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A NOVEL METHOD OF HIGH PRECISION HEIGHT DETERMINATION FOR INDUSTRIAL APPLICATIONS

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Abstract

This paper presents a theodolite-based method of high-precision height determination. Horizontal and vertical angular observations are made to a calibrated hidden point bar featuring several nominally collinear target points. The endpoint of the hidden point bar rests on the point of interest, but the bar itself can be in any orientation. The observations are processed in a least-squares solution that yields the three-dimensional coordinates of the point of interest in the theodolites reference system. Laboratory tests indicate that the accuracy of this method can reach the 0.2 mm level in height on standoff distances of 5 metres.

1. Introduction

A major logistical problem in industrial surveying is ensuring that points to be surveyed or monitored are visible. Industrial environments often feature confined spaces, piping and operating equipment that can hinder the establishment of direct lines-of-sight to target points of interest.

This problem is particularly acute when precision levelling is to be done, as not only must the there exist common horizontal lines of sight from the level to both the backsight and foresight levelling staves, but there must be sufficient vertical clearance directly above the target points to accommodate the levelling staff.

One solution to this problem is to dispose with direct levelling and resort to trigonometric levelling techniques (Rueger and Brunner, 1982). In this way, any convenient inclined line of sight from the instrument to a target positioned over the point of interest can be observed. However, this method requires the distance from the instrument to a target reflector to be precisely measured. Since the accuracy of electronic distance measurement is rarely better than 0.5 mm (Leica, 1998; Radovanovic and Teskey, 2002), this directly limits the accuracy of the height determination.

This paper presents a novel method of theodolite-based height determination which does not require distances to be measured to a target point. Instead, angular measurements are made to a specially calibrated hidden point bar, the end of which is in contact with the point to be measured. Since the distances between nominally collinear target points on the scale bar are known through calibration, observing three such target points is sufficient to determine the coordinates of the point of interest. Heights can in this way be determined to an accuracy of 0.2 mm, at a five metre standoff distance.

2. Height Determination Using the Calibrated Hidden Point (CHIP^{TM}) Bar

The Calibrated HIdden Point (CHIP^{TM}) bar, shown in Figure 1, was developed by the Precise Engineering and Deformation Surveys Group, of the Department of Geomatics Engineering at The University of Calgary to solve the problem of observing difficult to access points in industrial surveying.

The bar features at least three nominally-collinear target points, designed to allow high accuracy (<1") pointing using a theodolite. Targets are mounted along the length of the bar and their relative positions are carefully determined through calibration. By observing horizontal and vertical angles to the target points, the three-dimensional coordinates of the end of the bar (and thus the point of interest) can be determined.

In addition, two telescoping bipod legs allow the bar to rest in a variety of positions while still maintaining contact with the point of interest. This frees the operator to set up the bar in *any* orientation that allows sighting of at least three target points. Thus points behind corners can be observed (Figure 2), as can points surrounded by piping and obtruding machinery.

Finally, a digital thermometer allows for reading of the bar temperature to allow for expansion/contraction correction during operation.

2.1 Mathematical Model

Consider the case of three collinear points separated by measured distances, as shown in Figure 3. These points form a line which is described by an azimuth, θ, zenith angle, Z, and the coordinates of one of the points (A). Thus the following equations can be written :

$$x_i = x_A + d_{Ai} \sin\theta \cdot \sin Z \tag{1a}$$
$$y_i = y_A + d_{Ai} \cos\theta \cdot \sin Z \tag{1b}$$
$$z_i = z_A + d_{Ai} \cos Z \tag{1c}$$

Figure 1. CHIP^{TM} Bar.

Figure 2. CHIP^{TM} Bar in Operation.

