD but also the reliability conditions for the network.

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BIOGRAPHICAL NOTES

Mualla Yalçinkaya is an associate professor at Karadeniz Technical University (KTU), Turkey. She graduated from the Department of Geodesy and Photogrammetry Engineering in KTU in 1985. She got her MSc. degree in 1988 and got her Ph.D. degree in 1995 from the Department of Geodesy and Photogrammetry Engineering at KTU. Her research interests are Adjustment Computation, Deformation Surveys and Analyses, Geodetic Networks. She is a member of Chamber of Surveying Engineers.

Kamil Teke is a research assistant at Karadeniz Technical University (KTU), Turkey. He graduated from the Department of Geodesy and Photogrammetry Engineering at KTU in 2001. He got his MSc degree at KTU in August 2004. He is currently studying on his PhD thesis. His research interests are Geodetic Network Optimization, Deformation Analysis, Plate Tectonics. He is a member of Chamber of Surveying Engineers.

CONTACTS

Mualla Yalçınkaya
Karadeniz Technical University
Karadeniz Technical University, Department. of Geodesy and Photogrammetry Engineering
Trabzon
TURKEY
Tel. +90 462 3772797
Fax + +90 462 3280918
Email: mualla@ktu.edu.tr
Web: www.jeodezi.ktu.edu.tr/mualla

Kamil Teke
Karadeniz Technical University
Karadeniz Technical University, Department. of Geodesy and Photogrammetry Engineering
Trabzon
TURKEY
Tel. +90 462 3772763
Fax + +90 462 3280918
Email: k_teke@ktu.edu.tr
Web: www.jeodezi.ktu.edu.tr/kteke