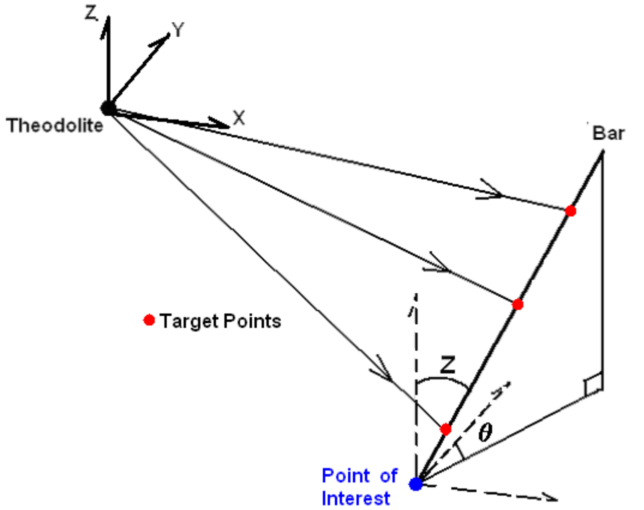

Figure 3. Principle of Positioning Using the CHIP™ Bar

where *x, y, z* are the coordinates of the ith point, x_A, y_A, z_A are the coordinates of the target point A, and d_{Ai} is the distance from the target point A to the ith point.

One zenith and horizontal angle measurement can be made to each target point using a conventional theodolite. For each observation pair the following equations can be written :

$$\alpha_i = \mathrm{atan}\left(\frac{x_i}{y_i}\right) \tag{2a}$$

$$z_i = \mathrm{acos}\left(\frac{z_i}{\sqrt{x_i^2 + y_i^2 + z_i^2}}\right) \tag{2b}$$

where it is assumed that the theodolite is located at the origin of the coordinate system and that the y axis of the system corresponds to the 0 degree angle reading on the horizontal circle.

Thus, for three target points, 6 observation equations can be formed, which allows a solution for the 3 unknown coordinates of the first target point and the azimuth and zenith angle of the bar orientation. Note that the inter-target distances d_{1i} are known from calibration and can either be treated as constants or as observations with very small variances. The latter case is implemented in the CHIP™ system, as it allows for propagation of the calibration accuracies into the estimates of accuracy of the final solution.

Finally, since the distance from the first point to the point of interest (which may not be visible) is known from calibration, the coordinates of the point of interest can be calculated from application of Eq. (1). If the azimuth of the 0° angle reading on the theodolite horizontal circle and the coordinates of the theodolite are known, the coordinates of the point of interest derived in the local system described above can be subsequently rotated and translated as appropriate.

2.2 Initial Estimate Generation

A least-squares solution to the above problem is non-linear with respect to the unknown parameters, and unfortunately requires fairly accurate initial estimates for convergence. Fortunately, Teskey et al present a closed-form, non-least-squares solution using observed horizontal and vertical angles which allows generation of initial estimates of sufficient accuracy.

With reference to Figure 4, the closed-form solution begins with determination of direction vectors from the theodolite (T) to the observed target points using

$$\mathbf{n_i} = \begin{bmatrix} n_{ix} \\ n_{iy} \\ n_{iz} \end{bmatrix} = \begin{bmatrix} \sin\alpha_i \sin z_i \\ \cos\alpha_i \sin z_i \\ \cos z_i \end{bmatrix} \tag{3}$$

The spatial angles ATB and BTC can then be determined using the dot product of required direction vectors, vis:

$$\psi_i = \text{acos}(n_i \cdot n_j) \tag{4}$$

Next, the angle BCT is be determined via

$$BCT = \text{atan}\left[\frac{\sin BTC}{\dfrac{(\overline{AB} + \overline{BC}) \cdot \sin ATB}{\overline{AB} \cdot \sin(ATB + BTC)} - \cos BTC}\right] \tag{5}$$

The distance from the theodolite to point A can then be determined via

$$\overline{TA} = \frac{(\overline{AB} + \overline{BC}) \cdot \sin BCT}{\sin(ATB + BTC)} \tag{6}$$

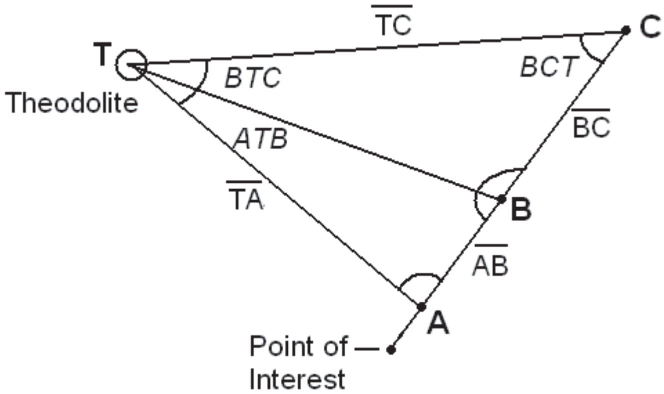

Figure 4. Geometry of Positioning for Closed Form Solution.

and similarly to point C

$$\overline{TC} = \frac{(\overline{AB} + \overline{BC}) \cdot \sin(\pi - ATB - BTC - BCT)}{\sin(ATB + BTC)} \quad (7)$$

Solution for these distances then allows for calculation of the coordinates point A and C by multiplication of the direction vectors to these points by the pertinent distances. The azimuth and zenith angle of the bar orientation is determined by inversing between point C and A. Finally, the initial estimates of the point of interest are calculated using the coordinates of point A, the orientation of the bar and the calibrated distance between the end of the bar and point A.

2.3 Effect of Non-Colinearity

The above derivation hinges on the assumption that the three target bar points and the point of interest are all colinear. Due to limitations in the manufacturing process, these points can be out of alignment by up to 1 millimetre. This problem is dealt with by establishing a reference line through points A and C, and determining the out-of-face (normal) and across-face deviations of the point-of-interest and point B (Figure 5). These values are acquired during the same calibration used to establish the inter-target distances.

The non-colinearity problem is resolved by requiring that the plane normal to the front face of the CHIP^{TM} bar includes the position of the theodolite (possible through orientation of the bar during operation). In this way, the direction of the *across-face* non-colinearity is in the direction of the cross-product between the direction vector to target point B and the vector corresponding to the bar orientation. Similarly, the *normal* non-colinearity direction is calculated by taking the cross-product of the direction of the across-face non-colinearity and the bar orientation vector.

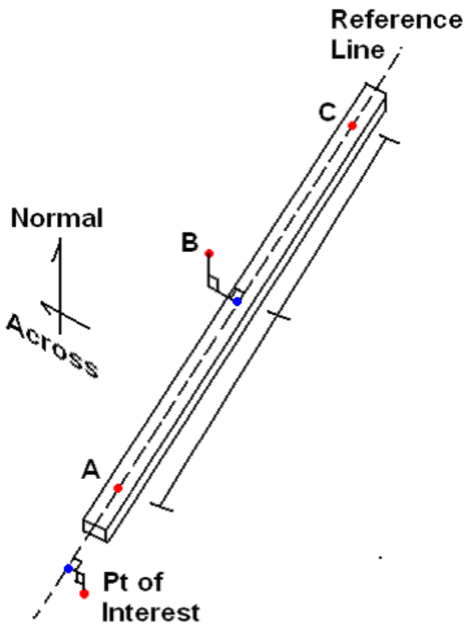

Figure 5. Non-Colinearity of Target Points and Point of Interest.

During the sequence of iterations in the least-squares adjustment, the current estimates of the bar orientation and location can be used to determine a correction to the horizontal and zenith angles measured to target point B (i.e. to determine the observations that *would* be made if target point B was in fact collinear). Once the solution converges, the bar orientation and location can be used in conjunction the normal and across-face non-colinearity parameters for the point of interest to determine its true location (which as well is not colinear with points *A*, *B* and *C*).

3. Performance Assessment of the CHIP™ Bar

A laboratory case study was conducted to assess the positioning performance of the CHIP™ bar. For eight different theodolite locations, the coordinates of a target station was determined using the CHIP™ bar in 5 different orientations (azimuth and zenith). Theodolite stand-off distance varied from 2-4.5 metres and zenith angle orientations of the bar varied from 1^o-20^o off of vertical.

Errors from the average position of the target stations were calculated in the x,y and z theodolite axis directions, with the 0^o horizontal angle direction set to coincide with the line between the theodolite and the reference point. These errors are plotted in Figure 6 along with the theoretical standard deviations calculated from the variance-covariance matrix of the solved parameters produced by the least-squares adjustment. Input standard deviations for the horizontal and vertical angles were set to 5 arc seconds.

Overall, the average standard deviations for the x,y, and z positioning errors is 0.24mm, 0.44mm, and 0.10mm, respectively. This indicates that the CHIP™ system is very suitable for precision height determination on short distances. The horizontal accuracy, unfortunately, is at the millimetre-level, and is not generally sufficient for high-precision work. The reason for the poorer horizontal accuracy along the y-axis implies that the scale of the positioning problem is significantly less accurate than the accuracy of the direction vector to the point-of-interest. Similarly, the x-direction accuracy is lower than the height direction since, in a nominally-vertical bar orientation, errors in the determination of the bar orientation parameters have less effect on the calculation of the height of the point of interest.

All positioning accuracies depend on theodolite standoff distance and bar orientation. The worst height determination geometry occurs when the bar is significantly off of vertical ($Z>10^o$) and is orientated directly away from the theodolite. Optimal height determination occurs when the bar is vertical, but the effect of non-verticality is minimized if the direction of tilt is 90^o to the line joining the theodolite and point of interest.

As the standoff-distance between the theodolite and point-of-interest increases, a smaller angle of arc is observed between the target points of maximum separation. Thus the "observability" of the scale-defining information (the fixed inter-target distances) is reduced due to poorer geometry and the uncertainty in scale determination thus increases with an associated degradation in overall positioning accuracy. The same effect occurs when the tilt of the bar is increased in a direction away from the theodolite.

4. Future Work

Research into understanding the behaviour of the CHIP™ system and improving achievable positioning accuracies is ongoing. For example, it is necessary to more directly understand how the performance of the CHIP™ system changes with variations in standoff distance and

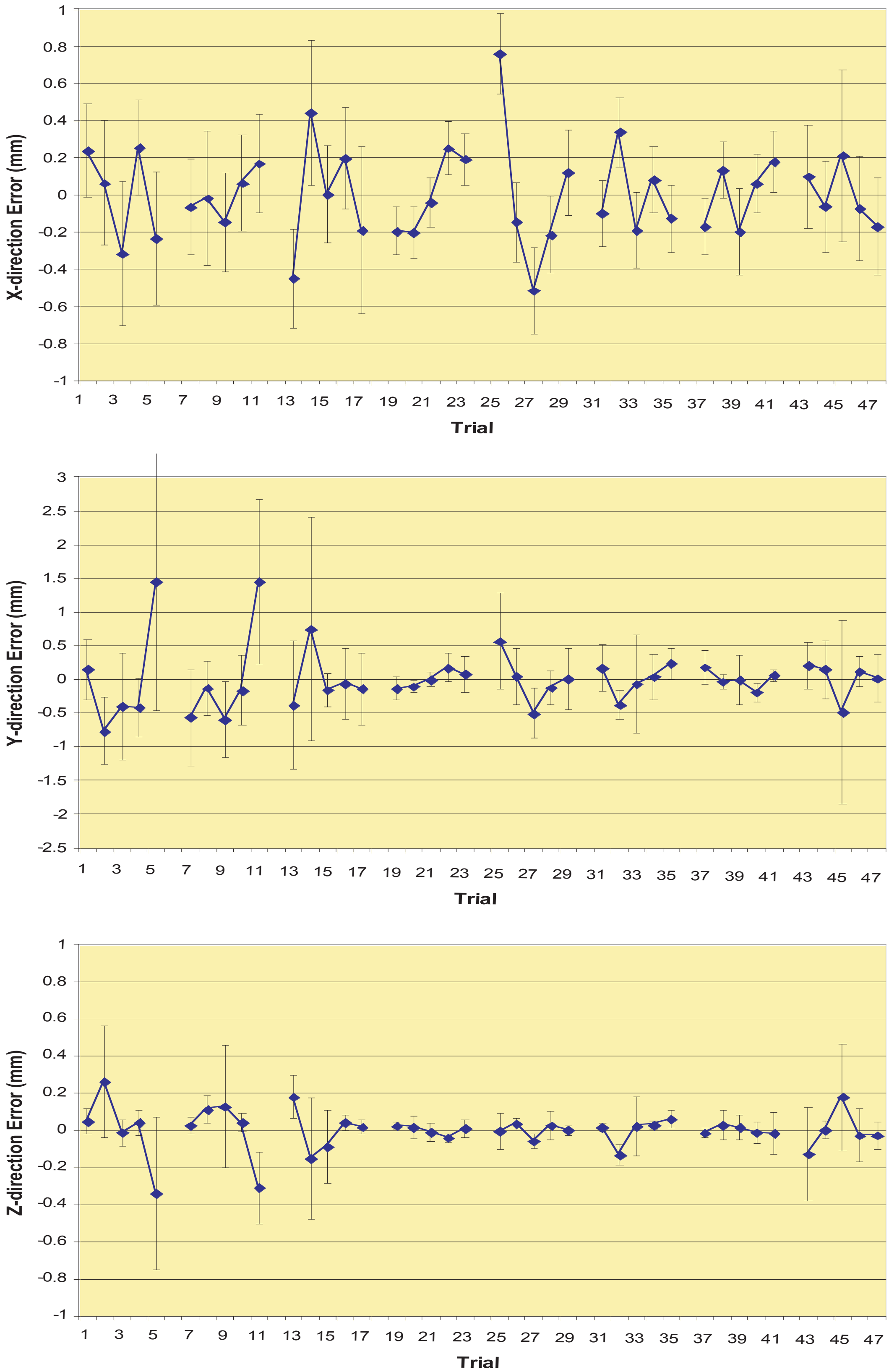

Figure 6. Deviations in derived positions using the CHIP^{TM} bar

bar orientation. The authors are currently performing simulations of many distance/orientation combinations to address this need.

In addition, since the scale is the most poorly determined quantity in the positioning problem, particularly on long standoff distances, it may be possible to significantly improve positioning accuracies by including distance observations available if an electronic tacheometer is used. Although electronic-distance measurement (EDM) observations are usually limited in accuracy to the 0.5mm level, the results in Figure 6 indicate that the y-direction (line-of-sight) errors are often in this range anyways. Thus the EDM observations may be very useful in determining the scale of the positioning problem, with the angular observations controlling the actual direction vector to the point-of-interest.

Finally, with the proliferation of electronic tacheometers with automatic target recognition and tracking, it is possible to automate the observation process such that the operator need only to roughly sight to one target point to initialize the observation process. The tacheometer can then accurately point to this point and then proceed to find the remaining target points using a search pattern. This development would significantly improve the operational application of the CHIP™ system.

5. Conclusions

This paper presented the development of a method of precise height determination that is useful in industrial scenarios where it may be difficult to directly observe or position a target directly vertically over a target point of interest. The method is based on making horizontal and vertical angular observations to target points on a bar in contact with the point of interest. The relative positions of the target points are determined via calibration and establish the scale of the positioning problem.

It was shown that on standoff distances less than 5 metres, the relative height of a point can be determined to within 0.2 mm. Horizontal positioning accuracies are significantly lower and not generally suitable for high-precision work. Positioning accuracies in all directions degrade quickly since scale is determined solely by angular measurements, but future work will focus on improving this behaviours via the inclusion of distances measured via EDMI.

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